1. B
2. B
3. C
4. B
5. B
6. C
7. D
8. A
9. D
10. B
11. B
12. E
13. B
14. C
15. D
16. C
17. E
18. D
19. D
20. E
21. B
22. C
23. C
24. B
25. B
26. B
27. D
28. D
29. B
30. E

1. B By Heron's Formula, A = $\sqrt{S(S-a)(S-b)(S-c)}$, where S is the semiperimeter (half of the perimeter) and a, b, and c are the sides. Plugging the values in, we obtain the answer to be $6\sqrt{6}$.
2. B This is the definition of a hyperbola.
3. C Since the line is $5x + 3y = 30$, the x intercept is 6 and the y-intercept is 10. This is a right triangle with legs of 6 and 10, so the area is $\frac{1}{2}bh$ or 30.
4. B By squaring both sides and dividing both sides by 16, this becomes $\frac{x^2}{16} + \frac{y^2}{4} = 1$ . Since there was a square root, y cannot be negative, so this graph is a semi ellipse with $a = 4$ and $b = 2$. The area is $\frac{1}{2}ab\pi$, or $4\pi$.
5. B In a parabola, the length of the latus rectum is twice the distance from the focus to the directrix. Applying the point to line formula, $\frac{|Aa+Bb-C|}{\sqrt{A^2+B^2}}$, the distance from the focus to the directrix is $\frac{1}{5}$, so the length of the latus rectum is $\frac{2}{5}$ .
6. C Graphing the points, we see that the convex quadrilateral is formed by the points in the order (1, 2), (2, 5), (6, 3), (3, -2) (NOT the given order, as doing that will result in a polygon that crosses over itself). Applying the shoestring method and remembering to divide by 2, the area is 18.
7. D This is the graph of a circle with radius 5.
8. A Rearranging as $x^3 - 1 = 0$, this can be factored through difference of cubes to obtain $(x-1)(x^2+x+1) = 0$. $x = 1$can be easily seen, and after applying the quadratic formula, the other two solutions are $-\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$. When the three points are graphed on the complex plane, an equilateral triangle with base $\sqrt{3}$ and height $\frac{3}{2}$ is formed, so the area is $\frac{3\sqrt{3}}{4}$.
9. D Completing the square and dividing yields $\frac{(x-1)^2}{12} + \frac{(y-3/2)^2}{9} = 1$. Thus, $b = 3$, and the conjugate axis is $2b$, or 6.
10. B This is a non-degenerate parabola, and parabolas have an eccentricity of 1.
11. B Since the conjugate axis is 12, this means $b = 6$. We know that $a^2 = b^2 + c^2$ and that $\frac{c}{a} = eccentricity = \frac{3}{5}$. Solving this yields $a = \frac{15}{2}$ and $c = \frac{9}{2}$. The area of an ellipse is given by $ab\pi$, so the area of this ellipse is $\frac{15}{2} \cdot 6 = 45\pi$.
12. E Since Nick has to go to $x = 8$ first, the point (10, 9) can be reflected over this line to mirror the path that he takes, which won't change the path length but will make it easier to see. The shortest path would be a straight line between the two points (6, 9) and (12, 1), which is 10.
13. B Completing the square for the parabola yields $x + 6 = (y-2)^2$, so the vertex is $(-6, 2)$. Substituting $x = 8 - y$ into the first equation yields $y = 5$ and $y = -2$, so and these give the intersection points $(3, 5)$ and $(10, -2)$. Using the shoestring method and remembering to divide by 2, the area is 42.

14. C We can use the Law of Cosines for this question. Substituting the values into the equation yields $KN = \sqrt{6^2 + 9^2 - 2(6)(9)cos\ 60^o} = 3\sqrt{7}$.
15. D Completing the square and dividing yields $\frac{(x-2)^2}{16} - \frac{(y-3)^2}{4} = 1$. This is a horizontal-oriented hyperbola with $a = 4$ and $b = 2$. The slopes of the asymptotes are thus $\pm\frac{1}{2}$, and they must pass through the center, $(2, 3)$. The only line that satisfies this is D.
16. C The best way to solve this problem is by graphing. Completing the square for both equations gives the circle to be $(x-1)^2 + (y+1)^2 = 9$ and the parabola to be $y + 4 = 4(x-1)^2$. When graphed it can be seen that the vertex of the parabola lies on the circle, and there are two other intersection points. Thus, the total number of intersection points is 3.
17. E Completing the square and dividing yields $\frac{(x+1)^2}{12} - \frac{(y+2)^2}{8} = 1$. This is a hyperbola with $a^2 = 12$ and $b^2 = 8$, which means that $c^2 = a^2 + b^2 = 20$, so $c = 2\sqrt{5}$. The distance between the foci is $2c$, or $4\sqrt{5}$.
18. D The length of this arc is one third of a circle, so it is $\frac{1}{3} \cdot 2\pi \cdot 12 = 8\pi$. When the sector is folded, this arc becomes the base of the new cone, so the circumference of the base is $8\pi$, meaning the base radius is 4. The sector radius of 12 becomes the slant height. Since the radius is 4 and the slant height is 12, the height of the cone is $\sqrt{12^2 - 4^2} = 8\sqrt{2}$, so the volume is $\frac{128\pi\sqrt{2}}{3}$.
19. D Completing the square and dividing yields $\frac{(x-4)^2}{72} + \frac{(y+7)^2}{48} = 1$, which is an ellipse with $a = 6\sqrt{2}$ and $b = 4\sqrt{3}$. The length of the latus rectum is given by $\frac{2b^2}{a}$, which simplifies to $8\sqrt{2}$. Alternatively, if the formula is not known, there is an alternate way: it is known that the latus rectum passes through the focus, so you can plug in $4 + c$ for $x$ (this is the $x$- coordinate of the focus) where $c^2 = a^2 - b^2$, solve for both solutions of $y$ (these are the endpoints of the latus rectum) and find the vertical distance between them.
20. E All of these are valid examples of degenerate conic sections. $x^2 + y^2 = 0$ is a point, $x^2 = 1$ is a pair of parallel lines, $x^2 - y^2 = 0$ is two intersecting lines, and $x^2 = 0$ is one line.
21. B Completing the square for the circle yields $(x-6)^2 + (y+3)^2 = 64$. The distance from the point (-3, 9) to the center of the circle, (6, -3), is 15. Subtracting the radius of 8 yields the shortest distance from the point (-3, 9) to the circle which is 7.
22. C Completing the square yields the parabola $-(x-5)^2 = y - 16$. The two roots of this are $x = 1, 9$. The vertex of the parabola is (5, -16). This is a triangle with base 8 and height 16, which has area 64.
23. C Graphing this equation shows that the area is a triangular region with base 10 and height 5. The area is 25.
24. B The point on the line closest to (9, -2) will be the point of intersection between this line and the perpendicular line containing (9, -2). The perpendicular line is $2x + 3y = 12$. The intersection of this line and $3x - 2y = 5$ is (3, 2). The sum of the coordinates is 5.

25. B A right triangle inscribed in a circle has its hypotenuse on a diameter. Therefore, the hypotenuse is 12. If this is treated as the base, the maximum height of the triangle is 6, meaning the maximum area is 36.

26. B It is best to plot this triangle on the coordinate grid with one vertex at the origin. The side lengths can be written as $\sqrt{50}, \sqrt{52}, \sqrt{82}$. By inspection, $52 = 4^2 + 6^2$, $82 = 9^2 + 1^2$, and $50 = 5^2 + 5^2 \text{ } or \text{ } 7^2 + 1^2$. Since 50 can be written as the sum of squares two different ways, we should deal with that side last because we do not definitively know how that side length will be satisfied. We can arbitrarily set our second point to be (1, 9) so that the side length from the origin to this point is $\sqrt{82}$ because $1^2+9^2$ is the only way to express 82. Our third point has to satisfy the remaining two sides. After some analysis, it can be determined that the third point is (-5, 5). Applying shoestring and remembering to divide by 2 yields an area of 25. It should be noted that you could've assigned the second coordinate to be any of $(\pm 1, \pm 9), (\pm 9, \pm 1), (\pm 4, \pm 6), or\ (\pm 6, \pm 4)$, as long as it satisfied one of the sides being $\sqrt{82}$ or $\sqrt{52}$, and you would've arrived at the same answer. This is due to symmetry and the fact that you can rotate the triangle and maintain the same area.

27. D By definition, the vertex of the parabola is directly in between the focus and the directrix. Since the equation of the directrix is $5x + 3y = 12$, the equation of the line perpendicular to it which passes through the vertex is $3x - 5y = 14$. Solving for the intersection of these two lines yields the point $(3, -1)$. Since the vertex is directly between the directrix and the focus, that means the vertex, $(8, 2)$, is the midpoint of the closest point on the directrix, $(3, -1)$,and the focus, $(a, b)$. Solving for this, the focus is $(13, 5)$, so the sum of the coordinates is 18.

28. D After completing the square, the equation becomes $\frac{(x-2)^2}{9} - \frac{(y-1)^2}{18} = 1$, which, when graphed, is a horizontal-oriented hyperbola with $a = 3$ and $b = 3\sqrt{2}$. Thus, the slopes of the two asymptotes are $\pm\sqrt{2}$. By drawing a horizontal line through both asymptotes, a triangle can be formed with sides in the ratio $\sqrt{3}$:$\sqrt{3}$:2, where the two congruent sides lie along the asymptotes, and the third side is parallel to the x-axis. The angle of interest is opposite the side with length 2. Applying Law of Cosines, the cosine of the acute angle is found to be $\frac{1}{3}$. Also, since the angle is acute, the cosine must be positive.

29. B This graph is an ellipse. Since the constant sum of the distances is 20, which is equal to $2a$, $a = 10$. The distance between the foci is $2c$, so $c = 6$. In an ellipse, $a^2 = b^2 + c^2$, so $b = 8$, and thus the area is $ab\pi = 80\pi$.

30. E Completing the square yields $(x - 3)^2 + (y - 1)^2 = -20$, which is degenerate because the sum of two squares must be nonnegative.