

1. B
2. C
3. B
4. A
5. E
6. B
7. C
8. C
9. C
10. D
11. C
12. B
13. D
14. D
15. C
16. B
17. B
18. D
19. E
20. B
21. C
22. A
23. B
24. B
25. A
26. A
27. D
28. C
29. B
30. A

1. B The probability that Terri wins on her turn is $\frac{3}{4}$ and for Avery is $\frac{1}{4}$. The probability Terri wins is $\frac{3}{4} + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) \dots = \frac{12}{13}$
2. C Every consecutive 4 powers of i sums to 0, so the whole sum is $i^{2022} + i^{2023} + i^{2024} = -i$
3. B $12 + 12 + 24 + 24 + 48 + 48 + 96 + 96 + 192 + 192 = 24(1 + 2 + 4 \dots + 2^4) = (24)(31) = 744$.
4. A Since the values are so high, it would be difficult to solve for a_{100} . $a_{101} = a_{100} + a_{99} + 1$ and $a_{100} = a_{99} + a_{98} + 1$. Plugging a_{100} in gets $a_{101} = 2a_{99} + a_{98} + 2$. Subtracting over gets the desired value of 2.
5. E The sequence classifies as none of the options.
6. B Using finite differences,
1: 1, 11, 26, 46, 71
2: 10, 15, 20, 25
3: 5, 5, 5
Since it is constant after 3 differences, the answer is 3.
7. C Let $4 + \frac{5}{4+\dots} = S$. Then $4 + \frac{5}{S} = S$ and $S^2 - 4S - 5 = 0$ so $S = 5, -1$. Since S must be positive $S = 5$.
8. C $\frac{1}{1-r} = 4r^2 - 12r + 7$. Multiplying both sides by $1-r$ and subtracting by 1 gets $-4r^3 + 16r^2 - 19r + 6 = 0$. Use rational root theorem to get $r = \frac{1}{2}, \frac{3}{2}, 2$. The last two are extraneous since they exceed 1 so the answer is 2.
9. C Let the sides be $a-d$, a , and $a+d$. Using the pythagorean theorem gets $a^2 - 2ad + d^2 + a^2 = a^2 + 2ad + d^2$ or $a = 4d$. The sides are $3d$, $4d$, and $5d$, so using the area gets $6d^2 = 96$ gets $d = 4$ and $5d = 20$.
10. D $\frac{1+5i}{3+2i} = \frac{\sqrt{26}}{\sqrt{13}} > 1$. The series diverges.
11. C The series telescopes $\frac{1}{n} - \frac{1}{n+1}$ so the workers' combined rate is $1 - \frac{1}{11} = \frac{10}{11}$ blocks per hour. This means it will take 33 hours to finish $\left(\frac{30}{11}\right)$
12. B $f(1) + f(-1) = 2046 = 2a(1+4+16+64+256) = 682a$. $a = 3$. Plug $f(1)$ in to get $2048 - b = (511)(3)$. $b = 515$. $5+1+5 = 11$.
13. D There are 5 single-digit odds so find the 48th digit of the 2-digit odds or the last digit of the 24th 2-digit odd. 11, 13, 15...57. So the answer is 7.
14. D $\frac{a}{1-r} = 6$ and $\frac{a^2}{1-r^2} = 144$ Squaring the first equation and dividing by the second gets $\frac{1-r^2}{(1-r)^2} = \frac{1}{4} = \frac{1+r}{1-r}$ Solving this gets $r = \frac{-3}{5}$ and $a = \frac{48}{5}$
15. C The highest and lowest possible scores are 6 and 57, and every score in between is possible, so the 20th number in that list is 25.
16. B Chris can get any score between 6 and 57, so $6+7+8+\dots+57 = 1638$.
17. B Break it up into $\frac{x^{2^x}}{4^x} + \frac{1}{4^x} = \frac{x}{2^x} + \frac{1}{4^x}$ Solve each one individually. $K = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} \dots$
 $\frac{1}{2}K = \frac{1}{4} + \frac{2}{8} \dots$ Subtracting the two gets $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \dots = \frac{1}{2}K$. $K = 2$. The second part is an infinite geometric series with sum $\frac{4}{3}$. $S = 2 + \frac{4}{3} 3S = 10$.

18. D Since RUIWEN is equiangular, it can be an equilateral triangle with 3 equilateral triangles cut out from the corners. Solving for each side of the big equilateral triangle, we get $b^2 + b^3 = 5a$ and $b^2 + b = 3a$. $3ab = 5a$. $b = \frac{5}{3}$ and $a = \frac{40}{27}$. The largest side length is $b^3 = \frac{125}{27}$ so 81 times that is 375.
19. E Using the infinite geometric series formula, $\frac{1}{1-r} \frac{1}{1-s} \frac{1}{1-t} = \frac{1}{\frac{1}{12}f(1)} = 6$
20. B The minimum possible value of $a+b+c$ would be when they are 4, 2, 1 or 1, 2, 4. This would mean n must be at least 5. $1+2+4+5=12$.
21. C The first circle has radius of 1, meaning an area of π . The second square has side length $\sqrt{2}$, so the second circle has radius of $\frac{\sqrt{2}}{2}$ and area of $\frac{\pi}{2}$ and so on. $\pi + \frac{\pi}{2} + \frac{\pi}{4} \dots = 2\pi$.
22. A To go up 6 terms, we multiply by 3, so to go from 9th term to 12th term would be $9\sqrt{3}$.
23. B $0+1+2+\dots+49 = \frac{(50)(49)}{2} = 1225$.
24. B Factoring $n^2 - 1$ into $(n - 1)(n + 1)$. Every other term cancels out.

$$\frac{2^2}{1 \cdot 3} \cdot \frac{3^2}{2 \cdot 4} \cdots \frac{n^2}{(n-1)(n+1)} = \frac{2 \cdot n}{1 \cdot (n+1)} = \frac{2n}{n+1} = \frac{200}{101}$$
25. A
$$\frac{n^2}{n^2 - 1} = 1 + \frac{1}{n^2 - 1} = 1 + \frac{1}{2} \left(\frac{1}{n-1} - \frac{1}{n+1} \right)$$

 Thus, the summation is

$$9 + \frac{1}{2} \left(1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} \cdots + \frac{1}{8} - \frac{1}{10} + \frac{1}{9} - \frac{1}{11} \right)$$

$$9 + \frac{1}{2} \cdot \left(1 + \frac{1}{2} - \frac{1}{10} - \frac{1}{11} \right) = \frac{531}{55}$$
26. A $S = \frac{1}{2} + \frac{1}{4} + \frac{2}{8} \dots S - \frac{1}{2}S - \frac{1}{4}S = \frac{1}{2} = \frac{1}{4}S$ $S=2$.
27. D $a_2=5, a_5=4$, so $a_8=3$ so the sum of the first fifteen terms is $(3)(15)=45$.
28. C Call the terms $a-11, a-9, a-7 \dots a-1, a+1 \dots a+9, a+11$. The sum of the terms would be $12a=372$, so $a=31$. The greatest term would be $31+11=42$.
29. B $\frac{2}{5} + \frac{1}{5} + \frac{1}{10} = \frac{\frac{2}{5}}{1-\frac{1}{2}} = \frac{4}{5}$ Since the question asks for the remainder, it is $\frac{1}{5}$
30. A There are 2 cases.
 i) n is odd
 $\rightarrow \frac{2024}{n}$ is an integer
 $\rightarrow n = 1, 11, 23, 253$
 ii) n is even
 $\rightarrow \frac{2024}{n}$ is half of an odd integer
 $\rightarrow n = 16 * (1, 11, 23, 253)$