

Directions: For questions involving rolling dice, flipping coins, or choosing from a deck of cards, assume that the objects are standard, unless otherwise noted.

Good luck! :)

- Three identical dice are rolled, then arranged in increasing order. How many possible outcomes are there?
A. 20 B. 56 C. 120 D. 216 E. NOTA
- How many ways are there to place 4 distinguishable objects in 4 indistinguishable boxes?
A. 5 B. 18 C. 35 D. 256 E. NOTA
- How many positive integers cannot be written in the form $5a + 9b$, where a and b are nonnegative integers?
A. 14 B. 16 C. 20 D. 31 E. NOTA
- What is the units digit of $2^{22} + 3^{33} + 4^{44} + 5^{55}$?
A. 0 B. 1 C. 8 D. 9 E. NOTA
- How many unique scores exist on a Mu Alpha Theta test? Assume a grading scale of 5 points for a correct answer, 1 for a blank answer, and 0 for an incorrect answer on a 30-question test.
A. 120 B. 132 C. 144 D. 145 E. NOTA
- Frank rolls three dice. What is the probability that the sum of the numbers on their faces will be a multiple of 3?
A. $8/27$ B. $1/3$ C. $10/27$ D. $11/27$ E. NOTA
- Frank rolls three dice. What is the probability that the product of the numbers on their faces will be a multiple of 3?
A. $8/27$ B. $13/27$ C. $17/27$ D. $19/27$ E. NOTA

8. A family has two children. Given that at least one of the children is female, what is the probability that the other one is also female?
A. $1/4$ B. $1/3$ C. $1/2$ D. $2/3$ E. NOTA
9. On the classic game show *Make a Deal in the Honky Mall*, four closed doors hide a car and three goats, with one object behind each door. You initially choose a random door out of four, after which the host opens two doors he knows has goats behind them. He then offers you a choice between your initial door and the remaining unopened door. If you choose to switch, what is the probability that you win the car?
A. $1/4$ B. $1/2$ C. $2/3$ D. $3/4$ E. NOTA
10. A dartboard is composed of three concentric circles of radii 2, 4, and 6. Assuming that my dart will hit the dartboard, what is the probability that my dart will land inside the circle of radius 4 but outside the circle of radius 2?
A. $\pi/36$ B. $1/9$ C. $1/3$ D. $4/9$ E. NOTA
11. Every positive integer from 1 to 500 inclusive is written out in a row. How many times does the digit 4 appear?
A. 150 B. 200 C. 250 D. 300 E. NOTA
12. Frank tosses two dice. What is the probability that their sum is a prime number?
A. $5/12$ B. $4/9$ C. $1/2$ D. $5/9$ E. NOTA
13. How many ways can a calico cat, a tortoiseshell cat, a tabby cat, and two indistinguishable humans sit in a row?
A. 20 B. 30 C. 60 D. 120 E. NOTA

14. Schrödinger's cat is in quite the predicament. It is trapped in a box and there is a 10% chance of it dying of natural causes in the next 10 minutes. Furthermore, in five minutes there is a 60% chance that gas will be released into its box, killing it immediately. However, if the gas is not released, there is a 20% chance that it will leak in anyways, but at a much lower concentration, and thus there is only a 40% chance of killing it. If I check on the cat in ten minutes, what is the probability it is dead? For the sake of simplicity, assume that gas cannot leak in if it is manually released, and the cat can still die from natural causes if the gas doesn't kill it.
- A. 0.6666 B. 0.6688 C. 0.6776 D. 0.6969 E. NOTA
15. The odds that the San Francisco 49ers will win the Super Bowl are 2:97. What is the Probability that the San Francisco 49ers win the Super Bowl this year?
- A. $\frac{2}{99}$ B. $\frac{2}{97}$ C. $\frac{97}{99}$ D. 1 E. NOTA
16. In how many consecutive zeroes does $2024!$ end in?
- A. 487 B. 498 C. 503 D. 606 E. NOTA
17. How many subsets of the set $\{F, H, L\}$ contain an even number of elements?
- A. 2 B. 4 C. 6 D. 8 E. NOTA
18. Frank flips n coins, while I flip $n + 1$ coins. What is the probability that I flip more heads than Frank?
- A. $\frac{1}{2}$ B. $\frac{1}{n}$ C. $\frac{1}{n+1}$ D. 1 E. NOTA
19. How many positive integers less than 1000 have exactly 3 positive factors?
- A. 11 B. 12 C. 13 D. 14 E. NOTA

20. How many distinguishable permutations are there of the word "MACHINA"?
- A. 720 B. 2520 C. 5040 D. 40320 E. NOTA
21. 11 people arrive at a party and shake hands with each other. Every person shakes everyone else's hand exactly once, and no one shakes their own hand. How many handshakes occurred?
- A. 22 B. 55 C. 110 D. 121 E. NOTA
22. A rare and unheard-of virus known as SARS-CoV-2 is beginning to circulate in the *homo sapiens* population. A test has been created that returns positive if the patient has the virus 99% of the time but will return a false positive for patients without the virus 1% of the time. You also know that 99% of the population does not have the virus. Given that your test result returned positive, what is the probability that you are infected by SARS-CoV-2?
- A. 1/100 B. 1/99 C. 1/2 D. 99/100 E. NOTA
23. At Buchholz High School, three science courses are offered – physics, chemistry, and biology. If 54 seniors take only one class, 13 take only 2 classes, and 4 take all three, how many seniors are there in total? Assume that seniors cannot take zero classes.
- A. 4 B. 45 C. 54 D. 71 E. NOTA
24. How many ways can four people rearrange themselves in a row of 4 seats such that none of them are seated in their original seat?
- A. 4 B. 6 C. 8 D. 9 E. NOTA
25. How many ways can those same four people rearrange themselves in a row of four seats if exactly one of them is to be seated in their original seat?
- A. 4 B. 6 C. 8 D. 9 E. NOTA

26. Frank has a well-shuffled deck of cards. He throws away the top 13 cards face-down. What is the probability that the next card is the Ace of Hearts?
- A. $\frac{1}{52}$ B. $\frac{1}{39}$ C. $\frac{1}{4}$ D. $\frac{1}{3}$ E. NOTA
27. The arrangements of the word "BLUEJAY" are listed out in alphabetical order starting with "ABEJLUY" as number 1. What is the 647th arrangement in the list?
- A. AYEBULJ B. AYEJULB C. AYEULBJ D. BLUEJAY E. NOTA
28. How many more 4-digit base 3 numbers are there than 4-digit base 2 numbers?
- A. 1 B. 16 C. 46 D. 65 E. NOTA
29. What is the coefficient of a^3bc^2 in the expansion of $(a + b + c)^6$?
- A. 1 B. 6 C. 30 D. 60 E. NOTA
30. The function $\lfloor x \rfloor$ rounds x to the nearest integer. What is the probability that $\lfloor x \rfloor + \lfloor y \rfloor = \lfloor x + y \rfloor$ if x, y are randomly chosen real numbers between 0 and 10?
- A. $\frac{1}{2}$ B. $\frac{3}{4}$ C. $\frac{7}{8}$ D. 1 E. NOTA