Mu School Bowl
Test #812
Question #0

Mu School Bowl
Test #812
Question #0

#0 Mu School Bowl
MAΘ National Convention 2024

Let $\{x\} = x - [x]$ denote the fractional part of x, and let $\log(x) = \log_{10} x$

Then, the numbers $\{\log(1)\}, \{\log(2)\}, \{\log(3)\} \dots, \{\log(9999)\}$ are listed from least to greatest. (In case of a tie, the numbers are listed from smallest argument to largest argument of the logarithm)

To clarify, the first few numbers are $\{\log(1)\}, \{\log(10)\}, \{\log(100)\}, \{\log(1000)\}, \{\log(1001)\} \dots$

If the position of a number is defined as the leftmost being 1, and the rightmost being 9999,

Let $A =$ the position of $\{\log(24)\}$

Let $B =$ the position of $\{\log(2024)\}$

Let $C =$ the argument of the logarithm in the 24^{th} position

Submit $A + B + C$.

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Submit $A + B + C$.

Mu School Bowl
Test #812
Question #1

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Question #1

#1 Mu School Bowl
MAΘ National Convention 2024

For the following questions, let the value of the limit be -1 if the limit does not exist. (Consider $\infty, -\infty$ to be the same thing as DNE)

A:

$$\lim_{x\to\infty} x\left(\sqrt{4x^2+1}-\sqrt{4x^2-1}\right)$$

B:

$$\lim_{x\to 0}\frac{\ln\left(\cos(2x)\right)}{\ln\left(\cos(x)\right)}$$

C:

$$\lim_{x\to\infty}\frac{1+x\sin(x)}{3+x\sin(x)}$$

D:

$$\lim_{n\to\infty}\frac{\sqrt{n^2-1}+\sqrt{n^2-4}+\sqrt{n^2-9}+\cdots+\sqrt{n^2-(n-1)^2}}{n^2}$$

E:

$$\lim_{x\to\infty}\frac{2x^2+3\sin(x)}{x^2}$$

Submit $ABCDE$.

#1 Mu School Bowl
MAΘ National Convention 2024

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$$\lim_{x\to\infty}\frac{2x^2+3\sin(x)}{x^2}$$

Submit $ABCDE$.

Mu School Bowl
Test #812
Question #2

Mu School Bowl
Test #812
Question #2

#2 Mu School Bowl
MAΘ National Convention 2024

Consider the polynomial $f(x) = x^4 - 4x^3 + x^2 + 5x - 1$ with real roots $r_1 < 0 < r_2 < r_3 < r_4 < 4$.

Let $A = (r_1 + r_2)(r_3 + r_4) + (r_1 + r_3)(r_2 + r_4) + (r_1 + r_4)(r_2 + r_3)$

Let $B = (r_1 + r_2 + r_3)(r_1 + r_2 + r_4)(r_1 + r_3 + r_4)(r_2 + r_3 + r_4)$

Let $C = \frac{1}{4-r_1} + \frac{1}{4-r_2} + \frac{1}{4-r_3} + \frac{1}{4-r_4}$

Let $D = r_1$

Submit $ABCD$.

#2 Mu School Bowl
MAΘ National Convention 2024

Consider the polynomial $f(x) = x^4 - 4x^3 + x^2 + 5x - 1$ with real roots $r_1 < 0 < r_2 < r_3 < r_4 < 4$.

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Let $C = \frac{1}{4-r_1} + \frac{1}{4-r_2} + \frac{1}{4-r_3} + \frac{1}{4-r_4}$

Let $D = r_1$

Submit $ABCD$.

Mu School Bowl
Test #812
Question #3

Mu School Bowl
Test #812
Question #3

#3 Mu School Bowl
MAΘ National Convention 2024

Let $N = 2^3 \cdot 3 \cdot 5 \cdot 6$

Let $A =$ the number of positive integral factors of N

Let $B =$ the number of positive integers less than 2024 that are relatively prime to N

If $\frac{m}{n} =$ the ratio of the sum of the positive integral factors of N that are a multiple of 12 to the sum of the positive integral factors of N that are a multiple of 6 in simplest form.

Let $C = m + n$.

If d_1, d_2 are positive integral factors of N

Let $D =$ the number of distinct possible values for $\frac{d_1}{d_2}$

Submit $A + B + C + D$.

#3 Mu School Bowl
MAΘ National Convention 2024

Let $N = 2^3 \cdot 3 \cdot 5 \cdot 6$

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Submit $A + B + C + D$.

Mu School Bowl
Test #812
Question #4

Mu School Bowl
Test #812
Question #4

#4 Mu School Bowl
MAΘ National Convention 2024

Let R be the region bounded by $y = 0$ and $y = -x^2 + 10x - 21$

Let $\frac{m}{n}$ = the area of R in simplest form.

$$A = m + n$$

Let B = the trapezoidal approximation of R using 4 equal subintervals in simplest form.

Let $\frac{p\sqrt{q}}{r}$ be the minimum distance between a point on R and $y = x$ in simplest form

$$C = p + q + r$$

Let $\frac{u\sqrt{v}}{w}\pi$ = the volume of the shape when R is rotated around $y = x$ in simplest form

$$D = u + v + w$$

Submit $A + B + C + D$.

#4 Mu School Bowl
MAΘ National Convention 2024

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Submit $A + B + C + D$.

Mu School Bowl
Test #812
Question #5

Mu School Bowl
Test #812
Question #5

#5 Mu School Bowl
MAΘ National Convention 2024

A piecewise function that is differentiable everywhere is given by:

$$f(x) = \begin{cases} -x^3 - Ax^2 + Bx + 5 & \text{if } x < -2 \\ Cx^2 + Ax + 7 & \text{if } -2 \leq x < 4 \\ Cx^3 + Bx^2 + 31 & \text{if } 4 \leq x \end{cases}$$

for unique values of A, B, and C.

$g(x) = D|x| + \arctan|x|$ is differentiable everywhere for a unique value of D.

Submit $ABCD$.

#5 Mu School Bowl
MAΘ National Convention 2024

A piecewise function that is differentiable everywhere is given by:

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for unique values of A, B, and C.

$g(x) = D|x| + \arctan|x|$ is differentiable everywhere for a unique value of D.

Submit $ABCD$.

Mu School Bowl
Test #812
Question #6

Mu School Bowl
Test #812
Question #6

#6 Mu School Bowl
MAΘ National Convention 2024

Let $f(x) = \frac{1+x}{1-2x}$ and $g(x) = \sqrt{x} \arctan \sqrt{x}$

Let $A = f'(0)$

Let $B = f''(0)$

Let $C = \lim_{x \to 0^+} g'(x)$

Let $D = m + n$, where $\lim_{x \to 0^+} g''(x) = -\frac{m}{n}$ in simplest form

Submit $A + B + C + D$.

#6 Mu School Bowl
MAΘ National Convention 2024

Let $f(x) = \frac{1+x}{1-2x}$ and $g(x) = \sqrt{x} \arctan \sqrt{x}$

Let $A = f'(0)$

Let $B = f''(0)$

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Let $D = m + n$, where $\lim_{x \to 0^+} g''(x) = -\frac{m}{n}$ in simplest form

Submit $A + B + C + D$.

Mu School Bowl
Test #812
Question #7

Mu School Bowl
Test #812
Question #7

#7 Mu School Bowl
MAΘ National Convention 2024

Consider the fully simplified expansion of

$$(w + x + 2y - 3z)^8$$

Let A be the sum of the coefficients

Let B be the number of terms

Let C be the number of terms such that the exponent on x is greater than the exponent on y.

Let D be the number of terms with a negative coefficient

Submit $A + B + C + D$.

#7 Mu School Bowl
MAΘ National Convention 2024

Consider the fully simplified expansion of

$$(w + x + 2y - 3z)^8$$

Let A be the sum of the coefficients

Let B be the number of terms

Let C be the number of terms such that the exponent on x is greater than the exponent on y.

Let D be the number of terms with a negative coefficient

Submit $A + B + C + D$.

Mu School Bowl
Test #812
Question #8

Mu School Bowl
Test #812
Question #8

#8 Mu School Bowl
MAΘ National Convention 2024

Let $A =$ minimum value of the function $a(x) = x^2 - 100x + 2024$

Let $B =$ minimum value of the function $b(x) = \frac{2x}{x^2+1}$

Let $C =$ minimum value of the function $c(x) = x^3 - 3x^2 - 9x + 4$ over the range $[-2,5]$

Let $D =$ minimum value of the function $d(x) = |x-1| + |x-2| + \cdots + |x-20|$

Submit $A + B + C + D$.

#8 Mu School Bowl
MAΘ National Convention 2024

Let $A =$ minimum value of the function $a(x) = x^2 - 100x + 2024$

Let $B =$ minimum value of the function $b(x) = \frac{2x}{x^2+1}$

Let $C =$ minimum value of the function $c(x) = x^3 - 3x^2 - 9x + 4$ over the range $[-2,5]$

Let $D =$ minimum value of the function $d(x) = |x-1| + |x-2| + \cdots + |x-20|$

Submit $A + B + C + D$.

Mu School Bowl
Test #812
Question #9

Mu School Bowl
Test #812
Question #9

#9 Mu School Bowl
MAΘ National Convention 2024

Let $P = (a, b)$ be a point on the graph of $(x-3)^2 + (y-4)^2 = 4$.

Let $H =$ the maximum value of a, let $h =$ the minimum value of a, let $K =$ the maximum value of b, let $k =$ the minimum value of b.

$$A = H + h + K + k$$

Let $B =$ the sum of all distinct values of a where point P lies on the line $y = x$

Let $N =$ the maximum value of $a^2 + b^2$, let $n =$ the minimum value of $a^2 + b^2$

$$C = N + n$$

Let O be the circle $(x+1)^2 + (y+1)^2 = r$. Given the two circles intersect,

Let $Q =$ the maximum value of r, let $q =$ the minimum value of r

$$D = Q + q$$

Submit $A + B + C + D$.

#9 Mu School Bowl
MAΘ National Convention 2024

Let $P = (a, b)$ be a point on the graph of $(x-3)^2 + (y-4)^2 = 4$.

Let $H =$ the maximum value of a, let $h =$ the minimum value of a, let $K =$ the maximum value of b, let $k =$ the minimum value of b.

$$A = H + h + K + k$$

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$$C = N + n$$

Let O be the circle $(x+1)^2 + (y+1)^2 = r$. Given the two circles intersect,

Let $Q =$ the maximum value of r, let $q =$ the minimum value of r

$$D = Q + q$$

Submit $A + B + C + D$.

Mu School Bowl
Test #812
Question #10

Mu School Bowl
Test #812
Question #10

#10 Mu School Bowl
MAΘ National Convention 2024

The n^{th} anti-Chebyshev expansion is defined as
$$\cos^n(x) = a_{n,0} + a_{n,1}\cos(x) + a_{n,2}\cos(2x) + \cdots + a_{n,n}\cos(nx)$$

For example, since
$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$
$$a_{2,0} = \frac{1}{2}, a_{2,1} = 0, a_{2,2} = \frac{1}{2}, a_{2,3} = 0, \ldots$$

If $a_{4,0}^2 + a_{4,1}^2 + a_{4,2}^2 + a_{4,3}^2 + a_{4,4}^2 = \frac{m}{n}$ in simplest form, let $A = m + n$. Hint: $\cos(x) = \frac{e^{ix}+e^{-ix}}{2}$.

Let $B =$
$$\sum_{n=0}^{\infty} \frac{a_{2n,0}}{4^n}$$

Given
$$\int_0^{\frac{\pi}{2}} \cos^{2n}(x)\,dx = \frac{\pi}{2} \cdot a_{2n,0}$$

Submit AB.

#10 Mu School Bowl
MAΘ National Convention 2024

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If $a_{4,0}^2 + a_{4,1}^2 + a_{4,2}^2 + a_{4,3}^2 + a_{4,4}^2 = \frac{m}{n}$ in simplest form, let $A = m + n$. Hint: $\cos(x) = \frac{e^{ix}+e^{-ix}}{2}$.

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$$\int_0^{\frac{\pi}{2}} \cos^{2n}(x)\,dx = \frac{\pi}{2} \cdot a_{2n,0}$$

Submit AB.

Mu School Bowl
Test #812
Question #11

Mu School Bowl
Test #812
Question #11

#11 Mu School Bowl
MAΘ National Convention 2024

All answers to these integrals are written in simplest form, all lowercase variables are integers, and all arguments of logarithms are minimized.

Let $A = a_1 + a_2 + a_3$, where

$$\int_0^1 \frac{X^2}{X^3+1}\,dX = \frac{a_1}{a_2}\ln a_3$$

Let $B = b_1 + b_2$, where

$$\int_0^1 X\sqrt{1-X^2}\,dX = \frac{b_1}{b_2}$$

Let $C = c_1 + c_2 + c_3$, where

$$\int_1^e 4X\ln X\,dX = c_1 + c_2 e + c_3 e^2$$

Let $D = d_1 + d_2$, where

$$\int_{-1}^2 \frac{3X-4}{X^2-X-6}\,dX = d_1 \ln d_2$$

Submit $A + B + C + D$.

#11 Mu School Bowl
MAΘ National Convention 2024

All answers to these integrals are written in simplest form, all lowercase variables are integers, and all arguments of logarithms are minimized.

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Submit $A + B + C + D$.

Mu School Bowl
Test #812
Question #12

Mu School Bowl
Test #812
Question #12

#12 Mu School Bowl
MAΘ National Convention 2024

Consider the graph of $(2x + 3y - 1)(3x + 2y + 6) = k$.

Let $A =$ the unique value of k for which this is not a hyperbola.

Let $B =$ Given an appropriate k that results in the graph passing through the origin, find the slope of the tangent line to the graph at the origin.

A 20-foot ladder is leaning against a wall. The top of the ladder is moving down the wall at a constant rate of 4 ft/min, and the bottom of the ladder is sliding directly away from the wall on the flat floor. At a particular time t, the top of the ladder is 16 feet from the ground.

Let $C =$ the distance (in feet) from the wall to the bottom of the ladder at time t.

Let $D =$ the rate (in ft/min) at which the base of the ladder is sliding away from the wall at time t.

Submit $A + BCD$.

#12 Mu School Bowl
MAΘ National Convention 2024

Consider the graph of $(2x + 3y - 1)(3x + 2y + 6) = k$.

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Submit $A + BCD$.

Mu School Bowl
Test #812
Question #13

Mu School Bowl
Test #812
Question #13

#13 Mu School Bowl
MAΘ National Convention 2024

Let $A =$

$$\int_1^2 \frac{4x^4}{x^3+x^2+x+1}\,dx$$

Let $B =$

$$\int_1^2 \frac{3x^3}{x^3+x^2+x+1}\,dx$$

Let $C =$

$$\int_1^2 \frac{2x^2}{x^3+x^2+x+1}\,dx$$

Let $D =$

$$\int_1^2 \frac{x}{x^3+x^2+x+1}\,dx$$

Submit $A+B+C+D$.

#13 Mu School Bowl
MAΘ National Convention 2024

Let $A =$

$$\int_1^2 \frac{4x^4}{x^3+x^2+x+1}\,dx$$

Let $B =$

$$\int_1^2 \frac{3x^3}{x^3+x^2+x+1}\,dx$$

Let $C =$

$$\int_1^2 \frac{2x^2}{x^3+x^2+x+1}\,dx$$

Let $D =$

$$\int_1^2 \frac{x}{x^3+x^2+x+1}\,dx$$

Submit $A+B+C+D$.

Mu School Bowl
Test #812
Question #14

Mu School Bowl
Test #812
Question #14

#14 Mu School Bowl
MAΘ National Convention 2024

Sock is interested in the sequence a_n, b_n such that $\left(1+\sqrt{2}\right)^n = a_n + b_n\sqrt{2}$ where a_n, b_n are integers.

Let $A = \lim_{n\to\infty} \frac{a_n}{b_n}$

Let $B = \sum_{n=1}^{\infty} \frac{a_n}{3^n}$

Another sequence Sock likes is the recursive sequence $c_0, = 0, c_1 = 0, c_n = c_{n-1} + 2c_{n-2} + 2$.

Let $C = \lim_{n\to\infty} \frac{c_{n+1}}{c_n}$

Let $D = \sum_{n=1}^{\infty} \frac{c_n}{3^n}$

Submit $A + B + C + D$.

#14 Mu School Bowl
MAΘ National Convention 2024

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Let $C = \lim_{n\to\infty} \frac{c_{n+1}}{c_n}$

Let $D = \sum_{n=1}^{\infty} \frac{c_n}{3^n}$

Submit $A + B + C + D$.