You will find a few standard notations and definitions used for this test below.

- As is standard for factorials, we define $n! = (n)(n-1)(n-2)(n-3)\ldots(2)(1)$.
- A *squarefree* positive integer will be a positive integer whose prime factorization contains no exponents greater than 1.
- Euler's totient function for positive integers, notated as $\varphi(n)$, will be the number of positive integers that are both less than or equal to n and relatively prime to n.

Good luck and have fun!

1. What is the remainder when 2024 is divided by 11?
   A. 0 B. 2 C. 5 D. 8 E. NOTA

2. Find the sum of the proper divisors of 76.
   A. 52 B. 64 C. 100 D. 140 E. NOTA

3. Compute the value of $\gcd(8398, 1729)$.
   A. 91 B. 221 C. 247 D. 323 E. NOTA

4. Find the value of $\text{lcm}(105, 182) \cdot \gcd(728, 420)$.
   A. 76440 B. 77350 C. 79540 D. 82450 E. NOTA

5. How many positive integers less than 2024 have an even number of positive integral divisors?
   A. 44 B. 45 C. 1979 D. 1980 E. NOTA

6. How many primes are less than 120?
   A. 30 B. 31 C. 32 D. 33 E. NOTA

7. Which of the following numbers is composite?
A. 26627 B. 38547 C. 45691 D. 53003 E. NOTA

8. How many integers are in the interval [1, 2024] and are relatively prime to 1001?
A. 568 B. 1396 C. 1456 D. 1458 E. NOTA

9. The Fibonacci sequence has initial terms $F_0 = 0$, $F_1 = 1$ and the recurrence relation $F_{n+1} = F_n + F_{n-1}$ for all integers $n \geq 1$. What is the remainder when F_{2024} is divided by 21?
A. 0 B. 5 C. 13 D. 18 E. NOTA

10. Sean can buy bowls of jajangmyeon in packs of 28 and 91. What is the largest number of bowls of jajangmyeon that he can't buy exactly using packs of 28 and 91?
A. 245 B. 252 C. 2429 D. 2436 E. NOTA

11. Samuel then shows Sean a store in Seoul that sells jajangmyeon in packs of 6. Now, Sean can buy bowls of jajangmyeon in packs of 6, 28, and 91. What is the largest number of bowls of jajangmyeon that he can't buy exactly using packs of 6, 28, and 91?
A. 141 B. 147 C. 267 D. 273 E. NOTA

12. How many terminal zeroes does $\binom{127}{39}$ have when expressed in base 2?
A. 1 B. 2 C. 3 D. 4 E. NOTA

13. Jeffrey used his lunch money to buy a huge stack of Pokemon cards. He tries to split the stack into equal groups. When he tries to split the stack into 6 groups, he's left with 1 card. When he tries to split the stack into 14 groups, he's left with 11 cards. When he tries to split the stack into 26 groups, he's left with 9 cards. What's the least possible number of cards that Jeffrey could have bought?
A. 319 B. 607 C. 828 D. 1075 E. NOTA

14. Find the value of

$$\sum_{d|30 \text{ and } d\in Z^+} \varphi(d)$$

which is the summation of $\varphi(d)$ for all positive divisors d of 30.

A. 22 B. 29 C. 30 D. 34 E. NOTA

15. Find the value of

$$\sum_{n=0}^{8} \varphi(3^n)$$

A. 2187 B. 3281 C. 6560 D. 6562 E. NOTA

16. Find the last 3 digits of 7^{2024}.

A. 201 B. 401 C. 601 D. 801 E. NOTA

17. Let S be the set of all integers n such that the value of the fraction $\frac{n^2-4n+43}{n-11}$ is an integer. Compute the summation

$$\sum_{n\in S} \frac{n^2-4n+43}{n-11}$$

A. 192 B. 288 C. 360 D. 576 E. NOTA

18. When 16! is computed, it results in $20\,ABC\,789\,888\,000$. A, B, C are digits, and the spacing is for the sole purpose of readability. Compute the value of $100A + 10B + C$.

A. 103 B. 229 C. 625 D. 922 E. NOTA

19. Consider the polynomial $f(x) = x^3 - 9x^2 + 23x - 15$. Find the sum of all integers n that make $|f(n)|$ a prime number (answer 0 if no such n exists).

A. -4 B. 0 C. 6 D. 10 E. NOTA

20. A positive integer n is chosen so there doesn't exist a tuple (a, b, c, d, e, f) of integers that satisfies the equation

$$a^4 + b^4 + c^4 + d^4 + e^4 + f^4 = n.$$

Which of the following could be a possible value of n? (Hint: consider x^4 modulo 16.)

A. 2964 B. 3127 C. 3270 D. 3489 E. NOTA

21. What is the smallest positive integer k such that $41k$ leaves a remainder of 1 when divided by 181?

A. 42 B. 53 C. 106 D. 124 E. NOTA

22. What is the remainder when $\binom{83}{58}$ is divided by 29?

A. 1 B. 6 C. 13 D. 24 E. NOTA

23. What is the tenth smallest natural number that has exactly 6 positive factors?

A. 52 B. 63 C. 68 D. 75 E. NOTA

24. Let $n = 36^{36} - 4^{36}$. What is the greatest power of 2 that divides into n?

A. 2^{74} B. 2^{75} C. 2^{76} D. 2^{77} E. NOTA

25. What is the remainder when 33! is divided by 37?

A. 18 B. 20 C. 31 D. 36 E. NOTA

26. Let $n = 8^6 + 9^8$. Given that n is squarefree and has exactly 3 prime factors, find the sum of the prime factors of n.

A. 1947 B. 3261 C. 5133 D. 6419 E. NOTA

27. What is the remainder when 12^{676} is divided by 125?

A. 58 B. 66 C. 99 D. 114 E. NOTA

28. Find the remainder when $\binom{44}{12}$ is divided by 10.

A. 1 B. 3 C. 7 D. 9 E. NOTA

29. Find the last nonzero digit of 25!

A. 2 B. 4 C. 6 D. 8 E. NOTA

30. How many integers n are there such that the fraction $\frac{2024}{n+73}$ is equal to an integer?

A. 3 B. 6 C. 16 D. 32 E. NOTA