All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means "None of the Above."

~~~~~~ *Good luck, and have fun!* ~~~~~~

1. Evaluate: $\lim_{x\to 0} \frac{e^{x-1}(e^x-1)}{(x+1)\ln(x+1)}$.

   A. $\frac{1}{2e}$ B. $\frac{1}{e}$ C. e D. $2e$ E. NOTA

2. Evaluate: $\lim_{x\to 0} (\sec x)^{\cot^2 x}$.

   A. 1 B. $\sqrt{2}$ C. $\sqrt{e}$ D. e E. NOTA

3. If $\lim_{x\to 0} \frac{e^{2x}-f(x)}{\frac{x^2}{2}-1+\cos x} = A$ for some cubic $f(x)$, find $f(3)$.

   A. 42 B. 46 C. 61 D. 67 E. NOTA

4. If $\int_0^1 \frac{dx}{(1+x^2)(1+\arctan x)} = \ln\left(A+\frac{\pi}{B}\right)$, find $A+B$.

   A. 3 B. 5 C. 6 D. 9 E. NOTA

5. If $\int_0^6 x^2 e^{-x/3}\, dx = A - \frac{B}{e^C}$, find $\frac{BC}{A}$.

   A. 4 B. 5 C. 8 D. 10 E. NOTA

6. If $\frac{1}{L} = \int_0^1 \frac{x^3(1-x^2)^2}{(1+x^2)^6}\, dx$, find L. *Hint: either $u = \tan x$ or $u = \tan\frac{x}{2}$ is possible.*

   A. 60 B. 120 C. 180 D. 240 E. NOTA
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For questions 7 through 9, use the following information. A partial derivative of a function of several variables is a derivative of that function with respect to a single variable, treating all other variables as constants. This is denoted D_x for a partial derivative with respect to x. A critical point is where all partial derivatives of the function are equal to zero.

7. Consider a function $f(x,y)=0$. If $x=x_0+\Delta x$ and $y=y_0+\Delta y$, then usage of partial derivatives gives $D_x f(x_0,y_0)\Delta x + D_y f(x_0,y_0)\Delta y = 0$. Find $\frac{dy}{dx}$ in terms of D_x and D_y.

A. $-\frac{D_x}{D_y}$ B. $-\frac{D_y}{D_x}$ C. $\frac{D_x}{D_y}$ D. $\frac{D_y}{D_x}$ E. NOTA

8. Find $\left\lfloor\frac{dy}{dx}\right\rfloor$ at the point $(3,4)$ if $x^2y(x+y)(4x-y)+2y=2024$.

A. -5 B. -4 C. -3 D. -2 E. NOTA

9. If (h,k) is the critical point of $f(x,y)=x^2+y^2+3xy-x-4y-2$, find $f(h,k)$.

A. -10 B. -1 C. 3 D. 8 E. NOTA

10. Identify the graph given by $r=\frac{1}{1+\sin\theta}$.

A. Line B. Parabola C. Ellipse D. Hyperbola E. NOTA

11. Find the area contained within the dimpled limaçon $r=3+2\cos\theta$.

A. 9π B. 11π C. 12π D. 13π E. NOTA

12. Find the total length of the graph of $r=4+4\cos\theta$.

A. 16 B. 8π C. $16+4\pi$ D. 32 E. NOTA

13. If the volume of the solid formed when the region between the graphs of $y=4x-x^2$ and $y=x$ is rotated over the x-axis is equal to $\frac{A\pi}{B}$, find $A+B$.

A. 69 B. 91 C. 113 D. 221 E. NOTA

14. If the volume of the solid formed when the region between the graph of $y = \sqrt{2x+1}$ and the x-axis for $x \in [0,4]$ is rotated over the y-axis is equal to $k\pi$, find $\lfloor k \rfloor$.

A. 32 B. 36 C. 39 D. 41 E. NOTA

15. Find the volume of the solid formed when the region between the graphs of $y = x^2$ and $y = \sqrt{x}$ is rotated over the line $x - y = 4$ is equal to $\frac{A\pi\sqrt{B}}{C}$ for squarefree B, find ABC.

A. 12 B. 15 C. 18 D. 24 E. NOTA

16. If $\langle x, y \rangle = \langle t^2 - 2t + 5, 2t^2 - 5t + 2 \rangle$, find $\frac{d^2y}{dx^2}$ at the point $(5,0)$.

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. NOTA

17. Find the arc length of the curve given by $\langle x, y \rangle = \langle 2\cos^2 t, 2\cos t \sin t \rangle$ for $0 \leq t \leq \pi$.

A. 2 B. π C. 4 D. 2π E. NOTA

18. If the surface area of the solid formed when the curve given by $\langle x, y \rangle = \langle 2t^3, 3t^2 \rangle$ for $0 \leq t \leq 1$ is rotated about the x-axis is equal to $\frac{A+B\sqrt{C}}{D}\pi$ for squarefree C, find $A + B + C + D$.

A. 19 B. 31 C. 32 D. 55 E. NOTA

19. Let $f(x)$ be the third-degree Taylor polynomial of $y = \tan x$ at $x = \frac{\pi}{4}$. Find $f\left(\frac{\pi}{4} + 1\right)$.

A. $\frac{20}{3}$ B. $\frac{41}{6}$ C. $\frac{23}{3}$ D. DNE E. NOTA

20. Find the sum of the digits of $\left|\sum_{m=1}^{30}\sum_{n=1}^{60} \tan\frac{mn\pi}{61}\tan\frac{m(n+1)\pi}{61}\right|$.

Hint: Use the tangent subtraction formula to create something that telescopes.

A. 9 B. 12 C. 14 D. 15 E. NOTA

21. Determine the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+4n+4)}{n^3-8n}$.

A. Absolutely convergent
B. Bifurcates
C. Conditionally convergent
D. Divergent
E. NOTA

22. For a function $y = f(x)$ that contains the point $(1,1)$, $xy\frac{dy}{dx} = 1$. If $f(a) = 3$, find $\lfloor\sqrt{a}\rfloor$.

A. 2 B. 3 C. 5 D. 7 E. NOTA

23. The function $y = f(x)$ passes through the points $(0, 2e-1)$ and $(1, e^2)$ and satisfies the differential equation $y'' - 3y' + 2y = 0$. Find $f(2)$.

A. $2e^3 - e^4$ B. 0 C. $e^4 - e^3$ D. $2e^4 - e^3$ E. NOTA

24. The function $y = f(x)$ passes through the point $(0,2)$ and satisfies the differential equation $(x^2+1)\frac{dy}{dx} + 2xy = x^3$. If $f(2) = \frac{T}{B}$, find $T + B$.

A. 7 B. 9 C. 11 D. 15 E. NOTA

25. Let the roots of the equation $x^3 + 3x^2 + 2x + 4 = 0$ be a, b, and c. Find $\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a}$.

A. $-\frac{11}{2}$ B. $-\frac{9}{2}$ C. $\frac{3}{2}$ D. $\frac{5}{2}$ E. NOTA

26. Evaluate: $\lim_{x\to 1} \frac{x^x - x}{1 - x + \ln x}$.

A. -2 B. -1 C. 1 D. 2 E. NOTA

27. Evaluate: $\int_0^{\pi/2} \frac{\sin\theta}{1+\sqrt{\sin 2\theta}}\, d\theta$.

A. $\frac{4-\pi}{4}$ B. $\frac{\pi-2}{4}$ C. $\frac{4-\pi}{2}$ D. $\frac{\pi-2}{2}$ E. NOTA

28. Find the slope of the line normal to $r = 2\theta\cos\theta$ when $\theta = \frac{\pi}{4}$.

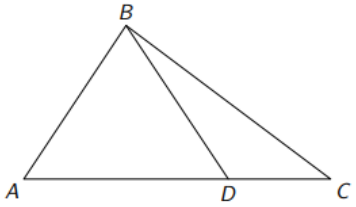
A. $\frac{4-\pi}{4}$ B. $\frac{\pi-2}{4}$ C. $\frac{4-\pi}{2}$ D. $\frac{\pi-2}{2}$ E. NOTA

29. The Laplace transform of a function $f(x)$ is $L\{f\}(s) = \int_0^{\infty} f(t)e^{-st}\, dt$. This tool is often used to solve differential equations. Find $L\{\sinh(2x)\}(3)$.

A. $\frac{4}{13}$ B. $\frac{2}{5}$ C. $\frac{4}{9}$ D. $\frac{2}{3}$ E. NOTA

30. No one really seems to know why the two AP-level Calculus classes are called "AB" and "BC". Maybe it's just because those are cool letters! One place all three of those letters show up is the Pythagorean Theorem, where in a right triangle with legs a and b and hypotenuse c, we can say $a^2 + b^2 = c^2$.

Anyways, in the below figure, $AB = BD = 5$, $BC = 7$, and $AC = 9$. Find the ratio $\frac{AD}{DC}$.



A. $\frac{15}{8}$ B. 2 C. $\frac{9}{4}$ D. $\frac{19}{8}$ E. NOTA