1. B
2. A
3. C
4. B
5. A
6. B
7. C
8. A
9. A
10. C
11. C
12. D
13. A
14. A
15. C
16. B
17. D
18. C
19. A
20. D
21. C
22. D
23. A
24. B
25. D
26. B
27. C
28. A
29. C
30. A

1. B We change the limit's bound from negative infinity to positive infinity by plug in –x for x. The limit turns into $\lim_{x\to\infty} \sqrt[3]{3x^2 - x^3 + 37} - \sqrt[3]{6x^2 - x^3 + 65}$. By completing the cube inside the cube root, we obtain $\lim_{x\to-\infty} \sqrt[3]{(-x+1)^3 + 36} - \sqrt[3]{(-x+2)^3 + 64}$. The limit approaches $-x + 1 - (-x+2) = -1$

2. A Stirling's approximation for n! when it approaches infinity is $\sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ Using the approximation, the limit simplifies to $\sqrt{2\pi}$.

3. C $y = x^{\frac{1}{5}}$ $\frac{dy}{dx} = \frac{1}{5}x^{-\frac{4}{5}}$ Plug in x=32. $\frac{dy}{dx} = \frac{1}{80}$ $33^{\frac{1}{5}} = 2 + 1 \times \frac{1}{80} = \frac{161}{80}$

4. B $y = \sqrt{2x + y}$ Plug in $x = \frac{3}{8}$, $y = \sqrt{\frac{3}{4} + y}$. Solving for y. $y^2 = \frac{3}{4} + y$, $\left(y - \frac{3}{2}\right)\left(y + \frac{1}{2}\right) = 0$ Due to the nature of the square roots, the solution for y must be positive and thus 3/2. Solving further for $\frac{dy}{dx} = \frac{2+\frac{dy}{dx}}{2\sqrt{2x+y}}$. Plug in x and y. $\frac{dy}{dx} = \frac{2+\frac{dy}{dx}}{2\sqrt{\frac{3}{4}+\frac{3}{2}}}$ $\frac{dy}{dx} = \frac{2+\frac{dy}{dx}}{3}$ $\frac{dy}{dx} = 1$.

5. A The denominator of the limit approaches infinity while the numerator is restricted by the domain of $\sin x$ to be between -1 and 1. The limit will approach 0.

6. B $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ $x_{n+1} = x_n - \frac{5x^4 - 8x^3}{20x^3 - 24x^2}$ After simplifying, $x_{n+1} = x_n - \frac{5x^2 - 8x}{20x - 24}$ $x_1 = 1 - \frac{-3}{-4} = \frac{1}{4}$. $x_2 = \frac{1}{4} - \frac{\frac{5}{4} - 8}{5 - 24} = \frac{49}{304}$.

7. C Solving for the y coordinate of the point. Plug in x=1/2. $y = \frac{3}{2}\frac{\pi}{6} = \frac{\pi}{4}$Taking derivatives. $\frac{dy}{dx} = 3\sin^{-1}x + 3\frac{x}{\sqrt{1-x^2}} = 3\frac{\pi}{6} + 3\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\pi}{2} + \sqrt{3}$. Using the slope and the point. $y - \frac{\pi}{4} = \left(\frac{\pi}{2} + \sqrt{3}\right)\left(x - \frac{1}{2}\right)$ After simplifying, $y = \left(\frac{\pi}{2} + \sqrt{3}\right)x - \frac{\sqrt{3}}{2}$.

8. A Take the natural log of both sides and then using L'Hopital's rule. $\ln y = \tan x \ \ln(2024\sin x) = \frac{\ln(2024\sin x)}{cotx}$ which results in $\frac{\infty}{\infty}$. Apply L'Hopital's rule, $\ln y = \frac{\frac{2024\cos x}{2024\sin x}}{-\csc x \cot x} = -\sin x = 0$

9. A $\frac{dy}{dx} = 3x^2 + 2x + 1$=6. Finding the line: $y - 4 = 6(x - 1)$ Simplifying to $y = 6x - 2$

10. C To find the Taylor series of $\tan^{-1}x$, take the derivative first to find $\frac{dy}{dx} = \frac{1}{1+x^2}$ View this as the sum of an infinite geometric series to find $\frac{dy}{dx} = 1 - x^2 + x^4 - \cdots$. Integrate both side to find $y = \tan x = x - \frac{x^3}{3}$ as the third degree approximation. Plug in 1/10 to get $\frac{299}{3000}$

11. C $C'(x) = 6a^2 - 30a + 36$ to The cost would be minimized at either end points or when C'(x)=0. $C'(x) = 6(x-2)(x-3)$ Plug in critical points and end points. C(0)=2024 C(2)=2052 C(3)=2051. The minimum cost is 2024 when 200 bottles are produced.

12. D The i and n are flipped from normal Riemann sum. Switch the n and i to get a normal Riemann Sum. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{3i+6n}$=$\lim_{i\to\infty}\frac{1}{n}\sum_{n=1}^{i}\frac{1}{3\frac{i}{n}+6} = \int_0^1 \frac{1}{3x+6}dx = \frac{1}{3}\ln(x+2) = \frac{1}{3}\ln\frac{3}{2}$

13. A The radius of convergence for $y = \tan x$ is $\frac{\pi}{2}$. Without having this memorized, we can realize that tan(x) only converges in intervals of π, making the radius $\frac{\pi}{2}$.

14. A Plug in the given x, we get $\frac{0}{0}$. Using Le'Hopital's rule, $\lim_{x\to 0}\frac{\int_0^{\sin x} e^t \sinh t\, dt}{2x} = \lim_{x\to 0}\frac{\cos x\, e^{\sin x}\sinh \sin x}{2}$=0

15. C $\lim_{h\to 0}\frac{f\left(\frac{\sqrt{2}}{2}-2h\right)-f\left(\frac{\sqrt{2}}{2}+3h\right)}{h} = \frac{0}{0}$ Using Le'Hopital's rule, $\lim_{h\to 0}\frac{f\left(\frac{\sqrt{2}}{2}-2h\right)-f\left(\frac{\sqrt{2}}{2}+3h\right)}{h} = \lim_{h\to 0}\frac{-2f\prime\left(\frac{\sqrt{2}}{2}-2h\right)-3f\prime\left(\frac{\sqrt{2}}{2}+3h\right)}{1} = -5f'\left(\frac{\sqrt{2}}{2}\right)$ $f'(x) = 4\frac{2x}{\sqrt{1-x^4}}$ Plug in $\frac{\sqrt{2}}{2}$, $-5f'(x) = -5\frac{8*\frac{\sqrt{2}}{2}}{\sqrt{1-\frac{1}{4}}} = \frac{-40\sqrt{6}}{3}$

16. B Draw the picture and get a right triangle with bottom leg of x and side leg of 4+1/2. Setting up an equation for the angle. $\theta = \tan^{-1}\frac{9/2}{x} - \tan^{-1}\frac{1/2}{x}$.Using expansion for tangent subtraction. $\tan\theta = \frac{4/x}{1+\frac{9/4}{x^2}} = \frac{4x}{x^2+\frac{9}{4}}$ The angle is at a maximum when $d\theta = 0$ and changes from positive to negative. Taking derivative of previous equation. $\frac{1}{1+\theta^2}d\theta = \frac{4x^2+9-4x(2x)}{\left(x^2+\frac{9}{4}\right)^2}$ Since the denominator is always positive, we only need to examine the numerator. $9 - 4x^2 = 0$ which changes from positive to negative at $x = \frac{3}{2}$.

17. D Differentiate implicitly, $4x\frac{dx}{dy} + 2y = -3x - 3y\frac{dx}{dy}$. Plug in the points, $4\frac{dx}{dy} - 4 = -3 + 6\frac{dx}{dy}$ $\frac{dx}{dy} = -\frac{1}{2}$.

18. C $y = r\sin\theta$ $x = r\cos\theta$ $\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{-r\sin\theta + r\prime\cos\theta}$ Since $\frac{\sqrt{2}}{2} = \sin\frac{\pi}{4} = \cos\frac{\pi}{4}$, $\frac{dy}{dx} = \frac{r'+r}{r'-r} = \frac{1-\csc^2\theta+\theta+\cot\theta}{1-\csc^2\theta-\theta-\cot\theta} = \frac{\frac{\pi}{4}}{-2-\frac{\pi}{4}} = \frac{-\pi}{\pi+8}$.

19. A Plug in x=1, we get that y=1. Differentiate implicitly, $\frac{5}{2}x^{3/2} + \frac{5}{2}y^{3/2}\frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{x^{\frac{3}{2}}}{y^{\frac{3}{2}}} = -1$ $\frac{d^2y}{dx^2} = -\frac{\frac{3}{2}\sqrt{x}y^{\frac{3}{2}}-\frac{3}{2}\sqrt{y}x^{\frac{3}{2}}\frac{dy}{dx}}{y^3} = -3$

20. D Differentiate $f'(x) = \frac{1+\frac{1}{2\sqrt{x+3}}}{2\sqrt{x+\sqrt{x+3}}} = \frac{1+\frac{1}{2*3}}{2\sqrt{9}} = \frac{7}{36}$

21. C $dv = Adh$ Radius of the surface can be found using similar triangles, with ratio to the entire bottle, $3 * \frac{\frac{1}{2}}{2} = \frac{3}{4}$ $dv = \frac{9\pi}{16}\frac{12}{60} = \frac{9\pi}{80}$.

22. D The limit does not approach from the negative side.

23. A $\lim_{x\to 27}\frac{x-27}{\sqrt[3]{x}-3} = \frac{0}{0}$ $\lim_{x\to 27}\frac{x-27}{\sqrt[3]{x}-3} = \frac{1}{\frac{1}{3}x^{-\frac{2}{3}}} = \frac{1}{\frac{11}{39}} = 27$

24. B $|5x+4-24| < \frac{1}{2024}$ $|5x-20| < \frac{1}{2024}$ $|x-4| < \frac{1}{10120}$

25. D $$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3in^2-7ni^2+1}{n^4} = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{3i}{n}-\frac{7i^2}{n^2}+\frac{1}{n^4} = \int_0^1 3x-7x^2\,dx = -\frac{5}{6}$$

26. B $\frac{dy}{dx} = \frac{\frac{1}{t}}{3t^2+2t} = \frac{1}{3t^3+2t^2}$ $\frac{d^2y}{dx^2} = \frac{-\frac{9t^2+4t}{(3t^3+2t^2)^2}}{3t^2+2t} = \frac{-13/25}{5} = -\frac{13}{125}$

27. C

Step	x	y	Slope	Δx	$y+\Delta x\frac{dy}{dx}$
1	0	1	2	0.25	$\frac{3}{2}$
2	$\frac{1}{4}$	$\frac{3}{2}$	$\frac{13}{4}$	0.25	$\frac{37}{16}$
3	$\frac{1}{2}$	$\frac{37}{16}$	$\frac{41}{8}$	0.25	$\frac{115}{32}$
4	$\frac{3}{4}$	$\frac{115}{32}$	$\frac{127}{16}$	0.25	$\frac{357}{64}$

28. A Since $\lim_{(x,y)\to(0,0)}\frac{x^4+2x^2y^2+y^4}{x^2+y^2+x^4y^4}$ Replace with polar coordinates r=0.

$\lim_{r\to 0}\frac{(r\cos\theta)^4+2(r\cos\theta)^2(r\sin\theta)^2+(r\sin\theta)^4}{(r\cos\theta)^2+(r\sin\theta)^2+(r\cos\theta)^4(r\sin\theta)^4} = \lim_{r\to 0}\frac{r^2((\cos\theta)^4+2(\cos\theta)^2(\sin\theta)^2+(\sin\theta)^4)}{(\cos\theta)^2+(\sin\theta)^2+r^6(\cos\theta)^4(\sin\theta)^4}$=0

29. C Take the natural log of both sides. $\ln f(x) = \frac{lnx}{2} + \sqrt{x+3} - \ln(\tan x)$

$f(1) = \frac{e^2}{\tan 1}$ $\frac{f'(x)}{f(x)} = \frac{1}{2x} + \frac{1}{2\sqrt{x+3}} - \frac{(\sec x)^2}{\tan x}$ $\frac{(\tan 1)f'(1)}{e^2} = \frac{1}{2} + \frac{1}{4} - \frac{(\sec 1)^2}{\tan 1}$

$f'(1) = \left(\frac{e^2}{\tan 1}\right)\left(\frac{3}{4} - \frac{(\sec 1)^2}{\tan 1}\right)$

30. A $f'(x) = \frac{8x}{1+4x^2} = \frac{4}{2} = 2$