1. B
2. B
3. B
4. C
5. D
6. B
7. D
8. B
9. D
10. C
11. B
12. A
13. C
14. E
15. A
16. C
17. B
18. A
19. A
20. B
21. B
22. C
23. D
24. D
25. C
26. A
27. C
28. C
29. D
30. B

1. B Consider the cases: you could have 3 of the same object, the same object twice and a different third object, or a different object every time. For the triple object case, there are 6 objects that could be repeated, so there are 6 possibilities. For the double object and different third object case, there are 6 ways to choose the object being repeated and 5 ways to choose the remaining object, totalling 30. For the 3 different objects case, it is the same as doing 6C3, which is 20. 6 + 30 + 20 = 56.
2. E Consider the cases: all four objects could be in one box, they could be split 3-1, 2-2, 2-1-1, or 1-1-1-1. In the first and last cases, there is only one way to put them all in a box or put them all in different boxes. In the 3-1 case, there are 4 different possibilities, one for each ball that is alone. In the 2-2 case, you are counting 4C2 objects to be in a different box, or 6 possibilities. However, the splits of AB/CD and CD/AB are both counted under the 4C2, so there are only 3 possibilities. In the 2-1-1 case, you choose 2 to be in a box (4C2), one to be in another box (2C1), and one to be in the remaining box (1C1). However, the two boxes with one ball each are interchangeable, so the sum must be divided by two, yielding 6. 1 + 4 + 3 + 6 + 1 = 15.
3. B The answer to this is found by the Chicken McNugget formula $(a-1)(b-1)/2$ for relatively prime a,b. Plug in 5 and 9 for a and b, resulting in 16. Alternatively, list numbers and cross out attainable numbers until you find every number above 31 is attainable using 5's and 9's.
4. C Units digits of powers cycle every 4 powers, but sometimes every 2 powers or stay the same no matter the power. Powers of two cycle between 2, 4, 8, and 6. Powers of four cycle between 4 and 6. Powers of three cycle between 3, 9, 7, and 1. Powers of 5 always end in 5. Since x^5 always ends in the same digit as x^1, we can say that 2^{22} will end in 4, 3^{33} will end in 3, 4^{44} will end in 6, and 5^{55} will end in 5. Summing these, our final answer will end in 8.
5. D The minimum score is 0; the maximum is 150. There are 151 scores betwixt them, but not every one of them is attainable – everything from 0 to 30 is achievable, since replacing an incorrect answer with a blank one adds 1 to the score. On the high end, scores are effectively counting down by 4 or 5 from 150. Listing some numbers, you can see that 149, 148, 147, 144, 143, and 139 are unattainable, meaning that there are 151 – 6 = 145 total attainable scores.
6. B Regardless of the first two rolls, there is a 1/3 chance that the final die will make the sum a multiple of 3.
7. D Since you only need one multiple of three to make the entire product a multiple of three, it is easier to look at the complement in this question – that is, the probability that none of the rolls produce a multiple of three. The probability of this on each roll is 2/3 and cubing it for the three die rolls yields an 8/27 chance that the product will not be a multiple of three. Subtracting this from the total probability of 1, the answer is 19/27.
8. B Consider the possibilities: a family could have 2 girls, 2 boys, an older boy and younger girl, or older girl and younger boy. Knowing that at least one is a girl eliminates the 2-boy possibility, resulting in 3 possibilities, only one of which has the other child being female.

9. D This is a variation of the famous Monty Hall problem, which you can look up online for many detailed solutions. For an intuitive solution, imagine this: instead of 4 doors, there are a billion. You still choose one door, and the host still reveals every unopened door except one. The chance that you picked the right door on your first try is 1/1000000000, forcing the other door to have a 999999999/1000000000 chance of containing the car, since probabilities must sum to 1. Interpolating to 4 doors, the answer will be ¾.
10. C The area inside the circle of radius 4 and outside the circle of radius 2 is $16\pi - 4\pi$, or 12π. Thus, the answer is $12\pi/36\pi$, or 1/3.
11. B Write each number as a 3-digit number, like 007 or 069. From there, 4 will appear 100 times in the hundreds digit (400-499), 50 times in the tens digit (040, 041, 042, … 449), and 50 times in the units digit (004, 014, 024, … 494). Therefore, it will appear 200 times in total.
12. A Drawing a 6x6 chart for all the possibilities when a die is rolled, you see the only prime numbers you can produce are 2, 3, 5, 7, and 11. There are 15 ways to produce a prime number out of 36 total, so 15/36, or 5/12.
13. C This is nearly the same as asking how many ways five distinguishable objects can be placed in a row, only two of the objects are indistinguishable from each other, so the count must be divided by two. 5!/2 = 60.
14. E The probability that gas is released and kills the cat is .6. The probability that gas is not released, leaks in anyways, and kills the cat is (.4)(.2)(.4) = .032. The sum of these two is the probability that the cat is killed by gas, which is .632. However, there is a (1 - .68)(.1) = 0.032 chance that the cat will not die from gas but from natural causes instead (you can't double-kill it!), totalling a 0.664 chance that the cat will be dead when I check on it.
15. A Odds are the relative probabilities of two events, so the probability that the 49ers win will be 2/(2 + 97) = 2/99.
16. C The number of consecutive zeroes at the end of a number is the same as the power of 10 in its prime factorisation, since multiplying a number by 10 adds a zero to the end of the number. Thus, look for the number of 5's in the prime factorisation, since 10 = 2 * 5, and there are going to be more 2's than 5's in the factorisation. The number of 5's can be found by dividing 2024 by 5 repeatedly, flooring the result, and adding the quotients, resulting in 404 + 80 + 16 + 3 = 503.
17. B Each element can be in the set or out of the set, meaning there are 2^3 total sets. There is one set with zero elements and 3 sets with 2 elements, so there are 4 sets with even numbers of elements.
18. A If my extra flip is heads, Frank needs strictly more heads than me to win. If my extra flip is tails, I need strictly more heads than Frank to win. By symmetry, the answer is ½.
19. A A number with 3 positive factors is the square of a prime (factors of 1, p, p^2). There are 11 primes from 2 to 31, so the answer is 11.
20. B This is 7!/2, since the two A's are interchangeable. The answer is 2520.
21. B Each person shakes the hand of 10 other people, meaning there are 11 * 10 = 110 total handshakes. However, since each handshake involves 2 people, this counts each handshake twice, so there are really only 55.

22. C Draw a 2-way table to contain the probabilities. Alternatively, notice that the probability of a false positive is (.01)(.99), which happens to equal the probability of a true positive, (.99)(.01), meaning that there is a 50% chance you have Covid-19.

23. D This is not a PIE question! Note that the question specifically enumerates that 54 seniors take *one* class, 13 take *two*, etc. This differs from a typical PIE question, which involves overcounting (for instance, a student could be counted in the physics pool, as well as the physics and chemistry pool). Thus, add the 4 categories: 54 + 13 + 4 = 71.

24. D This is equivalent to the number of derangements of 4 items, which is 9.

25. C This situation can be seen as choosing 1 element to held still, while the others are deranged. There are 4 elements that can be held still and 2 ways to derange 3 elements, totalling 8.

26. A The face-down cards have an equal chance of being any card, so they won't affect the possibilities for the remaining cards. Thus, the answer is still 1/52.

27. C There are 6! arrangements where A is the first letter, or 720. This means that the first letter must be A, since the list is in alphabetical order. Once A is confirmed as the first letter, 5! arrangements have B as the second letter, 5! have E as the second letter, and so on. From here, we can repeat this process to arrive at an answer of AYEULBJ.

28. C There are $2 * 3^3$ 4-digit base 3 numbers (the first digit can't be zero, as that would produce a 3-digit number) and $1 * 2^3$ 4-digit base 2 numbers. 54 – 8 = 46.

29. D The multinomial coefficient 6C(3, 1, 2) yields 60.

30. B Due to symmetry, this is the same as picking x, y from 0 to 1. Consider a 1 by 1 square with $0 \leq x < 1, 0 \leq y < 1$.

Case 1. If $x, y < 0.5$

The probability that $\backslash x/+\backslash y/=\backslash x+y/$ is if $x+y<0.5$. The probability is $\frac{1}{2}$

Case 2. One of x, y is greater than 0.5 and the other is less than 0.5

It is guaranteed that $\backslash x/+\backslash y/=\backslash x+y/$.

Case 3. If $x, y > 0.5$

This is same as case 1, the probability is $\frac{1}{2}$.

The total is $\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{2}=\frac{3}{4}$