1. In the MAΘ club, 6 people are wearing shorts and 4 people are wearing pants. If we select 3 people to make a team, what is the probability that at least one of them are wearing pants?

A. $\frac{5}{6}$ B. $\frac{9}{10}$ C. $\frac{14}{15}$ D. $\frac{98}{125}$ E. NOTA

2. An ellipse has foci at the points $(3,0)$ and $(0,5)$. If one of the $x-$ intercepts is at the origin, the other x-intercept has x-coordinate $\frac{m}{n}$ in simplest form. Find $m+n$.

A. 55 B. 59 C. 65 D. 69 E. NOTA

3. Evaluate

$$\int_0^{\frac{\pi}{4}} \sin^3(x)\cos^3(x)\ dx$$

A. $\frac{1}{24}$ B. $\frac{1}{18}$ C. $\frac{1}{12}$ D. $\frac{1}{6}$ E. NOTA

4. Evaluate

$$\lim_{x\to\infty}\left(\sqrt[3]{x^3-x^2+4}-\sqrt[3]{x^3+5x^2-x}\right)$$

A. -6 B. -4 C. -2 D. 0 E. NOTA

5. Let $A=\lim_{x\to 0}\frac{\sin(x^2)-x^2}{x^4},\ B=\lim_{x\to 0}\frac{\sin^2(x)-x^2}{x^4}$. Evaluate $A-B$.

A. $-\frac{1}{3}$ B. $-\frac{1}{6}$ C. $\frac{1}{6}$ D. $\frac{1}{3}$ E. NOTA

6. A positive integer $\overline{a_1a_2a_3 \ldots a_n}$ is increasing if $a_1<a_2<a_3<\cdots<a_n$ and is decreasing if $a_1>a_2>a_3>\cdots>a_n$. Let A be the number of increasing integers and let B the number of decreasing integers. Find $|A-B|$.

A. 0 B. 511 C. 512 D. ∞ E. NOTA

For the next two questions, let $f(x) = \frac{x^3}{x+1}$.

7. Evaluate

$$\int_0^1 f(x)dx$$

A. $\frac{2}{3} - \ln(2)$ B. $\frac{2}{3} + \ln(2)$ C. $\frac{5}{6} - \ln(2)$ D. $\frac{5}{6} + \ln(2)$ E. NOTA

8. Find the integer closest to $f(25)$.

A. 575 B. 576 C. 600 D. 601 E. NOTA

9. Approximate the root of

$$f(x) = \int_0^x t(t-2)\ dt$$

using two iterations of newton's method starting at $x_0 = 3$

A. 2 B. $\frac{8}{3}$ C. $\frac{10}{3}$ D. 4 E. NOTA

10. Edward is sending his love to Mr. Lu with the heart $r = 1 - \sin(\theta)$! Mr. Lu decided to test Edward if he knows everything about the function $r = 1 - \sin(\theta)$. Help Edward pass this test. What is the shape of the graph $r = \frac{1}{1-\sin(\theta)}$?

A. ellipse B. hyperbola C. parabola D. line E. NOTA

11. Find the difference between the maximum and minimum y-coordinate of the graph $r = 1 - \sin(\theta)$.

A. $2 + \frac{\sqrt{2}}{2}$ B. $\frac{9}{4}$ C. $\frac{5}{2}$ D. 3 E. NOTA

12. Find the difference between the maximum and the minimum x-coordinate of the graph $r = 1 - \sin(\theta)$.

A. $\frac{1}{2}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{3}{2}$ D. $\frac{3\sqrt{3}}{2}$ E. NOTA

13. Find the area common to both $r = 1 + \sin(\theta)$ and $r = 1 - \sin(\theta)$.
A. $\frac{\pi}{2} - 1$ B. $\pi - 2$ C. $\frac{3\pi}{2} - 4$ D. $2\pi - 6$ E. NOTA

14. Polynomial $x^3 - 5x^2 + 6x - k$ has the property that one of the roots is the product of the two others. Find the sum of all possible values of k.
A. 6 B. 12 C. 18 D. 24 E. NOTA

15. Triangle ABC with side lengths $3, 4, 5$ has circumcircle O. The side with length 3 splits O into region A and region B with region A containing the center of circumcircle O. One circle of maximum area that lies in region A is tangent to the side with length 3 and O. A second circle of maximum area that lies in region B is also tangent to the side with length 3 and O. Find the positive difference between the area of the two circles.
A. 4π B. 5π C. 6π D. 7π E. NOTA

16. Let $p_1 = 2, p_2 = 3, \ldots, p_{25} = 97$ be the first 25 primes. For how many $1 \leq i \leq 23$ is the average of p_i, p_{i+1}, p_{i+2} an integer?
A. 1 B. 2 C. 3 D. 4 E. NOTA

17. Timmy doesn't like that only Edward is getting tested, so he decides to have a battle with Edward over Mr. Lu. For this battle, Mr. Lu makes a special coin for them.

The special coin has the property that it has 50% chance of landing on heads, 40% of landing on tails, and a 10% chance of landing on the side. Timmy wins if the coin lands on heads, Edward wins if the coin lands on tails, and they flip the coin again if the coin lands on the side. What is the probability that Timmy wins?
A. $\frac{5}{9}$ B. $\frac{3}{5}$ C. $\frac{3}{4}$ D. $\frac{2}{3}$ E. NOTA

18. What is the expected number of coin flips to end this game? *Note: The game is referring to the game described in the previous question.*
A. $\frac{7}{2}$ B. $\frac{9}{2}$ C. 5 D. $\frac{11}{2}$ E. NOTA

19. Polynomial $f(x), g(x)$ satisfies

$$\frac{d}{dx}\big(f(x)+g(x)\big)=2x+4, \frac{d}{dx}\big(f(x)g(x)\big)=6x^2+4x+4, f(0)=4, g(0)=-2$$

Find $f(1)+g(-1)$.

A. -2 B. 1 C. 4 D. 5 E. NOTA

20. Let R be the region bounded by $x=0, x=1, y=0, y=\sqrt{4-x^2}$. Find the area of R.

A. $\frac{1}{2}+\frac{\pi}{3}$ B. $\frac{\sqrt{3}}{2}+\frac{\pi}{3}$ C. $1+\frac{2\pi}{3}$ D. $\sqrt{3}+\frac{2\pi}{3}$ E. NOTA

21. Let the answer of the previous question be A. Let V denote the volume of the solid formed when R is rotated around $x=2$. Then $\frac{V}{2\pi}$ can be written as

A. $2A-\frac{8}{3}+\sqrt{3}$ B. $A-\frac{8}{3}+\sqrt{3}$

C. $2A-\frac{4}{3}+\frac{\sqrt{3}}{2}$ D. $A-\frac{4}{3}+\frac{\sqrt{3}}{2}$ E. NOTA

22. Let $s_n(k)$ be the sum of digits of k when written in base n. Find the number of positive integers less than 2024 satisfying $s_2(k)=3$.

A. 162 B. 163 C. 164 D. 165 E. NOTA

23. Given $s_8(n)=2024$, which of the following is a possible value for $s_8(n+19)$?

A. 1168 B. 1278 C. 1331 D. 1470 E. NOTA

24. Evaluate

$$\int_{\frac{1}{2}}^{2} |\ln(x)|\,dx$$

A. $\frac{3\ln(2)}{2}-\frac{1}{2}$ B. $\ln(2)-\frac{1}{2}$ C. $\frac{5\ln(2)}{2}-\frac{3}{2}$ D. $3\ln(2)-\frac{1}{2}$ E. NOTA

25. If the equation $|\ln x| = ax + b$ has 3 roots in the ratio 1: 2: 3, find the sum of the three roots.

A. $6\ln(2)$ B. $3\ln(6)$ C. $3\sqrt{3}$ D. $\frac{12}{e}$ E. NOTA

26. A number $\overline{a_1 a_2 a_3 \ldots a_n}$ is pyramid if for some $1 \le i \le n$, $a_1 < a_2 < \cdots < a_i$ and $a_i > a_{i+1} > \cdots > a_n$. For example, 7, 192, 450 are pyramid whereas 11, 1324, 3365 are not. Find the number of 3-digit pyramid numbers.

A. 428 B. 432 C. 480 D. 484 E. NOTA

27. It is well known that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $|x| < 1$, then

$$\sum_{n=0}^{\infty} \frac{n^2}{5^n} = \frac{q}{r}$$

in simplest form. Find $q + r$.

A. 31 B. 37 C. 41 D. 47 E. NOTA

28. In triangle ABC, points X, Y, Z lie on AB, BC, CA respectively such that $\frac{AX}{XB} = \frac{BY}{YC} = \frac{CZ}{ZA} = k$. Given the area of triangle XYZ is $\frac{3}{4}$ the area of ABC, the smallest value of k can written in the form $a - \sqrt{b}$ for positive integers a, b. Find $a + b$.

A. 29 B. 30 C. 31 D. 32 E. NOTA

29. Polynomial $f(x)$ satisfies $f(x) + f(y) = f(x + y) + xy$. Given $f'(1) = 1$, evaluate $f(1)$.

A. 1 B. $\frac{3}{2}$ C. 2 D. $\frac{5}{2}$ E. NOTA

30. Function $f(x)$ satisfies $f(x) + 2f\left(\frac{1}{x}\right) = x + 3$ for all reals except for $x = 0$. The largest root of the function can be written in the form $\frac{a+\sqrt{b}}{c}$ in simplest form for positive integers a, b, c. Find $a + b + c$.

A. 19 B. 20 C. 21 D. 22 E. NOTA