1. B
2. C
3. C
4. B
5. A
6. B
7. D
8. B
9. C
10. A
11. A
12. D
13. B
14. A
15. C
16. E
17. B
18. D
19. B
20. B
21. D
22. B
23. C
24. A
25. C
26. D
27. C
28. B
29. D
30. C

1. B $\lim_{x\to\pi} e = e$.
2. C $f(x) = e^{-\frac{(x-1)^2}{8}} \to f'(x) = -\frac{1}{4}(x-1)e^{-\frac{(x-1)^2}{8}} \to f''(x) = -\frac{1}{4}e^{-\frac{(x-1)^2}{8}} + \frac{1}{16}(x-1)^2 e^{-\frac{(x-1)^2}{8}} = 0 \to (x-1)^2 = 4 \to x-1 = \pm 2 \to x = -1,3$. So 10
3. C $\frac{d}{dx}\left[\sin\left(x^3+2x+\frac{\pi}{3}\right)\right] = \cos\left(x^3+2x+\frac{\pi}{3}\right)(3x^2+2) \to$ the answer is $2\cos\left(\frac{\pi}{3}\right) = 1$.
4. B This is an ellipse with foci at (0,3) and (4,0). So twice the distance from the center to a focus is $2c = \sqrt{3^2+4^2} = 5 \to c = \frac{5}{2}$. Further, for this ellipse, the sum of the distances from the foci to the curve is $10 \to (a-c)+(a+c) = 2a = 10 \to a = 5$ is the length of the major axis. Since $c = \sqrt{a^2-b^2} \to \frac{25}{4} = 25 - b^2 \to b = \frac{5\sqrt{3}}{2}$ we have the minor axis as well, so the area is $\pi ab = \pi(5)\left(\frac{5\sqrt{3}}{2}\right) = \frac{25\pi\sqrt{3}}{2}$.
5. A $y^3 + x^2y - x^3 + 3 = 0 \to 3y^2y' + 2xy + x^2y' - 3x^2 = 0 \to y' = \frac{3x^2-2xy}{3y^2+x^2} = \frac{12-4}{3+4} = \frac{8}{7}$.
6. B $y = x^{(2x)^y} \to \ln(y) = (2x)^y \ln(x) \to \ln(ln(y)) = \ln(ln(x)) + y\ln(2x) \to \frac{1}{y\ln(y)}y' = \frac{1}{x\ln(x)} + \ln(2x)\,y' + \frac{y}{x} \to y' = \frac{\frac{1}{x\ln(x)}+\frac{y}{x}}{\frac{1}{y\ln(y)}-\ln(2x)} = \frac{y\ln(y)}{x\ln(x)}\cdot\frac{1+y\ln(x)}{1-y\ln(y)\ln(2x)}$.
7. D In general, $A\cos(\theta) + B\sin(\theta)$ has an amplitude $\sqrt{A^2+B^2}$. So the amplitude is 13, which makes the difference from maximum to minimum 26.
8. B Let $u = \ln(\ln(\ln(x))) \to du = \frac{1}{\ln(\ln(x))}\frac{1}{\ln(x)}\frac{1}{x}$. So $\int_{e^{e^e}}^{\infty} \frac{dx}{x\ln(x)\ln(\ln(x))\ln(\ln(\ln(x)))^{2024}} = \int_1^{\infty}\frac{1}{u^{2024}}du = \left[-\frac{1}{2023}\frac{1}{u^{2023}}\right]_1^{\infty} = \frac{1}{2023}$.
9. C $\int_0^{-24} f(x)dx = -\int_{-24}^{0} f(x)dx = \int_0^{24} f(x)dx = 10$ since the function is odd. So $\int_{-20}^{24} f(x)dx = 6 = \int_{-20}^{20} f(x)dx + \int_{20}^{24} f(x)dx = 0 + \int_{20}^{24} f(x)dx = \int_{20}^{24} f(x)dx$ again since the function is odd. Therefore $\int_0^{20} f(x)dx = \int_0^{24} f(x)dx - \int_{20}^{24} f(x)dx = 10 - 6 = 4$.
10. A $y = ax(a-x) = a^2x - ax^2 \to y' = a^2 - 2ax \to m = a^2 - 4a$. So the equation of the line is $y - 2a^2 + 4a = (a^2-4a)(x-2) \to$ The y-intercept is $y_0 = -2a^2 + 8a + 2a^2 - 4a = 4a \to \frac{dy_0}{dt} = 4\frac{da}{dt} = 12$.
11. A Let $u = \sqrt[3]{1-\sqrt[3]{x}} \to 1 - u^3 = \sqrt[3]{x} \to 1 - 3u^3 + 3u^6 - u^9 = x \to dx = (-9u^2 + 18u^5 - 9u^8)du \to \int_0^1 \sqrt[3]{1-\sqrt[3]{x}}dx = \int_1^0(-9u^3 + 18u^6 - 9u^9)du = \left[\frac{9}{4}u^4 - \frac{18}{7}u^7 + \frac{9}{10}u^{10}\right]_0^1 = \frac{9*35-18*20+9*14}{140} = \frac{81}{140}$.
12. D $\cot(2\theta) = \frac{A-C}{B} = \frac{9-4}{12} = \frac{5}{12} \to \cos(2\theta) = \frac{5}{13} = 1 - 2\sin^2(\theta) \to \sin^2(\theta) = \frac{4}{13} \to \sin(\theta) = \sqrt{\frac{4}{13}}$.

13. B For a tangent line to go through the origin, $2ax_0 = \frac{ax_0^2+b}{x_0} \rightarrow x_0 = \sqrt{\frac{b}{a}}$. The slope is therefore $2a\sqrt{\frac{b}{a}} = 2\sqrt{ab}$. The area is thus $\int_0^{\sqrt{\frac{b}{a}}} ax^2 + b - 2\sqrt{ab}x\,dx = \left[\frac{a}{3}x^3 + bx - \sqrt{ab}x^2\right]_0^{\sqrt{\frac{b}{a}}} = \frac{a}{3}\frac{b\sqrt{b}}{a\sqrt{a}} + b\sqrt{\frac{b}{a}} - \sqrt{ab}\frac{b}{a} = \frac{b\sqrt{ab}}{3a}$.

14. A $m = \frac{d}{dx}\left[2\cosh\left(\frac{x}{2}\right)\right]_{x=\ln(4)} = \sinh\left(\frac{\ln(4)}{2}\right) = \frac{e^{\ln(2)} - e^{\ln\left(-\frac{1}{2}\right)}}{2} = \frac{2-\frac{1}{2}}{2} = \frac{3}{4}$. The line going though $(0,2)$ with slope $-\frac{4}{3}$ is $y = -\frac{4}{3}x + 2$. Its intercepts are $(0,2)$ and $\left(\frac{3}{2}, 0\right)$. The distance between these points is $\sqrt{\frac{9}{4}+4} = \frac{5}{2}$.

15. C $P(5) = P(1,4) + P(2,3) + P(3,2) + P(4,1) = \frac{4}{36} = \frac{1}{9}$. $P(10) = P(4,6) + P(5,5) + P(6,4) = \frac{3}{36} = \frac{1}{12}$. So in order for Romir to win, he must roll a 5 on the first, or not a 5, then Sydney not a 10, then he rolls a 5, etc. So the desired probability is $P = \left(\frac{1}{9}\right) + \left(\frac{8}{9}\right)\left(\frac{11}{12}\right)\left(\frac{1}{9}\right) + \left(\frac{8}{9}\right)^2\left(\frac{11}{12}\right)^2\left(\frac{1}{9}\right) + \cdots = \frac{1}{9}\sum_{k=0}^{\infty}\left(\frac{8}{9}\frac{11}{12}\right)^k = \frac{1}{9}\sum_{k=0}^{\infty}\left(\frac{22}{27}\right)^k = \frac{1}{9}\frac{1}{1-\frac{22}{27}} = \frac{1}{9}\frac{27}{5} = \frac{3}{5}$.

16. E $\lim_{h\to 0}\int_{x-24h}^{x+20h}\frac{f(t)}{h}dt = \lim_{h\to 0}\frac{\int_0^{x+20h} f(t)dt + \int_{x-24h}^{0} f(t)dt}{h} = \lim_{h\to 0}\frac{\int_0^{x+20h} f(t)dt - \int_0^{x-24h} f(t)dt}{h} = 44\lim_{h\to 0}\frac{\int_0^{x+20h} f(t)dt - \int_0^{x-24h} f(t)dt}{44h} = 44\frac{d}{dx}\left[\int_0^x f(t)dt\right] = 44f(x)$.

17. B Let V be the event that a person is vaccinated, and let H be the event that a person is hospitalized. Then $P(V) = 0.9$ and $P(V|H) = 0.6$. Let $x = P(H|V)$, so that $P(H|{\sim}V) = Kx$. Then by Bayes' Law, $P(V|H) = \frac{P(V)P(H|V)}{P(V)P(H|V) + P({\sim}V)P(H|{\sim}V)} \rightarrow 0.6 = \frac{0.9x}{0.9x + 0.1Kx} = \frac{9}{9+K} \rightarrow 5.4 + 0.6K = 9 \rightarrow K = \frac{3.6}{0.6} = 6$.

18. D Let r be the radius of the marble. The amount of water needed will be the volume of the cylinder filled to a height of $2r$ minus the volume of the sphere, so $V = \pi 24^2(2r) - \frac{4}{3}\pi r^3 \rightarrow V' = 2\pi(24)^2 - 4\pi r^2 = 0 \rightarrow r^2 = \frac{1}{2}(24)^2 \rightarrow r = 12\sqrt{2}$.

19. B Let $k \geq 2$ be the number of consecutive odd integers. Then an integer N that can be expressed this way if $N = (2n+1) + (2n+3) + \cdots + (2n+2k-1) = 2nk + \sum_{i=1}^{k}(2i-1) = 2nk + 2\frac{k(k+1)}{2} - k = 2nk + k^2 = k(2n+k)$. Therefore, when N is even, either k is even $\rightarrow 2n+k$ is even OR $2n+k$ is even $\rightarrow k$ is even. Thus, N must be a multiple of four if it is even, and 2022 does NOT have this property. Note that if N is odd it must be composite, but none of these numbers are prime so everything else works.

20. B $P\left(|X| < \sqrt{3}\right) = P\left(-\sqrt{3} < X < \sqrt{3}\right) = P\left(-\sqrt{3} < 3\tan(\theta) < \sqrt{3}\right) = P\left(\arctan\left(-\frac{\sqrt{3}}{3}\right) < \theta < \arctan\left(\frac{\sqrt{3}}{3}\right)\right) = P\left(-\frac{\pi}{6} < \theta < \frac{\pi}{6}\right) = \frac{\frac{\pi}{6} - \left(-\frac{\pi}{6}\right)}{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)} = \frac{1}{3}$.

21. D Let X be the amount of time it takes Tiger to escape. Then $E[X] = E[X|Door\ 1]P(Door\ 1) + E[X|Door\ 2]P(Door\ 2) + E[X|Door\ 3]P(Door\ 3) = \frac{1}{3}(20) + \frac{1}{3}(24 + E[X]) + \frac{1}{3}(48 + E[X]) \to E[X] = \frac{20+24+48}{3} + \frac{2}{3}E[X] \to E[X] = 92$.

22. B $V = 2\pi \int_0^\infty xe^{-x^2}dx = 2\pi \int_0^\infty \frac{1}{2}e^{-u}du = \pi[-e^{-u}]_{u=0}^{u\to\infty} = \pi$.

23. C $V = \pi \int_0^\infty \left(e^{-x^2}\right)^2 dx = \pi \int_0^\infty e^{-2x^2}dx = \pi \int_0^\infty \frac{1}{\sqrt{2}}e^{-u^2}du = \frac{\pi}{\sqrt{2}}\left(\frac{\sqrt{\pi}}{2}\right) = \sqrt{\frac{\pi^3}{8}}$.

24. A First consider $I_n = \int_0^\infty x^{2n}e^{-x^2}dx$. Note that $I_0 = \frac{\sqrt{\pi}}{2}$ based on the information from the previous question. Then using integration by parts, $I_n = \int_0^\infty x^{2n}e^{-x^2}dx = \left[\frac{1}{2n+1}x^{2n+1}e^{-x^2}\right]_0^{\to\infty} - \int_0^\infty \frac{1}{2n+1}x^{2n+1}(-2x)e^{-x^2}dx = \frac{2}{2n+1}\int_0^\infty x^{2n+2}e^{-x^2}dx = \frac{2}{2n+1}I_{n+1} \to I_n = \frac{2n-1}{2}I_{n-1} \to I_n = \frac{2n(2n-1)}{4n}I_{n-1} = \frac{2n(2n-1)(2n-2)(2n-3)}{4^2n(n-1)}I_{n-2} = \cdots = \frac{(2n)!}{4^n n!}I_0 = \frac{(2n)!}{4^n n!}\frac{\sqrt{\pi}}{2}$. Thus: $\int_0^\infty e^{-x^2}\cos(\sqrt{2}x)\,dx = \int_0^\infty e^{-x^2}\sum_{n=0}^\infty \frac{(-1)^n(\sqrt{2}x)^{2n}}{(2n)!}dx = \sum_{n=0}^\infty \frac{(-1)^n(\sqrt{2})^{2n}}{(2n)!}\int_0^\infty x^{2n}e^{-x^2}dx = \sum_{n=0}^\infty \frac{(-1)^n 2^n}{(2n)!}I_n = \sum_{n=0}^\infty \frac{(-1)^n 2^n}{(2n)!}\frac{(2n)!}{4^n n!}\frac{\sqrt{\pi}}{2} = \frac{\sqrt{\pi}}{2}\sum_{n=0}^\infty \frac{\left(-\frac{1}{2}\right)^n}{n!} = \sqrt{\frac{\pi}{4e}}$.

25. C If H_n is the event that the first n flips are heads, then we want $P(H_{25}|H_{24}) = \frac{P(H_{25}\cap H_{24})}{P(H_{24})} = \frac{P(H_{25})}{P(H_{24})}$. In general, $P(H_n) = \sum_{i=0}^{2023} P(H_n|Coin\ i)P(Coin\ i) = \sum_{i=0}^{2023}\left(\frac{i}{2023}\right)^n\left(\frac{1}{2024}\right) \approx \lim_{N\to\infty}\sum_{i=0}^{N}\left(\frac{i}{N}\right)^n\left(\frac{1}{N}\right) = \int_0^1 x^n dx = \frac{1}{n+1}$. So $\frac{P(H_{25})}{P(H_{24})} \approx \frac{\frac{1}{26}}{\frac{1}{25}} = \frac{25}{26}$.

26. D $f(x) = \sum_{n=0}^\infty a_n x^n = 2 + 3x + \sum_{n=2}^\infty a_n x^n = 2 + 3x + \sum_{n=0}^\infty a_{n+2}x^{n+2} = 2 + 3x + \sum_{n=0}^\infty (a_{n+1} - a_n)x^{n+2} = 2 + 3x + \sum_{n=0}^\infty a_{n+1}x^{n+2} - \sum_{n=0}^\infty a_n x^{n+2} = 2 + 3x + x\sum_{n=0}^\infty a_{n+1}x^{n+1} - x^2\sum_{n=0}^\infty a_n x^n = 2 + 3x + x(f(x) - 2) - x^2 f(x) = 2 + x + f(x)(x - x^2) \to f(x)(1 - x + x^2) = 2 + x \to f(x) = \frac{2+x}{1-x+x^2}$

27. C $\lim_{x\to\infty}\frac{x\ln|x-4|}{|x|\ln|x^2-16|} = \lim_{x\to\infty}\frac{\ln|x-4|}{\ln|x-4|+\ln|x+4|} = \frac{1}{2}$ and $\lim_{x\to-\infty}\frac{x\ln|x-4|}{|x|\ln|x^2-16|} = \lim_{x\to-\infty} -\frac{\ln|x-4|}{\ln|x-4|+\ln|x+4|} = -\frac{1}{2}$, so there are two horizontal asymptotes. Note that $\lim_{x\to 0^\pm}\frac{x\ln|x-4|}{|x|\ln|x^2-16|} = \lim_{x\to 0^\pm} \pm\frac{\ln|x-4|}{\ln|x^2-16|} = \pm\frac{1}{2}$ so this is not a vertical asymptote, and $\lim_{x\to\pm 4}\frac{x\ln|x-4|}{|x|\ln|x^2-16|} = \lim_{x\to\pm 4} \pm\frac{\ln|x-4|}{\ln|x^2-16|} = \lim_{x\to\pm 4} \pm\frac{\frac{1}{x-4}}{\frac{1}{x-4}+\frac{1}{x+4}} \to \lim_{x\to 4}\frac{1}{1+\frac{x-4}{x+4}} = 1$ & $\lim_{x\to -4} -\frac{\frac{x+4}{x-4}}{\frac{x+4}{x-4}+1} = 0$ so these are not vertical asymptotes. However, there will be vertical asymptotes when $\ln|x^2 - 16| = 0 \to x^2 - 16 = \pm 1 \to x = \pm\sqrt{15}, \pm\sqrt{17}$. So, the total is 6,

28. B From Taylor's Theorem, $x - \frac{x^3}{6} \le \sin(x) \le x \to \frac{k}{n^2} - \frac{k^3}{6n^6} \le \sin\left(\frac{k}{n^2}\right) \le \frac{k}{n^2} \to \sum_{k=1}^n\left(\frac{k}{n^2} - \frac{k^3}{6n^6}\right) \le \sum_{k=1}^n \sin\left(\frac{k}{n^2}\right) \le \sum_{k=1}^n \frac{k}{n^2} \to \frac{n(n+1)}{2n^2} - \frac{n^2(n+1)^2}{24n^6} \le \sum_{k=1}^n \sin\left(\frac{k}{n^2}\right) \le$

$\frac{n(n+1)}{2n^2}$. Since the lower and upper bounds both limit to ½, by the Squeeze Theorem so does the desired limit. $\sin(\theta) = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6}$.

29. D $\int_{\frac{1}{2}}^{1} \frac{1}{x^a(1-x)^b}\,dx = \int_{\frac{1}{2}}^{1} \frac{1}{x^{a+b}} \frac{x^b}{(1-x)^b}\,dx = \int_{\frac{1}{2}}^{1} \frac{1}{x^2}\left(\frac{x}{1-x}\right)^b dx$. Let $u = \frac{x}{1-x} \rightarrow x = \frac{u}{u+1} = 1 - \frac{1}{u+1} \rightarrow dx = \frac{1}{(u+1)^2}du$. Then $\int_{\frac{1}{2}}^{1} \frac{1}{x^2}\left(\frac{x}{1-x}\right)^b dx = \int_1^\infty \left(\frac{u+1}{u}\right)^2 (u)^b \frac{1}{(u+1)^2}du = \int_1^\infty \frac{1}{u^{2-b}}du = \int_1^\infty \frac{1}{u^a}du = \left[\frac{u^{1-a}}{1-a}\right]_{u=1}^{u\rightarrow\infty}$. Since $a > b \rightarrow 2a > a + b = 2 \rightarrow a > 1$, $\left[\frac{u^{1-a}}{1-a}\right]_{u=1}^{u\rightarrow\infty} = 0 - \frac{1}{1-a} = \frac{1}{a-1}$.

30. C $\lim_{x\rightarrow-\infty}\left(1 + \frac{20}{|x|}\right)^{-24x} = \lim_{x\rightarrow\infty}\left(1 + \frac{20}{x}\right)^{24x} = e^{(20)(24)} = e^{480}$.