Useful information: $2024 = 45^2 - 1 = 44 \cdot 46$.
NOTA means "None of the Above." Good luck!

1. Find:

$$\int \frac{1}{1+x^2}\,dx$$

A. $-\operatorname{arccot}(x) + C$ B. $-\arcsin(x) + C$
C. $-\arccos(x) + C$ D. $-\arctan(x) + C$ E. NOTA

2. Evaluate:

$$\int_1^{\infty} \frac{\arctan(x)}{x^3}\,dx$$

A. $\frac{\pi-2}{4}$ B. $\frac{12-\pi}{24}$ C. $\frac{5\pi-12}{24}$ D. $\frac{1}{2}$ E. NOTA

3.

$$I = \int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx$$

I can be expressed in the form $\frac{\pi}{K} - \frac{1}{H}$ for positive integers K and H. Find $K + H$.

A. 12 B. 16 C. 20 D. 28 E. NOTA

4. Evaluate:

$$\int_0^3 x\sqrt{9-x^2}\,dx$$

A. 9 B. 12 C. 15 D. 18 E. NOTA

5. Evaluate:

$$\int_0^2 x^3\sqrt{4-x^2}\,dx$$

A. $\frac{8}{15}$ B. $\frac{16}{15}$ C. $\frac{32}{15}$ D. $\frac{64}{15}$ E. NOTA

6. This question will be dedicated to 2020, the year everything stopped because of COVID-19. Let $f(x) = (2)x^3 + (0)x^2 + (2)x + (0)$. Find the volume of the solid formed by rotating the region bounded by the graphs of $f(x)$, the x-axis, and $x = 1$ about the y-axis.

A. $\frac{8\pi}{15}$ B. $\frac{16\pi}{15}$ C. $\frac{32\pi}{15}$ D. $\frac{64\pi}{15}$ E. NOTA

7. Let R be the region bounded by $y = \sqrt{x}$, $x = a$, and the x-axis for $a > 0$, such that the volume of the solid formed by rotating R about the x-axis equals the volume of the solid formed by rotating R about the y-axis. The value of a that accomplishes this can be written in the form $\frac{m}{n}$ for relatively prime positive integers m and n, find $m + n$.

A. 41 B. 89 C. 147 D. 205 E. NOTA

8. Evaluate:

$$\int_0^{\frac{\pi}{3}} \tan^4(x)\, dx$$

A. $\frac{\pi}{3}$ B. $2\sqrt{3} - \frac{\pi}{3}$ C. $\frac{\pi}{2}$ D. $2\sqrt{3} - \frac{\pi}{2}$ E. NOTA

9. For $i = \sqrt{-1}$, let

$$\ln A = i \int_{\ln(2)}^{\ln(3)} \tan(ix)\, dx$$

If A can be written in the form $\frac{m}{n}$ for relatively prime positive integers m and n, find $m + n$.

A. 5 B. 7 C. 9 D. 11 E. NOTA

10.

$$I = \int_{\frac{2}{\sqrt{3}}}^{\infty} \frac{\sqrt{4 + x^2}}{x^4}\, dx$$

I can be expressed in the form $\frac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

A. 19 B. 25 C. 31 D. 37 E. NOTA

11. Evaluate:

$$\ln A = \int_0^{\ln(5)} \frac{e^x - 1}{e^x + 1}\, dx$$

If A can be written in the form $\frac{m}{n}$ for relatively prime positive integers m and n, find $m + n$.

A. 4 B. 14 C. 24 D. 34 E. NOTA

12. Evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{1}{9\sin^2(x) + 4\cos^2(x)}\, dx$$

A. $\frac{\pi}{24}$ B. $\frac{\pi}{18}$ C. $\frac{\pi}{12}$ D. $\frac{\pi}{6}$ E. NOTA

13. Evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{1}{\sin(x) + \cos(x)}\, dx$$

A. $\frac{\sqrt{2}}{2}\ln\left(1+\sqrt{2}\right)$ B. $\ln\left(1+\sqrt{2}\right)$
C. $\sqrt{2}\ln\left(1+\sqrt{2}\right)$ D. $2\ln\left(1+\sqrt{2}\right)$ E. NOTA

14. For $f(x) = 2x^3 + 2x + 4$, let $g(x)$ be the inverse of $f(x)$. Evaluate:

$$\int_8^{24} \frac{1}{g^2(x)}\, dx$$

A. -5 B. 1 C. 7 D. 13 E. NOTA

15. Evaluate:

$$\int_{-51}^{-39} (x^2 + 90x + 2024)dx$$

A. 126 B. 128 C. 130 D. 132 E. NOTA

16. Evaluate:

$$\lim_{n\to\infty} \sum_{i=0}^{n} \frac{1}{\sqrt{i^2 + 3n^2}}$$

A. $\frac{1}{2\sqrt{3}}\ln(3)$ B. $\frac{1}{3}\ln(3)$ C. $\frac{1}{2}\ln(3)$ D. $\frac{1}{\sqrt{3}}\ln(3)$ E. NOTA

17. Evaluate:

$$\int_0^{12} \left(\lfloor x \rfloor + \sqrt{\{x\}}\right) dx$$

where $\lfloor x \rfloor$ represents the greatest integer that is less than or equal to x and $\{x\}$ represents the fractional part of x such that $x = \lfloor x \rfloor + \{x\}$ and $0 \leq \{x\} < 1$.

A. 70 B. 72 C. 74 D. 78 E. NOTA

18. Evaluate:

$$\int_1^e \frac{\ln(x)}{x^2}\, dx$$

A. 1 B. $\frac{e-2}{e}$ C. $\frac{e+2}{2}$ D. 2 E. NOTA

19. Evaluate:

$$\int_0^1 \frac{1}{(1+x^2)\sqrt{1-x^2}}\,dx$$

Hint: $x = \frac{1}{u}$.

A. $\frac{\pi}{8}$ B. $\frac{\pi\sqrt{2}}{8}$ C. $\frac{\pi}{4}$ D. $\frac{\pi\sqrt{2}}{4}$ E. NOTA

20. Evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{1+\sin(x)}{1+\cos(x)}\,dx$$

Hint: $\tan\left(\frac{x}{2}\right) = u$.

A. $\frac{1}{2}+\ln(2)$ B. $\frac{1}{2}+\ln(4)$ C. $1+\ln(2)$ D. $1+\ln(4)$ E. NOTA

21. Evaluate:

$$\int_0^{\frac{\pi}{2}} (\sin^3(x)+\cos^3(x))dx$$

A. $\frac{4}{3}$ B. 2 C. $\frac{8}{3}$ D. 4 E. NOTA

22.

$$f(a) = \int_0^1 x^a(1-x)^2 dx$$

Let:

$$S = \sum_{a=1}^{\infty} f(a)$$

S can be written in the form $\frac{m}{n}$ for relatively prime positive integers m and n. Find $m+n$.

A. 3 B. 7 C. 11 D. 13 E. NOTA

23. Let $f(x) = \int_3^x \frac{1}{\sqrt{t+1}}\,dt$. $g(x)$ is the inverse of $f(x)$. Compute $g'(86)$.

A. 43 B. 44 C. 45 D. 46 E. NOTA

24. Katie has trouble reading. For some positive integer k, let $f(x) = (\cos x)^k$. Mr. Lu asks her to approximate the value of $\int_0^1 f(x)dx$ using the 2nd degree Maclaurin approximation for $f(x)$. Instead, she simply computes the value of $\int_0^1 x^k dx$, misreading the $\cos x$ as x. However, she gets the same answer regardless. What value of k accomplishes this?

A. 4 B. 5 C. 6 D. 7 E. NOTA

25. Let $M(k)$ be the value of the midpoint approximation of the following integral using $n = 2$ equal subintervals:

$$\int_0^{\pi} \sin^{2k}(x)\,dx$$

Evaluate:

$$\sum_{k=0}^{\infty} M(k)$$

A. $\pi\sqrt{2}$ B. 2π C. $2\pi\sqrt{2}$ D. 4π E. NOTA

26. $$I = \int_2^7 \frac{1}{x\sqrt{x^2-1}}\,dx$$

Then $\tan(I)$ can be expressed in the form $\frac{r\sqrt{s}}{t}$ for relatively prime positive integers r and t and squarefree s. Find $r + s + t$.

A. 19 B. 22 C. 25 D. 32 E. NOTA

27. $f(x)$ is an odd function. If $\int_{-2}^{6} f(x)dx = 4$, $\int_{-6}^{11} f(x)dx = 3$, and $\int_{-1}^{11} f(x)dx = 5$, find $\int_{-2}^{-1} f(x)dx$.

A. -2 B. -1 C. 1 D. 2 E. NOTA

28. Evaluate:

$$\int_1^2 \frac{2x+1}{x^3+x^2}\,dx$$

A. $\ln\left(\frac{3}{2}\right) - \frac{1}{2}$ B. $\ln\left(\frac{3}{2}\right) + \frac{1}{2}$ C. $\ln\left(\frac{4}{3}\right) - \frac{1}{2}$ D. $\ln\left(\frac{4}{3}\right) + \frac{1}{2}$ E. NOTA

29. The x-coordinate of the centroid of the region bounded by the graph of $y = -2x^3 - 2x + 4$ and the coordinate axes can be expressed in the form $\frac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

A. 29 B. 66 C. 103 D. 140 E. NOTA

30. Samuel loves this integral! Find:

$$\int x\cos(x)\,dx$$

A. $x\sin(x) + \cos(x)$ B. $x\sin(x) - \cos(x)$
C. $x\cos(x) + \sin(x)$ D. $x\cos(x) - \sin(x)$ E. NOTA