1. B
2. D
3. A
4. C
5. C
6. B
7. E
8. B
9. E
10. C
11. C
12. A
13. D
14. D
15. A
16. C
17. B
18. C
19. C
20. C
21. E
22. B
23. C
24. C
25. A
26. A
27. A
28. A
29. D
30. D

1. B By the Pythagorean Theorem (3^2 + 5 = 14), ACD and BCD are right triangles. The only way for a right triangle to be inscribed in a circle is if its hypotenuse goes through the circle's center. Therefore, the diameter of circle w must be equal to the hypotenuse of the right triangles (square root of 14). The radius of the circle is (sqrt(14))/2 and the area is 14/4 = 3.5 or **B**.
2. D The numerator factors as $(x-3)(x+1)$ and the denominator factors as $(x-3)(x+1)(x-4)$. There are points of removable discontinuity (holes) at both x = 3 and x = -1, and choices A and B can be eliminated. Additionally, because the degree of the polynomial in the denominator is greater than that of the numerator, there is an additional horizontal asymptote at y=0. There is no asymptote at y=1 (D)
3. A We are seeking all n where n is $8^4 < n^2 < 4^8$. We can take the square root to get $8^2 < n < 4^4 = 64 < n < 256$. Thus, n is all positive integers from 65 to 255, meaning there are $255 - 64 = 191$ perfect squares.

4. C The three intersection points are (0,0), (5, 5sqrt6/2), and (5,0). The circumcenter can be found by calculating the intersection of two perpendicular bisectors of sides in the triangle; x = 5/2 and y = 5rt6/4 intersect at (5/2, 5rt6/4). The circumradius can be calculated by finding the distance from a vertex of the triangle to the circumcenter, or $\sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{5\sqrt{6}}{4}\right)^2} = \sqrt{\frac{125}{8}}$. The area is $\frac{125\pi}{8}$. (A**)**
5. C We will map the parabolic path that the ball passes through to the coordinate plane. Let the parabola that the ball follows be the function f(x) = Ax^2 + Bx + C. Using the information from the problem, the parabolic trajectory of the ball passes through the points (0,6), (10, 8), and (50, 6). Using three equations to solve for the three variables in $y = A\text{x}^2 + \text{Bx} + \text{C}$, we realize that the ball's path can be mapped by $y = -\frac{1}{200}x^2 + \frac{1}{4}x + 6$. The maximum height is at the vertex, where x = 25. Plug in to get 73/8. **(C)**
6. B The formula for the sum of the first n cubes is $\left(\frac{n(n+1)}{2}\right)^2$. We're looking for the sum of the first 16 cubes minus the sum of the first 4 cubes, to get $\left(\frac{16*17}{2}\right)^2 - \left(\frac{4*5}{2}\right)^2 = (8*17)^2 - (2*5)^2 = 18496 - 100 = 18396$. **B**

7. E $2024 = 44 * 46 = 2^3 * 11 * 23$, all of which are prime, so 2021 has $4 * 2 * 2 = 16$ positive integral factors $(1,2,4,8,11,22,23,44,46,88,92,184,253,506,1012,2024)$. **E**

8. B This is a 72-72-36 triangle. Without loss of generality, let YD = XD = XZ = x and DZ = 1. By similar triangles ($\triangle XDZ \approx \triangle YZX$), $\frac{x}{1} = \frac{x+1}{x}$. Solve the equation to get the following quadratic in terms of x: $x^2 - x - 1 = 0$ which has solutions at $\frac{1\pm\sqrt{5}}{2}$. Since side lengths are exclusively positive, the answer is **B.**
9. E Scale the triangle produced in question 8 to find AC=AD=4*r +4 and AB=AE=4r. Thus, the sum is 2 (4*r*+4) + 2*r* = 16*r* +8 **(E)**

10. C This is an ellipse with foci at A and B and a major axis of length 12. The center of the ellipse is (0,10) at the midpoint of AB. Thus, c=5, a= 6, b=$\sqrt{11}$. The formula for the length of the latus rectum is $\frac{2b^2}{a}$. Plug in values to get **C**.

11. C The inverted matrix is $\begin{vmatrix} \frac{3}{2} & -\frac{5}{2} \\ -1 & 2 \end{vmatrix}$.The transposed matrix is $\begin{vmatrix} 4 & 2 \\ 5 & 3 \end{vmatrix}$. The sum is $\begin{vmatrix} \frac{11}{2} & -\frac{1}{2} \\ 4 & 5 \end{vmatrix}$. The determinant is $\frac{55}{2} - \left(-\frac{8}{2}\right) = \frac{59}{2}$. **(C)**

12. A Let $x = 3 + \frac{1}{3+\frac{1}{3+\cdots}}$. We can rewrite this as $x = 3 + \frac{1}{x}$, which gives us the equation $x^2 - 3x - 1 = 0$. Using the quadratic formula, we get $x = \frac{3+\sqrt{13}}{2}$, since x cannot be negative.

13. D First, we must express the number 42313_5 in base 10. We get $3 + 5 * 1 + 5^2 * 3 + 5^3 * 2 + 5^4 * 4 = 3 + 5 + 75 + 250 + 2500 = 2833$. We must then express this number in base 8, so we first divide 2833 by 8 to get 354 with a remainder of 1. We then divide 354 by 8 to get 44 with a remainder of 2. We then divide 44 by 8 to get 5 with a remainder of 4. Since 5 is less than 8, we now have our answer of 5421.

14. D Effectively, this summation is calculating the sum of the elements from the 0^{th} to 10^{th} row of Pascal's triangle. The sum of the elements in each row is 2^i. Thus, $2^0 + 2^1 + 2^2 + 2^3 \ldots + 2^{10} = 2^{11} - 1 = 2047$ **(D)**

15. A A cross section of the cone would form a 5-5-8 triangle. The inradius, r, can be calculated by solving: $\frac{r(5+5+8)}{2} = \frac{1}{2} * 3 * 8 \longrightarrow r = \frac{4}{3}$. The formula for surface area of a sphere is $4r^2\pi$ = $64pi/9$. **(A)**

16. C The prime factorization of 360 is $2^3 * 3^2 * 5$. The prime factorization of 40 is $2^3 * 5$. Because the GCD of 360 and *k* is 40, *k* must be 1) a multiple of 40 and 2) be relatively prime to 3^2. The numbers that satisfy this condition and are within the closed interval are {40, 80, 160, 200, 280, 320}. Therefore, there are 6 values of *k* that satisfy the initial conditions.

17. B Let c be the current of the river in yards per minute. Mr. Lu's rate going to school is 4 + c. His rate coming back is 4 – c. Because $(rate)(time) = distance$ we can construct the following equation:

$$\frac{15}{4+c} + \frac{15}{4-c} = 8$$

Solve to find that $c = 1$. Therefore, the speed of the current in feet per minute is 3 per minute. (B)

18. C There are intersections of two of the three lines at (3/4, 3/4), (-1,-1), and (1, -1). The centroid of three points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is at $(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3})$. The centroid of these three points is at (1/4, -5/12) **C**

19. C Let us set x = 3^y. Then, $\log_3 \frac{y}{4} = \log_9 \frac{y}{2} \rightarrow \log_3 y - \log_3 4 = \log_9 y - \log_9 2$. Thus, $\frac{1}{2}\log_3 y = \frac{3}{2}\log_3 2 \rightarrow y = 8$. 3^8 has a units digit of 1. **C**

20. C By equal tangents, MA+OT = AO + MT. Plugging in given lengths, we achieve: 4+7=6+MT. Therefore, MT=5. **C**

21. E This factors into two lines that intersect at (0, -4/3): (x+3y+4)(x-3y-4) = 0. Therefore, it is a degenerate conic. **E**

22. B Notice that 2016 is divisible by 2, 3, and 7. Therefore the number of positive integers that are less than or equal to and relatively prime to 2, 3, and 7 is (2016)(1/2)(2/3)(6/7) = 576.There are only two integers between 2017 and 2024 that are divisible by neither 2, 3, or 7 (2017 and 2021). 576 +2 = 578 **(B)**

23. C Draw a picture. MO is of length 8, as it is a diameter of the circle. LMO is a 30-60-90 triangle: LO=4, LM=4rt3, MO=8. Therefore, the area of the rectangle is 16rt3.

24. C This is a casework problem. Each of the six terms in the expression can attain a value of 1 or -1 depending on whether the variable or product of variables is positive or negative. There are a few cases for the parity of o, p, and s. Note that at the variables must not all be positive or negative because they sum to zero. -2 + 0 = **C**.

Parity of (o,p,s)	Value of Expression
(+, +, -) or any permutation	-2
(-, -, +) or any permutation	0

25. A The formula for volume of a tetrahedron in terms of sidelength a is $a^3/6\sqrt{2}$. An alternative formula for volume of the tetrahedron is 1/3(area of base) (height). The formula for area of the tetrahedron's base is (3rt3*a^2/4).

$$\frac{a^3}{6\sqrt{2}} = \left(\frac{1}{3}\right)\left(\frac{a^2\sqrt{3}}{4}\right)(h)$$

Thus, the height in terms of a is $\frac{\sqrt{6}a}{9}$. Because $\frac{a^3}{6\sqrt{2}} = 18\sqrt{2} \longrightarrow a = 6$. Thus, the height is $2\sqrt{6}$. **A**

26. A The maximum area traces out an ellipse with 2a = 50 and 2c = 40. Because $a^2 - c^2 = 15^2 = b^2 \rightarrow b = 15$. The area of the ellipse is $ab\pi = 375\pi$.

27. A For all odd n, $f_n = n$. For all even n, $f_n = \frac{n-1}{2}$. The sum of f_n for all odd n can be written as 1 + 3 + 5 + … + 19 = 100. The sum of f_n for all even n can be written as (1/2)(-1 + 1 + 3 + 5 + … 19) = 99/2. **A (299/2)**

28. A The fastest way to do this problem is to use base 7 multiplication. $1234_7 * 4_7 = 5302_7$. $2345_7 * 2_7 = 5023_7$. Thus, ABCDE = 13325 **A.**

29. D The decimal $.\bar{n}_{n+1}$always equals 1 for any n >=1. Therefore, $a_i = 1$ and $b_i = 1$ for all values of i in the summation. 2 * 8 = 16.

30. D We can rewrite x to be $x = \log_6 12 = \log_6 6 + \log_6 2 = 1 + \log_6 2 = 1 + \frac{1}{\log_2 6}$. Let $y = \log_2 6$. Then $x = 1 + \frac{1}{y}$, so $xy = y + 1$, $xy - y = 1$, $y(x-1) = 1$, and finally $y = \frac{1}{x-1}$. **D**