1. B
2. B
3. C
4. D
5. B
6. B
7. B
8. C
9. A
10. E
11. B
12. A
13. B
14. D
15. D
16. D
17. A
18. C
19. A
20. D
21. A
22. C
23. D
24. E
25. D
26. C
27. D
28. A
29. C
30. E

1.	B	2023 is divisible by 7 2021 is divisible by 43 2019 is divisible by 3 2017 is prime.
2.	B	Solving for x in the equation $.2 * 90 + .3 * 70 + .1 * 100 + .2 * 85 + .2 * x \geq 80$ gives $x \geq 70$, thus Andrew's minimum score on the final is $x = 70$.
3.	C	Trivially, A and B must be large and D must be small. (Try other values if you don't trust me) Logically, E must be the larger value out of C and E. Putting all this together, the answer is $\frac{7 \cdot 6 - 4}{3} + 5 = \frac{53}{3}$
4.	D	Let B$C = x$ $\rightarrow BE = \frac{x}{2}, BD = \frac{5x}{8}$ by angle bisector theorem. $DE = \frac{x}{8} = \frac{3}{2} \rightarrow \therefore x = 12$.
5.	B	Substituting $x = 2, -2$ gives $$f(2) + g(2) = 21$$ $$f(-2) + g(-2) = -f(2) + g(2) = -\frac{1}{3}$$ $$\rightarrow 2f(2) = \frac{64}{3} \rightarrow f(2) = \frac{32}{3}.$$
6.	B	Let x be Mimo's cat age. Katie's age corresponds to $16 + x$, as Mimo was 0 years old when Katie was exactly 16. Mimo's age $M(x)$ is as following. $M(1) = 15, M(2) = 24, M(x) = 16 + 4x$ when $x \geq 2$. Solving for $16 + 4x = 2(16 + x)$ gives $x = 8 \rightarrow 16 + x = 24$. *Note that when $x < 2$, Katie's age is never exactly twice of Mimo's.
7.	B	The system $$x \equiv 1 \equiv -5 \ (mod\ 6)$$ $$x \equiv 2 \equiv -5 \ (mod\ 7)$$ $$x \equiv 6 \equiv -5 \ (mod\ 11)$$ has solution $x = -5 \left(mod\ (6 * 7 * 11 = 462)\right) = 457$.
8.	C	Sum of all possible products is $(1 + 2 + 3 + 4 + 5 + 6)(1 + 2 + 3 + 4 + 5 + 6) = 441$. The expected product is $\frac{441}{36} = \frac{49}{4}$.
9.	A	The remainder when an integer n is divided by 9 is equal to the sum of digits of n. $1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 = 165 = 3$ modulo 9.
10.	E	$$f(x) = 2x + 3 + \frac{4}{2x + 1}$$ Plugging in 2024 gives $$f(2024) = 4051 + \frac{4}{4049} \rightarrow [f(2024)] = 4051.$$

11.	B	Letting $u = 2x + 1$, we get $$f(u) = u + 2 + \frac{4}{u} \geq 2 + 2\sqrt{u \cdot \frac{4}{u}} = 6$$ when $u > 0$. Note that this also tells us that if u is negative, the maximum value of $f(u)$ is $2 - 2 = 0$. Thus, the smallest positive value in the range of $f(x)$ is 6.
12.	A	Let b and c be the side lengths with reversed digits. Let $b = 10p + q, c = 10q + p$, and $$a^2 = c^2 - b^2 = (c + b)(c - b) = 11(p + q)9(q - p).$$ So $99\|a^2 \to 33\|a$ If $a = 33, b = 56, c = 65$ satisfies the conditions If $a = 66, (p + q)(q - p) = 44,$ which is impossible since p and q are digits. If $a = 99, s$ince $c > a$, it is impossible to have a 2-digit hypotenuse.
13.	B	The function $\sqrt{-x^2 + 4x - 3}$ has domain [1,3] and the function $\sqrt{-x^2 + 7x - 12}$ has domain [3,4] The only number in both of these domains is 3. This function will be the point (3,0)
14.	D	The amount of bamboo they will eat in one hour is $$\frac{1}{\log_2(2)} + \frac{1}{\log_3(2)} + \frac{1}{\log_4(2)} = \log_2(2) + \log_2(3) + \log_2(4) = \log_2(24)$$ So, the amount of time it will take to eat 10 bamboo treats is $$\frac{10}{\log_2(24)} = \frac{10\log(2)}{\log(24)} \approx \frac{3}{.9 + .477} = \frac{3}{1.377} \approx 2.18.$$ The closest answer choice is 2.2.
15.	D	Assume n has k digits. The information given is $$10^k + n = 8n + 1 \to 10^k - 1 = 7n.$$ Thus, we need $10^k \equiv 1\ (mod\ 7)$. Checking powers of 10, we see the smallest positive value of k is $k = 6$. Therefore $$n = \frac{10^6 - 1}{7} = 142857$$ The sum of the digits is 27.
16.	D	WLOG assume $a \leq b \leq c$. Since the sum is even, $a = 2$. Then the equations become $$b + c = 20,$$ $$2(b + c) + bc = 131,$$ $$\to bc = 91.$$ So $b = 7, c = 13$. $$abc = 182 \to 11$$
17.	A	$$2^{20} - 2^{11} + 1 = (2^{10} - 1)^2 = 1023^2 = 3^2 11^2 31^2$$ $$\to 2 + 2 + 2 = 6$$
18.	C	For 3 lines to split the xy-plane into 6 regions, either i) they intersect at 1 point ii) 2 lines are parallel and the other is a transversal.

		i) $x+2y=5, 2x+y=4 \to (x,y)=(1,2)$ $4*1+k*2=6 \to k=1$ ii) $\frac{1}{2}=\frac{4}{k} \to k=8$ or $\frac{2}{1}=\frac{4}{k} \to k=2.$ The sum is 11.
19.	A	Since triangle CDA and triangle CBE are similar, $\frac{CD}{DA}=\frac{CB}{BE}$ which means $AD=6$. Let $AC=3x$ and $AB=4x$ (the ratio is due to Angle Bisector Theorem). Applying Stewart's Theorem gives $$16x^2\cdot 6+9x^2\cdot 8=14\cdot 8\cdot 6+14\cdot 6^2$$ $$\to x^2=7$$ $$AC^2=9x^2=63.$$
20.	D	Since $A_n=p*q^n+r$, we know that $(A_n-r)=q(A_{n-1}-r)$. The recurrence relation allows us to infer $q=4$. Now we solve for r using the recurrence as well $$\to -3r=-1\to r=\frac{1}{3}$$ $$\to A_n=p\cdot 4^n+\frac{1}{3}$$ Since $A_1=2, 4p+\frac{1}{3}=2\to p=\frac{5}{12}$. $p+q+r=\frac{19}{4}$, giving us a final answer of 23.
21.	A	Dividing the equation by x gives $x-4+\frac{1}{x}=0\to x+\frac{1}{x}=4$. Squaring both sides gives $x^2+\frac{1}{x^2}+2=16\to x^2+\frac{1}{x^2}=14$. Squaring both sides again, we get $x^4+\frac{1}{x^4}+2=196$. $$\therefore x^4+\frac{1}{x^4}=194.$$
22.	C	Let $a_i=2^{p_i}3^{q_i}$ Then the sequence satisfies $$0\le p_1\le p_2\le p_3\le 3$$ $$0\le q_1\le q_2\le q_2\le 5$$ We can perform stars and bars on $(p_1, p_2-p_1, p_3-p_2, 3-p_2)$ since they are all non-negative and they add up to 3 The number of possible sequences of p_n is $\binom{6}{3}=20$. Similarly, the number of possible sequences of q_n is $\binom{8}{3}=56$. The number of possible sequences of a_n is $20*56=1120$.
23.	D	At the sides of the rectangle $(x=\pm p, y=\pm q)$, it will be tangent to the ellipse, meaning the quadratic will have a double root with respect to x or y. i) With respect to x $$(2q)^2-4(2q^2-9)=36-4q^2=0\to y=q=\pm 3$$ ii) With respect to y

		$$(2p)^2-4\cdot 2(p^2-9)=72-4p^2=0\rightarrow x=p=\pm 3\sqrt{2}$$ The area is $6\cdot 6\sqrt{2}=36\sqrt{2}$.
24.	E	Note that the right-hand side is equal to $\frac{a_1a_2\dots a_{n_7}}{7^n-1}$ (just like how we do it in base 10). So first, we have to find the minimal n such that $18\|7^n-1$. It is not hard to check that $n=3$ is the minimal n. Doing mods with 2 and 9 will do the job. So $\frac{7}{18}=\frac{95}{342}=\frac{a_1a_2a_{3_7}}{7^3-1=342}$. Now the only thing left to do is convert 95 in base 7. $$95=98-3=200_7-3_7=164_7$$ We have $a_1=1, a_2=6, a_3=4, n=3$, giving us a final answer of $$1+6+4+3=14.$$
25.	D	In the 5 turns, each panda has to be chosen once with one repeat. There are 4 ways to choose the repeated panda and $\frac{5!}{2!}$ ways to arrange the 5 selections. This gives us a probability of $$\frac{4\cdot\frac{5!}{2!}}{4^5}=\frac{240}{4^5}=\frac{15}{2^6}.$$ So, our final answer is $15+6=21$.
26.	C	WLOG Let the equilateral triangle have side length 2, with vertices $(-1,0)$, $(1,0)$, $(0,\sqrt{3})$. Then, the circumcenter has coordinate $(0,\frac{1}{\sqrt{3}})$. The equation of the line equidistant from the circumcenter and $(-1,0)$ is $$(x+1)^2+y^2=x^2+\left(y-\frac{1}{\sqrt{3}}\right)^2$$ $$\rightarrow 2x+1=-\frac{2y}{\sqrt{3}}+\frac{1}{3}$$ which has an x-intercept of $-\frac{1}{3}$. Thus, by symmetry the area of the region where the pandas are closer is $$3\cdot\frac{\left(\frac{2}{3}\right)^2\sqrt{3}}{4}=\frac{\sqrt{3}}{3}$$ whereas the total area is $$\frac{2^2\sqrt{3}}{4}=\sqrt{3}$$ $$1-\frac{\frac{\sqrt{3}}{3}}{\sqrt{3}}=\frac{2}{3}$$
27.	D	Let E_1 be the expected number of turns until the game ends if the number is in the form $2k^3$. Let E_2 be the expected number of turns until the game ends if the number is in the form $4k^3$

		$$E_1 = 1 + \frac{E_1}{3} + \frac{E_2}{3}$$ $$E_2 = 1 + \frac{E_2}{3}$$ $$\rightarrow E_2 = \frac{3}{2}, E_1 = \frac{9}{4}.$$ Expected number= $E_1 = \frac{9}{4} \rightarrow 13$
28.	A	Note that due to the symmetry about x^2, if r is a root to $f(x)$, $\frac{1}{r}$ must be as well. Since there are at least 3 positive roots, all of them must be positive. *If there was a negative root $r, \frac{1}{r}$ would also be negative, so there wouldn't be 3 positive roots. Also, since there are only 3 distinct roots, the roots must be $r, \frac{1}{r}, 1,1$. Knowing 1 is a root, we get $$f(1) = 2a + b + 2 = 0 \rightarrow 2a + b = -2$$ From the information given, $a + b = \frac{11}{5}$, $$a = -\frac{21}{5}$$ From Vieta's, $r + \frac{1}{r} + 2 = \frac{21}{5}$. The sum of the 3 distinct roots is $$r + \frac{1}{r} + 1 = \frac{16}{5}$$
29.	C	The following equation geometrically describes the sum of the distance between the points $(x, 0)$ and $(-4, 1)$ and the distance between the points $(x, 0)$ and $(2,7)$. This is a standard problem; you can reflect the point $(-4,1)$ across the x-axis to get $(-4, -1)$, then make the points $(-4, -1), (x, 0), (2,7)$ collinear. $$\frac{7}{2 - x} = \frac{8}{6}$$ $$\rightarrow 42 = 16 - 8x$$ $$\rightarrow x = -\frac{13}{4} \rightarrow \lvert x \rvert = \frac{13}{4} \rightarrow 13 + 4 = 17$$
30.	E	$$10 \leq n^2 < 100$$ means that $$4 \leq \lvert n \rvert \leq 9$$ This gives us $6 * 2 = 12$ integers.