1. A
2. B
3. A
4. C
5. D
6. B
7. C
8. D
9. E
10. C
11. B
12. D
13. C
14. D
15. B
16. B
17. A
18. E
19. E
20. B
21. A
22. D
23. A
24. A
25. C
26. E
27. D
28. A
29. B
30. D

1.	A	$1 - P(\text{all 3 no pants}) = 1 - \frac{6\ C\ 3}{10\ C\ 3} = 1 - \frac{20}{120} = \frac{5}{6}$
2.	B	A property of an ellipse is that the sum of the distance from the foci is a constant. If $(x, 0)$ is the x −intercept, $$8 = \sqrt{x^2 + 25} + (x - 3)$$ $$11 - x = \sqrt{x^2 + 25}$$ $$x^2 - 22x + 121 = x^2 + 25$$ $$\rightarrow x = \frac{48}{11}$$ $$48 + 11 = 59$$
3.	A	Since $\cos^2(x) = 1 - \sin^2(x)$ $$\int_0^{\frac{\pi}{4}} \cos(x)(\sin^3(x) - \sin^5(x))\, dx$$ Setting $u = \sin(x)$, $$I = \int_0^{\frac{\sqrt{2}}{2}} u^3 - u^5\, du = \frac{u^4}{4} - \frac{u^6}{6} = \frac{1}{16} - \frac{1}{48} = \frac{1}{24}$$
4.	C	As $x \rightarrow \infty, x^3 - x^2 + 4 \sim \left(x - \frac{1}{3}\right)^3, x^3 + 5x^2 - x \sim \left(x + \frac{5}{3}\right)^3$ Thus, the limit is approximately $\left(x - \frac{1}{3}\right) - \left(x + \frac{5}{3}\right) = -2$
5.	D	Subtracting the two limits gives $$\lim_{x \to 0} \frac{\sin(x^2) - \sin^2(x)}{x^4}$$ Using the Taylor series approximation of $\sin(x) \sim x - \frac{x^3}{6}$, $$\sin(x^2) \sim x^2 - \frac{x^6}{6}$$

		$$\sin^2(x) \sim x^2 - \frac{x^4}{3}$$ $$\lim_{x\to 0} \frac{\frac{x^4}{3}}{x^4} = \frac{1}{3}$$
6.	B	If $a_1 a_2 \ldots a_n$ is an decreasing integer, $a_n a_{n-1} \ldots a_1$ has to be a increasing integer except for when $a_n = 0$ There are a total 2^9 subsets of the set $\{1,2,3,4,5,6,7,8,9\}$ and each subset corresponds to a decreasing number ending with 0. For example the subset $\{1,3,5,7\} \rightarrow 75310$. However, the empty subset does not create a number, thus $$512 - 1 = 511$$
7.	C	$$\frac{x^3}{x+1} = x^2 - x + 1 - \frac{1}{x+1}$$ $$\int_0^1 x^2 - x + 1 - \frac{1}{x+1} = \frac{x^3}{3} - \frac{x^2}{2} + x - \ln(x+1)$$ $$= \frac{1}{3} - \frac{1}{2} + 1 - \ln(2) = \frac{5}{6} - \ln(2)$$
8.	D	Plugging in 25 to $x^2 - x + 1 - \frac{1}{x+1}$, $$f(x) = 625 - 25 + 1 - \frac{1}{26} \sim 601$$
9.	E	$$x_1 = 3 - \frac{f(3)}{f'(3)} = 3$$ $$x_2 = 3 - \frac{f(3)}{f'(3)} = 3$$
10.	C	The polar equation has eccentricity 1, thus a parabola
11.	B	$$y = r\sin(\theta) = \sin(\theta) - \sin^2(\theta)$$ The maximum is at the vertex, $\frac{1}{4}$ The minimum is clearly at $\theta = \frac{3\pi}{2}$, -2

		Difference = $\frac{9}{4}$
12.	D	$x = r\cos(\theta) = \cos(\theta) - \sin(\theta)\cos(\theta) = \cos(\theta) - \frac{1}{2}\sin(2\theta)$. Taking the derivative gives $-\sin(\theta) - \cos(2\theta) = -\sin(\theta) - 1 + 2\sin^2(\theta)$ which is 0 at $\sin(\theta) = -\frac{1}{2}, 1$ Plugging in these values, the minimum is $-\frac{3\sqrt{3}}{4}$, maximum is $\frac{3\sqrt{3}}{4}$
13.	C	Due to symmetry the area common will be 4 times the area of $1 - \sin(\theta)$ from 0 to $\frac{\pi}{2}$. $$\int_0^{\frac{\pi}{2}} (1 - \sin(\theta))^2 = 1 - 2\sin(\theta) + \sin^2(\theta) = \frac{\pi}{2} - 2 + \frac{\pi}{4} = \frac{3\pi}{4} - 2$$ $$\left(\frac{3\pi}{4} - 2\right) * 4 * \frac{1}{2} = \frac{3\pi}{2} - 4$$
14.	D	Let the roots be a, b, ab. From vietas, $$a + b + ab = 5$$ $$ab + ab(a + b) = 6$$ Let $a + b = s, ab = p$ $$s + p = 5$$ $$p + sp = 6 \rightarrow p + p(5 - p) = 6$$ $$p^2 + 6p + 6 = 0$$ $$p = -3 \pm \sqrt{3}$$ Since $k = a^2b^2 = p^2$ $$k = 12 \pm 6\sqrt{3}$$ Sum = 24

15.	B	Let $AB = 3, BC = 4, AC = 5.$ Let M be the midpoint of AB. The diameter of the larger circle is $\frac{5}{2} + 2 = \frac{9}{2}$ The diameter of the smaller circle is $\frac{5}{2} - 2 = \frac{1}{2}$ $$\frac{81 - 1}{16} = 5$$
16.	B	Since $p_i + p_{i+1} + p_{i+2} = 0 \ (mod\ 3)$ Either one of the three has to be a multiple of 3 or $p_i = p_{i+1} = p_{i+2} \ (mod\ 3)$ Since there is only multiple of 3 that is prime (3,5,7) is a triplet and for the second case, we only have to check $(a, a + 6, a + 12)$ since all primes are odd other than 2. Checking this case, there is only 1 triplet $(47, 53, 59)$.
17.	A	The ratio of the probability that Timmy wins to the probability that Edward wins is proportional to their chances, $$\frac{5}{10} : \frac{4}{10} = 5 : 4$$ Thus, $\frac{5}{5+4} = \frac{5}{9}$
18.	E	Expected number $= \frac{1}{p}$ since this is a geometric distribution, $\frac{1}{\frac{9}{10}} = \frac{10}{9}$
19.	E	$$f(x) + g(x) = x^2 + 4x + c_1$$ $$f(x)g(x) = 2x^3 + 2x^2 + 4x + c_2$$ Plugging in $x = 0$, $$c_1 = 2$$ $$c_2 = -8$$ Also since the product is degree 3 and sum is degree 2, either f is a quadratic and g is linear or vice versa. Case 1. $f(x) = x^2 + ax + 4, g(x) = 2x - 2$

		If this is true, $a = 2$. Since $(x^2 + 2x + 4)(2x - 2) = (2x^3 + 2x^2 + 4x - 8)$, this case works. Case 2. $f(x) = 2x + 4, g(x) = x^2 + ax - 2$ If this is true $a = 2$, Since $(x^2 + 2x - 2)(2x + 4) \neq 2x^3 + 2x^2 + 4x - 8$, this case does not work. $$f(1) = 7, g(-1) = -4$$ $$\rightarrow 3$$
20.	B	$$\int_0^1 \sqrt{4 - x^2}$$ Letting $x = 2\sin(u)$ $$\int_0^{\frac{\pi}{6}} 4\cos^2(u) = 2 + 2\cos(2u) = 2u + \sin(2u) = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$$
21.	A	Using the shell method $$\frac{V}{2\pi} = \int_0^1 (2 - x)\sqrt{4 - x^2} = 2A - \int_0^1 x\sqrt{4 - x^2}$$ The second integral is a simple u-sub with $u = x^2$ $$\frac{1}{2}\int_0^1 \sqrt{4 - u} = \frac{1}{3}(2 - \sqrt{3}) = \frac{8}{3} - \frac{3\sqrt{3}}{3} = \frac{8}{3} - \sqrt{3}$$
22.	D	Since $2047 = 2^{11} - 1$ has 11 digits, we just have to choose 3 of the 11 digits $$11\ C\ 3 = 165$$
23.	A	Similarly to base 10, the sum of digits modulo 7 is conserved in base 8. $$8^n a_n + 8^{n-1} a_{n-1} + \cdots 8a_1 + a_0 = a_n + a_{n-1} + \cdots + a_1 + a_0\ (mod\ 7)$$ $$2024 + 19 = 6\ (mod\ 7)$$ Out of the answer choices 1168 is the only number mod 7.
24.	A	$$\int_{\frac{1}{2}}^1 -\ln(x) + \int_1^2 \ln(x) = -\frac{1}{2}\ln(2) + \frac{1}{2} + 2\ln(2) - 1 = \frac{3}{2}\ln(2) - \frac{1}{2}$$

25.	C	Graphing $\|\ln(x)\|$, the smallest root must be between 0 and 1 and the other two must be greater than that since it crosses through a linear graph 3 times. Let the 3 roots be $r, 2r, 3r$. $$-\ln(r) = ar + b$$ $$\ln(2r) = 2ar + b$$ $$\ln(3r) = 3ar + b$$ Subtracting the last 2 equations, $$\ln\left(\frac{3}{2}\right) = ar$$ Subtracting the first 2 equations $$\ln(2r^2) = ar = \ln\left(\frac{3}{2}\right)$$ Thus, $r^2 = \frac{3}{4} \to r = \frac{\sqrt{3}}{2} \to 6r = 3\sqrt{3}$
26.	E	Case 1: first digit is the max If the first digit is n, there are n choose 2 ways to get the last 2 digits Using hockey stick $2\ C\ 2 + 3\ C\ 2 \ldots 9\ C\ 2 = 10\ C\ 3 = 120$ Case 2: second digit is the max If the second digit is n, there are $(n-1)n$ ways to choose the first and last digit Note that this is just twice the first case $= 240$ Case 3: third digit is the max If the last digit is n, there are $n - 1$ choose 2 ways to get the first 2 digits Using hockey stick, the sum is $9\ C\ 3 = 84$ $$120 + 240 + 84 = 444$$
27.	D	To get to the form $n^2/5^n$ we will take the derivative of the equation given $$1 + 2x + 3x^2 \ldots = \frac{1}{(1-x)^2}$$

		Multiplying both sides by x, $$x+2x^2+3x^3\ldots=\frac{x}{(1-x)^2}$$ Taking the derivative of both sides, $$1+4x+9x^2\ldots=\frac{1+x}{(1-x)^3}$$ Substituting $x=\frac{1}{5}$ gets $1+\frac{4}{5}+\frac{9}{25}\ldots=\frac{75}{32}$ which is 5 times the equation given, $$15+32=47$$
28.	A	Using law of sines on triangle XAZ, $$[XAZ]=AX\cdot AZ\cdot\frac{\sin(\theta)}{2}$$ $$[ABC]=AB\cdot AC\cdot\frac{\sin(\theta)}{2}$$ $$\frac{[XAZ]}{[ABC]}=\frac{AX}{AB}\cdot\frac{AZ}{AC}=\frac{k}{(k+1)^2}$$ Using this on triangle YBZ, ZCX $$\frac{3k}{(k+1)^2}=\frac{1}{4}$$ $$\rightarrow k^2-10k+1=0\rightarrow k=5\pm\sqrt{24}$$
29.	B	Substituting $x=0, y=0, f(0)=0$ Also, using the definition of derivatives, $$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{f(h)-xh}{h}=-x+c$$ Thus, $f(x)=-\frac{x^2}{2}+cx$ Since $f'(1)=1, -1+c=1\rightarrow c=2$

		$$f(x) = -\frac{x^2}{2} + 2x \rightarrow f(1) = \frac{3}{2}$$
30.	D	Substituting $\frac{1}{x}$ for x $$f\left(\frac{1}{x}\right) + 2f(x) = \frac{1}{x} + 3$$ Solving this system of equation for $f(x)$, $$f(x) + 2\left(\frac{1}{x} + 3 - 2f(x)\right) = x + 3$$ $$3f(x) = \frac{2}{x} + 3 - x = \frac{-x^2 + 3x + 2}{x}$$ The roots are at $$\frac{3 \pm \sqrt{17}}{2} \rightarrow 22$$