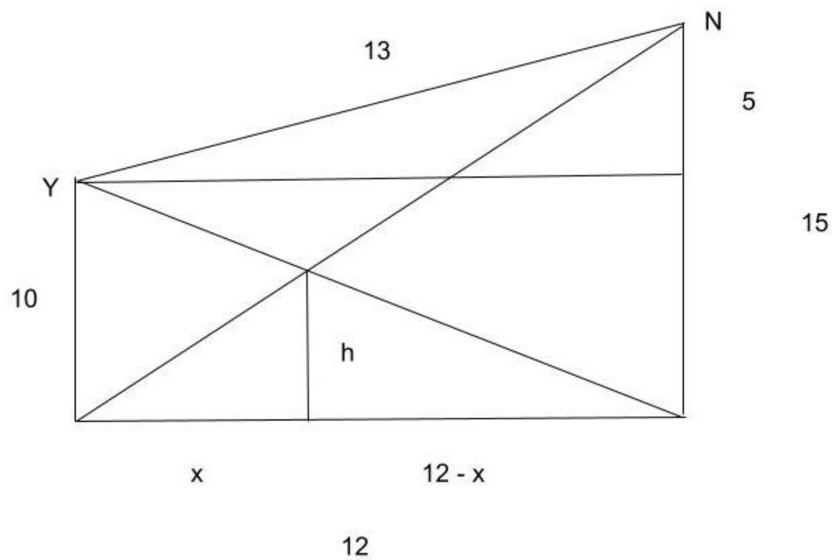
1. D
2. A
3. D
4. B
5. C
6. A
7. B
8. C
9. A
10. D
11. D
12. B
13. B
14. A
15. B
16. A
17. C
18. C
19. C
20. A
21. D
22. D
23. A
24. E
25. D
26. D
27. E
28. E
29. B
30. A

1. D We can say $3^x = a$, making the equation $a + 3^2 * a - 10 = 9a^2 + a - 10 = (9a + 10)(a - 1) = 0$. Now we have $3^x = -\frac{10}{9}, 1$. 3 to any power cannot be a negative value, so the sum of the solution of $3^x = 1$.
2. A By the Descartes Rule of Signs, if we count the number of sign changes, there are 4, meaning at most, there are 4 positive real roots. To find the number of negative real roots, we plug in -x for x, and we get $-x^5 - ax^4 - bx^3 - cx^2 + dx + e$. There is only one sign change so at most we have 1 negative real root. Add them up, and we get at most 5 real roots.
3. D First we can factor out the 2, and then use synthetic division to factor this. We get $(x - 3)(x - 2)(x + 1)(x^2 + 4) = 0$. The real roots are 3, 2, and -1. Their product is -6.
4. B Draw a table to organize the information, C*A = T:

Concentration	Amount	Total
$\frac{3}{10}$	10	3
1	x	x
$\frac{6}{10}$	$10 + x$	$3 + x$

The water level decreases to 40% at the end, so the dandelion extract level is at 60%. Solving the equation gives us $3(10 + x) = 5(3 + x) \to 30 + 3x = 15 + 5x \to 2x = 15 \to x = \frac{15}{2}$.
5. C After adding the dandelion extract from the previous problem, Kaeya has a total of $10 + \frac{15}{2} = \frac{35}{2}$ liters of juice. After he drinks 5 liters, he has $\frac{35}{2} - 5 = \frac{25}{2}$ liters remaining. The percentage of dandelion extract is still the same, so there will be $\frac{3}{5} * \frac{25}{2} = \frac{15}{2}$ of dandelion extract in this part. Rosaria's 10 liters has 3 liters of dandelion extract, and Diluc's 5 liters that he gives Rosaria has 3/2 liters of dandelion extract. We can divide the total amount of dandelion extract that Rosaria's juice will have by the total amount of juice Rosaria has to get the answer: $\frac{\frac{15}{2}+3+\frac{3}{2}}{\frac{25}{2}+10+5} = \frac{12}{\frac{55}{2}} = \frac{24}{55}$.

6. A



We can use similar triangles now to get 2 equations: $\frac{h}{10} = \frac{12-x}{12}, \frac{h}{15} = \frac{x}{12}$. Solving, we get $x = \frac{4}{5}h \rightarrow 12h = 120 - 10\left(\frac{4}{5}h\right) \rightarrow 20h = 120 \rightarrow h = 6$.

7. B We can rewrite this equation. $a + b + c = 25 \rightarrow a + b = 25 - c, a + c = 25 - b, b + c = 25 - a$. $\frac{1}{a+b} + \frac{1}{a+c} + \frac{1}{b+c} = \frac{1}{25-c} + \frac{1}{25-b} + \frac{1}{25-a} = \frac{3*625-50(a+b+c)+ab+ac+bc}{25^3-625(a+b+c)+25(ab+bc+ac)-abc}$. This may seem daunting at first, but most of these are obtainable using Vieta's Formulas. $a + b + c = 25, ab + bc + ac = -10, abc = -40$. Now this equation just becomes $\frac{625-10}{-250+40} = -\frac{615}{210} = -\frac{41}{14}$.

8. C Singular means determinant is 0. $(x-3)(2-2x+x) - \big((x+2)(1-x)+3\big) = (x-3)(2-x) - (-x^2 - x + 5) = -x^2 + 5x - 6 + x^2 + x - 5 = 0 \rightarrow 6x = 11 \rightarrow x = \frac{11}{6}$.

9. A The center must be where the medians intersect → $\left(0, \frac{\sqrt{3}}{3}\right)$. Now that we know the center, we just have to find the radius, which we can do by drawing a radius and an altitude of the triangle. Radius = $\frac{2\sqrt{3}}{3}$. Equation = $x^2 + \left(y - \frac{\sqrt{3}}{3}\right)^2 = \frac{4}{3}$.

10. D We can use logarithm rules to solve this: $\log\left(\frac{19x^2+21}{x-1}\right) = \log(20x) \rightarrow 19x^2 + 21 = 20x^2 - 20x \rightarrow x^2 - 20x - 21 = 0 \rightarrow (x+1)(x-21) = 0 \rightarrow x = 21$.

11. D We can set the fraction = x, in which case it becomes $x = \frac{3}{2+x} \rightarrow x^2 + 2x = 3 \rightarrow x^2 + 2x - 3 = 0 \rightarrow (x-1)(x+3) = 0 \rightarrow x = 1$.

12. B For it to have a double zero, the discriminant must be 0 → $(k-4)^2 - 4(-4k) = 0 \rightarrow k^2 + 8k + 16 = 0 \rightarrow (k+4)^2 = 0 \rightarrow k = -4$.

13. B Let $a = \lfloor x \rfloor, b = \{x\}, c = \lfloor y \rfloor, d = \{y\}$.

$$a + 2c + 2d = 9.6$$
$$3a + 3b + c = 14.1$$

Since $0 \leq b, d < 1, 2d = .6, 1.6 \rightarrow d = .3, .8$ and $3a = .1, 1.1, 2.1 \rightarrow a = \frac{.1}{3}, \frac{1.1}{3}, .7$

Thus, the equations becomes $a + 2c = 8, 9$ and $3a + c = 12, 13, 14$. Solving for a, c gives the equations $(4, 2)$ and $(3, 3)$. The respective (x, y) are (4.03 ... ,2.8) and $(3.7, 3.3)$.

14. A The directrix of an ellipse = $\frac{a}{e}$, where e is the eccentricity. $Area = \pi ab = 24\pi \rightarrow ab = 24$. $Latus\ Rectum = \frac{2b^2}{a} = \frac{16}{3} \rightarrow 3b^2 = 8a, a = \frac{24}{b} \rightarrow b^3 = 64 \rightarrow b = 4, a = 6, c = 2\sqrt{5}$. $Directrix = \pm\frac{6}{(\frac{\sqrt{5}}{3})} = \pm\frac{18\sqrt{5}}{5}$.

15. B We first begin by simplifying this equation, and putting it in a neater form → $-20y = x^2 - 16x - 36 \rightarrow -20y + 100 = x^2 - 16x + 64 \rightarrow -20(y - 5) = (x - 8)^2$. Maximum is 5. Taking a few steps back, we can find the minimum by setting y = 0. $x^2 - 16x - 36 = 0 \rightarrow (x - 18)(x + 2) = 0 \rightarrow x = 18 \rightarrow 5 - 18 = -13$.

16. A If we write out the possible equations → $x + y = 2, -x - y = 2, x - y = 2, -x + y = 2$. Forms a square with side length $2\sqrt{2}$, so the area will be 8.

17. C We can use partial fraction decomposition to split this up into two fractions: $\frac{10}{(x-2)(x+3)} \rightarrow \frac{A}{x-2} + \frac{B}{x+3} = \frac{10}{(x-2)(x+3)} \rightarrow A(x + 3) + B(x - 2) = 10$. We can now plug in -3 and 2 to erase one variable (A or B) and solve for the other one. We get A = 2, and B = -2. Now we have summation of $\frac{2}{x-2} - \frac{2}{x+3} = \frac{2}{1} - \frac{2}{6} + \frac{2}{2} - \frac{2}{7} + \frac{2}{3} - \frac{2}{8} + \frac{2}{4} - \frac{2}{9} + \frac{2}{5} - \frac{2}{10} + \frac{2}{6} - \cdots$ From this point it telescopes, so answer $= 2 + 1 + \frac{2}{3} + \frac{2}{4} + \frac{2}{5} = \frac{137}{30}$.

18. C If $Albedo = B, Aether = A, Sucrose = S$, we are told that $B = \frac{1}{2}(A + 50), B = 30(S - 10) \rightarrow 2B = A + 50, B = 30S - 300 \rightarrow 60S - 600 = A + 50$

We are also told that A + 20 is divisible by S, and A > 1000

$$\frac{A + 20}{S} = \frac{60S - 630}{S} = 60 - \frac{630}{S}.$$

Since this is an integer, S must be a factor of 630.Plugging in the answer choices tells us that $S = 35, S - 10 = 25$. Plugging in the numbers, $A = 60 * 35 - 650 = 1450 > 1000$. Thus, all requirements are met.

19. C As the question suggests, let $\log(x) = a, \log(y) = b, \log(z) = c$.

$$ab - a - b = -\frac{1}{3}$$
$$bc - b - c = -\frac{5}{8}$$
$$ca - c - a = 3$$

As the equations suggest, proceed with SFFT.

$$(a - 1)(b - 1) = \frac{2}{3}$$
$$(b - 1)(c - 1) = \frac{3}{8}$$
$$(c - 1)(a - 1) = 4$$

Multiplying the equations and square rooting gives $(a - 1)(b - 1)(c - 1) = 1$. (Note that it could be -1 but the resulting sum is smaller). Using this fact, we get the values

$$a-1=\frac{8}{3}, b-1=\frac{1}{4}, c-1=\frac{3}{2} \rightarrow a+b+c=\frac{11}{3}+\frac{5}{4}+\frac{5}{2}=\frac{89}{12}$$

20. A If Yoon Se-ri arrives at 12:00, Ri Jeong Hyeok can arrive between 12 and 12:30, if she arrives at 1:30, he has to arrive between 1:30 and 2. Same concept applies for Yoon Se-ri, but in 15 minute increments. Drawing the square and calling the side length 1 will give us a region that is the probability $\rightarrow 1-\left(\frac{1}{2}*\frac{3}{4}*\frac{3}{4}\right)-\left(\frac{1}{2}*\frac{7}{8}*\frac{7}{8}\right)=\frac{43}{128}$.

21. D The amount of true time passed is $6+24k$ hours where k is the number of days passed since the first measurement.
The amount of time lost by the clock is $42+12\cdot 60\cdot n$ minutes where n is the number of half-days passed since the first measurement.

Thus $x=\frac{42+720n}{6+24k}=\frac{120n+7}{4k+1}. \rightarrow x(4k+1)=120n+7$. Taking both sides mod 4, $x=3\ (mod\ 4)$. 3 is unachievable since the right hand side is not a multiple of 3. 7 is achievable when $n,k=0$. 11 is achievable when $k=19, n=7$.

22. D Aether's rate is 30 mph, let's call the distance around Mondstadt d. rt = d, so d = (15)(2) = 30. The total distance traveled for both laps is 30 miles. By 5:30, Noelle's already traveled $\frac{1}{2}(15)=\frac{15}{2} miles$. Aether catches up at a rate of 30 – 15 = 15 mph. He catches up in 30 minutes, so it is 6 AM now, and one lap has been completed by now. They do the second lap at Aether's initial rate, which is 30 mph, so they finish in ½ hour longer, meaning they finish at 6:30 AM.

23. E We can use triangle inequality to get possible solutions for x. $If\ x+2\ was\ the\ base \rightarrow 3x+1+x+2>3x+1, 3x+1+3x+1>x+2 \rightarrow x>-2, x>0; if\ 3x+1\ was\ the\ base \rightarrow x+2+3x+1>x+2, x+2+x+2>3x+1 \rightarrow x>-\frac{1}{3}, x<3$. If either condition is satisfied, the triangle exists. Thus as long as both $x+2$ and $3x+1$ are positive, the triangle exists.

24. E Base of logarithm cannot be 1, and it must be greater than 0. The entire logarithm must be greater than 0 because it is in a square root in the denominator of a fraction. This means x + 3 cannot be equal to 1. This gives us the inequality $x-2>0\ and\ x-2\neq 1 \rightarrow x>2, x\neq 3. x+3\neq 1\ gives\ us\ x\neq -2,$ which is outside of the domain we currently have, so we don't have to worry about it.

25. D Call $\frac{\log(a)}{\log(b)}=log_b a=x \rightarrow x+\frac{6}{x}=5 \rightarrow x^2-5x+6=0 \rightarrow (x-2)(x-3)=0 \rightarrow x=2,3$.

26. D Call the whole thing x, then we have $x=\sqrt{72+x} \rightarrow x^2-x-72=0 \rightarrow (x-9)(x+8)=0 \rightarrow x\ can\ only\ be\ positive, so\ x=9$.

27. E Simplifying gives us $\frac{\sqrt{10}}{\sqrt{13}}-4\left(\frac{5}{\sqrt{5}}\right)=\frac{\sqrt{130}}{13}-4\sqrt{5}=\frac{\sqrt{130}-52\sqrt{5}}{13}$.

28. E Divide by 4 and complete the square should be the first step. We get $x^2+6x+9+y^2-8y+16>49 \rightarrow (x+3)^2+(y-4)^2>49$. This gives us more clarity about what this figure is, which is a circle. But the inequality makes the area be everything outside of the circle, which is infinity.

29. B Call money spent = D, Mora = M, Characters = C → $\frac{D}{MC} = k \rightarrow \frac{5}{(12)(3000000)} = \frac{1}{9600000} = k \rightarrow \frac{D}{(36)(45000000)} = \frac{1}{9600000} \rightarrow D = 225.$

30. A Volume = $\frac{4}{3}\pi r^3 = 36\pi \rightarrow r = 3 \rightarrow SA = 4\pi r^2 = 36\pi.$