For each of the questions on this test, E. NOTA should be chosen if None Of The Answers given are correct. *Good luck and have fun!*

1. Find $\lim_{x\to\pi} e$.

A. π B. e C. 1 D. i E. NOTA

2. Find the sum of the squares of the x-values of the points of inflection of

$$f(x) = e^{-\frac{(x-1)^2}{8}}$$

A. 6 B. 8 C. 10 D. 18 E. NOTA

3. Find the slope of the graph of $f(x) = \sin\left(x^3 + 2x + \frac{\pi}{3}\right)$ at $x = 0$..

A. 0 B. ½ C. 1 D. 2 E. NOTA

4. Find the area in the complex plane enclosed by the locus of complex numbers z satisfying:

$$|z-3| + |z-4i| = 10$$

A. $\frac{75\pi}{4}$ B. $\frac{25\pi\sqrt{3}}{2}$ C. 50π D. $50\pi\sqrt{3}$ E. NOTA

5. Find the slope of the line tangent to the graph of $y^3 + x^2y - x^3 + 3 = 0$ at the point $(2, 1)$.

A. $\frac{8}{7}$ B. $\frac{7}{8}$ C. $-\frac{8}{7}$ D. $-\frac{7}{8}$ E. NOTA

6. Which of the following is an expression for y' if $y = x^{(2x)^{x^{(2x)^{\cdot^{\cdot^{\cdot}}}}}}$?

A. $\frac{y\ln(y)}{x\ln(x)} \cdot \frac{1+y\ln(x)}{1+y\ln(y)\ln(2x)}$ B. $\frac{y\ln(y)}{x\ln(x)} \cdot \frac{1+y\ln(x)}{1-y\ln(y)\ln(2x)}$

C. $\frac{x\ln(x)}{y\ln(y)} \cdot \frac{1+y\ln(x)}{1+y\ln(y)\ln(2x)}$ D. $\frac{x\ln(x)}{y\ln(y)} \cdot \frac{1+y\ln(x)}{1-y\ln(y)\ln(2x)}$ E. NOTA

7. Find the positive difference between the maximum and minimum values of

$$f(x) = 5\cos(3x - 1) + 12\sin(3x - 1) + 7$$

A. 10 B. 13 C. 24 D. 26 E. NOTA

8. Evaluate:

$$\int_{e^{e^e}}^{\infty} \frac{dx}{x\ln(x)\ln(\ln(x))\ln(\ln(\ln(x)))^{2024}}$$

A. $-\frac{1}{2023}$ B. $\frac{1}{2023}$ C. $-\frac{1}{2024}$ D. $\frac{1}{2024}$ E. NOTA

9. If $\int_0^{-24} f(x)dx = 10$, $\int_{-20}^{24} f(x)dx = 6$, and $f(x)$ is odd, what is $\int_0^{20} f(x)dx$?

A. -16 B. -4 C. 4 D. 16 E. NOTA

10. The value of $a > 0$ in the equation $y = ax(a - x)$ is changing at a rate of three units per time. Ignoring units, what is the rate of change of the y-intercept of the line tangent to the graph of this equation at $x = 2$ when $a = 2024$?

A. 12 B. 4 C. -4 D. -12 E. NOTA

11. Evaluate: $\int_0^1 \sqrt[3]{1 - \sqrt[3]{x}}dx$.

A. $\frac{81}{140}$ B. $\frac{22}{70}$ C. $\frac{12}{28}$ D. $\frac{121}{140}$ E. NOTA

12. Let θ be the acute angle the graph of the directrix of $9x^2 + 12xy + 4y^2 + x + y - 1 = 0$ makes with the x-axis. Find $\sin(\theta)$ if the above curve is a parabola with an axis of symmetry having negative slope.

A. $\frac{12}{13}$ B. $\frac{5}{13}$ C. $\sqrt{\frac{7}{26}}$ D. $\sqrt{\frac{4}{13}}$ E. NOTA

13. The region R is bounded by the y-axis, $y = ax^2 + b$, and by the line of positive slope that is tangent to $y = ax^2 + b$ and that also goes through the origin. If a and b are positive real numbers, find the area of R.

A. $\frac{b\sqrt{ab}}{2a}$ B. $\frac{b\sqrt{ab}}{3a}$ C. $\frac{b^2\sqrt{ab}}{2a^2}$ D. $\frac{b^2\sqrt{ab}}{3a^2}$ E. NOTA

14. A line $\mathcal{L}$ is drawn from the vertex $(0,2)$ of the graph of $y = 2\cosh\left(\frac{x}{2}\right)$ so that it is perpendicular to the line tangent to the graph of $y = 2\cosh\left(\frac{x}{2}\right)$ at $x = \ln(4)$. Find the distance between the x- and y-intercepts of $\mathcal{L}$.

A. $\frac{5}{2}$ B. $\frac{5}{4}$ C. $\frac{3}{2}$ D. $\frac{3}{4}$ E. NOTA

15. Romir and Sydney alternate rolling a pair of fair dice, stopping when either Romir rolls a sum of 5 or when Sydney rolls a sum of 10. If Romir rolls first, what is the probability that the final roll is made by Romir?

A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. $\frac{3}{5}$ D. $\frac{3}{4}$ E. NOTA

16. Evaluate: $\lim\limits_{h\to 0} \int_{x-24h}^{x+20h} \frac{f(t)}{h}\,dt$.

A. 0 B. $f(x)$ C. $20f(x)$ D. $24f(x)$ E. NOTA

17. In a certain population, 90% of people are vaccinated against a certain illness. For that same population, 60% of patients hospitalized with this illness are vaccinated. If the probability of an unvaccinated person being hospitalized is K times the probability of a vaccinated person being hospitalized, find K.

A. 3 B. 6 C. 9 D. 12 E. NOTA

18. Yusuf places a spherical marble into a cylindrical container with an open top, and then pours water into the container until the marble is completely submerged. Find the radius of the marble (in cm) that maximizes the volume of water needed if the radius of the cylindrical container is 24 cm and its height is 100 cm.

A. $4\sqrt{2}$ B. $6\sqrt{2}$ C. $9\sqrt{2}$ D. $12\sqrt{2}$ E. NOTA

19. Which of the following numbers is NOT expressible as the sum of at least two consecutive odd positive integers?

A. 2021 B. 2022 C. 2023 D. 2024 E. NOTA

20. Jayden is standing on the point $(0, 3)$, and wishes to shine a laser at a random point on an infinite wall located along the x-axis. To do this, he chooses an angle θ uniformly at random from the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, and positions his laser so it makes an angle of θ with the y-axis (positive values of θ point towards the positive x-axis). If X is the x-coordinate of the point on the wall he shines his laser towards, find the probability that $|X| < \sqrt{3}$.

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{\sqrt{3}}$ D. $\frac{1}{2}$ E. NOTA

21. Tiger is trapped in a room with three doors. Behind one of the doors is a path that, when he travels it for 20 minutes, will lead to freedom. Behind the second door is a path that, after 24 minutes, will lead back to the same room, but the doors will be randomly permuted. Behind the third door is a path that, after 48 minutes, will lead back to the same room, but again the doors will be randomly permuted. If, whenever Tiger is in the room, he has an equal likelihood of picking any one of the three doors, what is the expected length of time it will take Tiger to escape?

A. 44 B. 68 C. 72 D. 92 E. NOTA

22. Consider the region in the first quadrant bounded by the x- and y-axes and $y = e^{-x^2}$. Find the volume when this region is revolved around the y-axis.

A. $\frac{\pi}{2}$ B. π C. 2π D. π^2 E. NOTA

23. Consider the region in the first quadrant bounded by the x- and y-axes and $y = e^{-x^2}$. Find the volume when this region is revolved around the x-axis. You may need the fact that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$.

A. $\frac{\sqrt{\pi}}{2}$ B. $\sqrt{\frac{\pi}{2}}$ C. $\sqrt{\frac{\pi^3}{8}}$ D. $\sqrt{\frac{\pi^3}{2}}$ E. NOTA

24. Evaluate: $\int_0^{\infty} e^{-x^2} \cos(\sqrt{2}x)\, dx$.

A. $\sqrt{\frac{\pi}{4e}}$ B. $\sqrt{\frac{\pi}{2e}}$ C. $\sqrt{\frac{\pi e}{2}}$ D. $\frac{\sqrt{\pi e}}{2}$ E. NOTA

25. Aaron has a box of 2024 coins. Each coin has a different probability of flipping heads, with the i^{th} coin flipping heads with probability $i/2023$ for $i = 0, 1, \dots, 2023$. Aaron selects a coin from the box uniformly at random, and flips that coin 24 times. It comes up heads on all 24 flips. Which of the following fractions is closest to the probability that the coin will come up heads on Aaron's 25th flip? Assume all flips of the coin are independent.

A. $\frac{23}{24}$ B. $\frac{24}{25}$ C. $\frac{25}{26}$ D. $\frac{26}{27}$ E. $\frac{27}{28}$

26. Define a sequence recursively with $a_{n+2} = a_{n+1} - a_n$, $a_0 = 2$, and $a_1 = 3$.
Find an expression for $f(x) = \sum_{n=0}^{\infty} a_n x^n$ where the series converges.

A. $\frac{2+2x}{1+x-x^2}$ B. $\frac{2+x}{1+x-x^2}$ C. $\frac{2+2x}{1-x+x^2}$ D. $\frac{2+x}{1-x+x^2}$ E. NOTA

27. Find the sum of the number of horizontal and vertical asymptotes of the function

$$f(x) = \frac{x \ln|x-4|}{|x| \ln|x^2 - 16|}$$

A. 2 B. 4 C. 6 D. 8 E. NOTA

28. If $\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(\frac{k}{n^2}\right) = \sin(\theta)$ for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, then find θ.

A. 0 B. $\frac{\pi}{6}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$ E. NOTA

29. If $a + b = 2$ and $a > b$ are both positive real numbers, then evaluate:

$$\int_{\frac{1}{2}}^{1} \frac{1}{x^a (1-x)^b}\, dx$$

A. $\frac{b+2}{a}$ B. $\frac{b+1}{a-1}$ C. $\frac{2}{a}$ D. $\frac{1}{a-1}$ E. NOTA

30. Evaluate: $\lim_{x \to -\infty} \left(1 + \frac{20}{|x|}\right)^{-24x}$.

A. e^{44} B. e^{-44} C. e^{480} D. e^{-480} E. NOTA