1. C
2. B
3. C
4. C
5. A
6. A
7. D
8. B
9. E
10. C
11. A
12. E
13. C
14. A
15. B
16. D
17. C
18. B
19. D
20. B
21. D
22. B
23. D
24. D
25. E
26. B
27. E
28. D
29. C
30. C

1. C $\log_3 4 + \log_9 3 + \log_{27} \frac{1}{64} = \log_3 4 + \frac{1}{2} + \log_3 \frac{1}{4} = \frac{1}{2} + \log_3 1 = \frac{1}{2}$

2. B $(11^{1+\log_{121} 64 + \log_{11} 3})^2 = (11 * 11^{\log_{11} 8} * 11^{\log_{11} 3})^2 = (11 * 8 * 3)^2 = 69696$

3. C $49^{5x+8} = 343^{6x-5};\ 7^{10x+16} = 7^{18x-15}; 10x + 16 = 18x - 15; 8x = 31; x = \frac{31}{8}$

4. C $\sum_{p=1}^{126} \log_2 \frac{p+2}{p+1} = \log_2 \frac{3}{2} + \log_2 \frac{4}{3} + \cdots + \log_2 \frac{128}{127} = \log_2 \frac{128}{2} = \log_2 64 = 6$

5. A Plug in 1 for $a, b, c,$ and $d.$ $(3 + 4 - 9 + 5)^3 = 27$

6. A $7^1 \to 7;\ 7^2 \to 9;\ 7^3 \to 3;\ 7^4 \to 1.$ The units digit is 1 when the power is divisible by 4. The last two digits of the power are 48, so the power is divisible by 4, making the units digit 1.

7. D $\frac{1}{\log_p 3 + \log_{(p^3)} 9} = 2; \frac{1}{\log_p (3 * \sqrt[3]{9})} = \frac{1}{\log_p 3^{\frac{5}{3}}} = \frac{3}{5} \log_3 p = 2;\ \log_3 p = \frac{10}{3};\ \log_3 p^3 = 3 \log_3 p = 10$

8. B $\prod_{n=1}^{105} \log_{(m+2)}(m + 3) = \log_3 4 * \log_4 5 * \ldots * \log_{107} 108 = \log_3 108 = 3 + \log_3 4$

9. E $\log_{11}(\log_3(\log_7 a))\ ;\ \log_3(\log_7 a)) > 0\ ;\ \log_7 a > 1; a > 7$

10. C $\log_5\left(\sqrt{17} - 2\sqrt{3}\right) = x;\ \log_5\left(\sqrt{17} + 2\sqrt{3}\right) + x = \log_5(17 - 12) = 1; x = 1 - M$

11. A $\sqrt{24 - 6\sqrt{15}} = \sqrt{24 - 2\sqrt{135}} = \sqrt{(\sqrt{15} - \sqrt{9})^2} = \sqrt{15} - \sqrt{9}; 15 * 9 + 15 + 9 = 159$

12. E $\log a^{\log a} - 5 \log a + 6 = (\log a)^2 - 5 \log a + 6 = 0;\ \log a = y; y^2 - 5y + 6 = (y - 3)(y - 2) = 0; y = 2, 3 \to a = 1000, 100.\ 1000 + 100 = 1100$

13. C $\begin{vmatrix} \text{Log}_2 \sqrt[3]{3} & \log_5 \frac{1}{49} \\ \log_{\sqrt{7}} 25 & \log_9 \frac{1}{16} \end{vmatrix} = \frac{1}{3} \log_2 3 * (-2 \log_3 2) - (-2 \log_5 7 * 4 \log_7 5) = -\frac{2}{3} + 8 = \frac{22}{3}$

14. A $2021 * 2021^{2021} = 2021^{2022}$

15. B $x^2 + 5x + 5 = 1;\ x^2 + 5x + 4 = (x + 4)(x + 1) = 0; x = -4, -1;\ x^2 - 6x + 8 = (x - 4)(x - 2) = 0; x = 2, 4;\ x^2 + 5x + 6 = (x + 3)(x + 2) = 0;\ x = -3, -2;\ -3$ is extraneous because it gives you $-1.$ $-4 - 1 + 4 + 2 - 2 = -1$

16. D $\log_{63} 420 = \frac{\log 420}{\log 63} = \frac{1+a+b+c}{2b+c}$

17. C $\log_2(x^2-2x)=3;\ x^2-2x=8;\ x^2-2x-8=0; (x-4)(x+2)=0; x=-2$ is extraneous because the log of a negative number does not exist. Thus, $x=4$

18. B $2x^3+3x^2-11x-6>0; (2x+1)(x-2)(x+3)>0;$ Critical numbers. $(-3,-\frac{1}{2}) \cup (2,\infty)$

19. D ${}_9C_3(2x^2)^3(-\frac{1}{x})^6 = \frac{9*8*7}{3*2*1} * 8 * 1 = 672;$

20. B $\frac{\frac{\log a}{\log b}}{\frac{\log a}{\log c}} = \log_b c = \frac{25}{24};\ b^{\frac{25}{24}} = c;\ c^{\frac{24}{25}} = b; c^{-\frac{1}{25}} = \frac{b}{c};\ \frac{c}{b} = c^{\frac{1}{25}}; n = \frac{1}{25}$

21. D $\log_5(x^{\log_5 x}) = \log_5\left(\frac{x^4}{125}\right);\ (\log_5 x)^2 = 4\log_5 x - 3;\ \log_5 x = y;\ y^2-4y+3 = (y-3)(y-1)=0; y=1,3;\ x=5,125. \log_5(5*125)=4$

22. B $\log 5 = 1-\log 2 = 0.7; 420*0.7 = 294$

23. D $(1)(0)+(9)(1)+(90)(2)+(900)(3)+(9000)(4)+(1)(5) = 38894$

24. D $(3^x-3^{-x})^3 = 27^x - 3*3^x + 3*3^{-x} - 27^{-x} = 27^x - 27^{-x} - 3(3^x-3^{-x}) = 125;\ 27^x - 27^{-x} = 125+15 = 140$

25. E $x^{\frac{1}{12}(5+\frac{1}{7})}y^{\frac{1}{12}(\frac{1}{2}-20)}z^{\frac{1}{12}(-\frac{1}{3}-30)}$; All of the answers have a positive exponent for the y-term, when in reality it is negative, so E. NOTA

26. B $2i^{84} - 3i^{57} + 5i^{22} + 7i^{69} - 11i^{422} = 2-3i-5+7i+11 = 8+4i$

27. E $5x = 8y+9; 3y = 2x+3;$ Solve. $x=-51, y=-33;$ which are extraneous

28. D $\log_a 4a + \log_{16a} 64a = \frac{62}{21} = 2 + \log_a 4 + \log_{16a} 4 = 2 + \frac{1}{\log_4 a} + \frac{1}{\log_4 16a};\ \frac{1}{\log_4 a} + \frac{1}{2+\log_4 a} = \frac{20}{21}$. Solve. $\log_4 a = \frac{3}{2}; a = 8; d = a*64 = 512$

29. C $(2^4)^{10}?(3^3)^{10};\ 3^3 > 2^4; X > W;\ 3^{30}?5^{25};\ (3^6)^5?(5^5)^5;\ 5^5 > 3^6;\ Y > X;\ (7)^{15}?(3^2)^{15};\ 7<9;\ X>Z; (7^3)^5?(2^8)^5;\ 7^3 > 2^8;\ Y > X > Z > W$

30. C Let $x = 2^a, y = 2^b$. The expression becomes $\left(\frac{1}{a}+\frac{8}{b}\right)(a+b)$. Making a common denominator gives $\frac{9ab+8a^2+b^2}{ab} = 9 + \frac{8a^2+b^2}{ab}$. Dividing the top and the bottom by ab gives $9+8\left(\frac{a}{b}\right)+\left(\frac{b}{a}\right)$. Using AM-GM, the minimum is $9+4\sqrt{2} = 9+\sqrt{32}$.
*Note that this works because $a, b > 0$