1. B
2. D
3. D
4. B
5. A
6. D
7. B
8. C
9. C
10. C
11. A
12. D
13. B
14. C
15. D
16. B
17. C
18. D
19. B
20. B
21. B
22. C
23. A
24. B
25. A
26. C
27. D
28. C
29. C
30. E

1. B The graph of the cubic bounds 2 regions of equal area with the x-axis. One region is in the 1st quadrant and the other region is in the 3rd quadrant. We will solve for the area of the region in the 1st quadrant in terms of a and then double it to get the total area. We'll then take the derivative with respect to time. The area of the region in the 1st quadrant is:

$$\int_0^{\sqrt{a}} (0) - (x^3 - ax)\ dx = \frac{1}{2}ax^2 - \frac{1}{4}x^4 \Big|_0^{\sqrt{a}} = \frac{a^2}{4}$$

That means the total area is $2 * \frac{a^2}{4} = \frac{a^2}{2}$. Taking the time derivative and using chain rule gives us $\frac{d}{dt}\left(\frac{a^2}{2}\right) = a * \frac{da}{dt} = 3 * 2 = 6(B)$ after plugging in $a = 3$ and $\frac{da}{dt} = 2$.

2. D The volume of a cube is the side length of the cube raised to the 3rd power and the surface area of a cube is 6 times the area of one of its faces. Letting the side length of the cube be x:

$$6x^2 = x^3 \rightarrow x = 6(D)$$

3. D We know the formula for the volume of a cone, and we are given an equation in terms of r and h. Letting the volume of the cone be V, we have $V = \frac{\pi}{3}r^2h$. Using $r + h = 18 \rightarrow h = 18 - r$, we get that $V = \frac{\pi}{3}r^2(18 - r) = 6\pi r^2 - \frac{\pi}{3}r^3$ after substitution. To maximize $V = 6\pi r^2 - \frac{\pi}{3}r^3$, we take the derivative with respect to r and get $\frac{dV}{dr} = 12\pi r - \pi r^2 = \pi r(12 - r)$. Since $\frac{dv}{dr}$ goes from positive to negative at $r = 12$, the volume is maximized when $r = 12$. This gives $h = 6$ and the volume as $\frac{\pi}{3}r^2h = \frac{\pi}{3}(12)^2(6) = 288\pi(D)$

4. B Imagine any of these rectangles that have 2 vertices on the horizontal asymptote of $y = f(x)$ and 2 vertices on the graph of $y = f(x)$. The area of that rectangle won't change if we reflect the graph of $y = f(x)$ over the x-axis and shift it up by 1 unit. Doing that transformation would transform the graph from $y = \frac{x^2-1}{x^2+1}$ to $y = \frac{2}{x^2+1}$ and the horizontal asymptote from $y = 1$ to $y = 0$ (to calculate the horizontal asymptote(s) of $y = \frac{x^2-1}{x^2+1}$, we see what the y-value approaches as x approaches positive and negative infinity. In both cases, the y-value approaches 1, so the horizontal asymptote is $y = 1$). Thus, we solve an easier problem that asks us to find the maximum area of a rectangle with 2 vertices on the x-axis ($y = 0$) and 2 vertices on the graph of $y = \frac{2}{x^2+1}$. Letting $\left(a, \frac{2}{a^2+1}\right)$ for $a > 0$ be a vertex of the rectangle, the rectangle's area is $(2a)\left(\frac{2}{a^2+1}\right) = \frac{4a}{a^2+1}$. We take the derivative with respect to a to get $\frac{(a^2+1)4-4a(2a)}{(a^2+1)^2} = \frac{4-4a^2}{(a^2+1)^2}$. This expression goes from positive to negative at $a = 1$, so $a = 1$ must maximize the area of the rectangle. Thus, the maximum area of this rectangle is $\frac{4a}{a^2+1}$ evaluated at $a = 1$: $\frac{4(1)}{(1)^2+1} = 2(B)$

5. A With all edges having the same length, the surface area of the pyramid consists of 4 equilateral triangles and a square base. We let the edge length be x, the surface area

be A, and the volume be $V. A = x^2 + 4\left(\frac{x^2\sqrt{3}}{4}\right) = (\sqrt{3}+1)x^2. V = \frac{Bh}{3} = \frac{x^2h}{3}$. The height can be calculated using a right triangle consisting of a hypotenuse made up of an edge that isn't on the square base, a leg made up of half a diagonal of the base, and a leg made up of the height of the pyramid. The hypotenuse would be x and one leg would be $x/\sqrt{2}$, meaning the height would be $x/\sqrt{2}$. $V = \frac{x^2h}{3} = \frac{x^3}{3\sqrt{2}}$. Setting $A = V$, we get $(\sqrt{3}+1)x^2 = \frac{1}{3\sqrt{2}}x^3 \rightarrow x = 3\sqrt{2}(\sqrt{3}+1) = 3\sqrt{6}+3\sqrt{2} = \sqrt{54}+\sqrt{18}$. So $m = 54, n = 18$, and $m + n = 72(A)$

6. D The line is tangent to both circles and form right angles $\angle BAO_1$ and $\angle ABO_2$. Call point X the point on BO_2 such that $O_1X||AB$. Then $ABXO_1$ is a rectangle with $BX = AO_1 = 3. BX + XO_2 = BO_2 = 4 \rightarrow 3 + XO_2 = 4 \rightarrow XO_2 = 1. O_1O_2 = 3 + 4 = 7$ because the 2 circles are externally tangent. We see that O_1XO_2 is right triangle with $(O_1X)^2 + (XO_2)^2 = (O_1O_2)^2 \rightarrow O_1X = \sqrt{(7)^2 - (1)^2} = 4\sqrt{3}$. $O_1X = AB$ since $ABXO_1$ is a rectangle, so $AB = 4\sqrt{3}$. Now we can find the area of quadrilateral ABO_2O_1. It's a trapezoid with height AB and bases of AO_1 and BO_2. $AB = 4\sqrt{3}, AO_1 = 3$, and $BO_2 = 4$. The area of the trapezoid is $\frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(AB)(AO_1 + BO_2) = \frac{1}{2}(4\sqrt{3})(3+4) = 14\sqrt{3}$(D)

7. B Let's take a cross-section that goes through the center of the sphere and is perpendicular to the base of the cone. This cross-section then just depicts a circle inscribed in an isosceles triangle. This triangle would have a base that is the same length as the diameter of the base of the cone and altitude to the base that is the same length as the height of the cone. This triangle would thus have a base of length 14, an altitude to the base of length 24, and legs of length $\sqrt{7^2 + 24^2} = 25$. Note that the radius of the sphere equals the radius of the circle that is inscribed in such a triangle. We know that $A = rs$ where A is the area of the triangle, r is the radius of the inscribed circle, and s is the semi-perimeter of the triangle. $A = \frac{Bh}{2} = \frac{(14)(24)}{2} = 168$ and $s = \frac{14+25+25}{2} = 32$ so $A = rs \rightarrow 168 = r(32) \rightarrow r = \frac{21}{4} = \frac{m}{n} \rightarrow m = 21$ and $n = 4 \rightarrow m + n = 25(B)$

8. C The rectangle formed by those 4 vertices has length $2c$ where c is the focal radius of the ellipse and width $\frac{2b^2}{a}$ where a is half the length of the major axis and b is half the length of the minor axis. We are told that this rectangle is a square, meaning $\frac{2b^2}{a} = 2c \rightarrow b^2 = ac$. In an ellipse, it is well known that $a^2 = b^2 + c^2 \rightarrow b^2 = a^2 - c^2$. Substituting, we get that $b^2 = ac \rightarrow a^2 - c^2 = ac \rightarrow 1 - \left(\frac{c}{a}\right)^2 = \left(\frac{c}{a}\right)$ after diving both sides of the equation by a^2. The eccentricity of the ellipse is $e = \frac{c}{a}$. Substituting, we get $1 - e^2 = e \rightarrow e^2 + e - 1 = 0 \rightarrow e = \frac{-1\pm\sqrt{5}}{2}$. Since c and a are both positive, we know that $e \geq 0$. That means $e \neq \frac{-1-\sqrt{5}}{2}$ and $e = \frac{\sqrt{5}-1}{2}(C)$

9. C
$$y = x^2 + 1 \rightarrow y' = 2x$$
This tells us that the slope of the tangent line to $y = f(x)$ at point $(a, f(a))$ is $2a$. Thus, L_1 can be written as $y = 2ax + b$ for some b representing the y-intercept

of L_1. We can solve for b in terms of a because we know the point $(a, f(a)) = (a, a^2+1)$ lies on line L_1. This tells us $(a^2+1) = 2a(a) + b \rightarrow b = 1 - a^2$. This tells us that in terms of a, L_1 can be represented as $y = 2ax + 1 - a^2$. Region R is bounded above by $y = f(x)$ and below by L_1 from $x = 0$ to $x = a$. Thus, the area of region R is $\int_0^a (x^2+1) - (2ax + 1 - a^2)\ dx = \int_0^a (x-a)^2\, dx = \frac{1}{3}(x-a)^3 \Big|_0^a = \frac{a^3}{3}$. The y-intercept of L_1 is $1 - a^2$, so the sum of the area of R and the y-intercept of L_1 is $\frac{a^3}{3} + 1 - a^2$. To minimize this expression, we take the derivative with respect to a to get $a^2 - 2a = a(a-2)$. This expression goes from negative to positive at $a = 2$, so $a = 2(C)$ maximizes the sum of the area of R and the y-intercept of L_1.

10. C Using shell method:

$$V_1 = 2\pi \int_0^a x\big((x^2+1) - (2ax^2 + 1 - a^2)\big)\ dx$$
$$= 2\pi \int_0^a x(x-a)^2\, dx$$
$$= 2\pi \int_0^a x^3 - 2ax^2 + a^2 x\, dx$$
$$= 2\pi\Big(\frac{1}{4}x^4 - \frac{2}{3}ax^3 + \frac{a^2}{2}x \Big|_0^a\Big) = \frac{\pi}{6}a^4.$$

When $a = 2$, this is equal to $\frac{8\pi}{3}(C)$

11. A Note that the condition in the problem is possible because as a tends towards infinity, line L_1 intersects the y-axis at lower and lower points. We found in question 9 that the area of region R is $\frac{a^3}{3}$. Note that the half of region R below the x-axis resembles a right triangle, and that this triangle has area $\frac{1}{2}\left(\frac{a^3}{3}\right) = \frac{a^3}{6}$. The area of this triangle can also be represented in terms of a in another way using the x and y-intercepts of line L_1. One leg of this right triangle has length (x-intercept of L_1) and the other leg has length $(0) -$ (y-intercept of L_1) $= -$(y-intercept of L_1). Thus, the area of this triangle is also $-\frac{1}{2}$(y-intercept of L_1)(x-intercept of L_1) $=$ $-\frac{1}{2}(1-a^2)$(x-intercept of L_1). Using the equation of line L_1, we can solve for the x-intercept by letting $y = 0 \rightarrow 0 = 2ax + 1 - a^2 \rightarrow x = \frac{a^2-1}{2a}$. Substituting, the area of the triangle is $-\frac{1}{2}(1-a^2)\left(\frac{a^2-1}{2a}\right) = \frac{(a^2-1)^2}{4a}$. But his value also equals $\frac{a^3}{6}$, meaning $\frac{a^3}{6} = \frac{(a^2-1)^2}{4a} \rightarrow \frac{2}{3}a^4 = a^4 - 2a^2 + 1 \rightarrow a^4 - 6a^2 + 3 = 0 \rightarrow a^2 = \frac{6\pm\sqrt{24}}{2} = 3 \pm \sqrt{6}$. However, $a^2 \neq 3 - \sqrt{6}$ because $3 - \sqrt{6} < 1$ and $a^2 = 3 - \sqrt{6}$ would mean that $0 < a < 1$. This would make the y-intercept of $1 - a^2 > 0$, meaning the x-axis wouldn't even cross into region R. a must be greater than 1 for the problem condition to even be possible, so $a^2 \neq 3 - \sqrt{6}$ and $a^2 = 3 + \sqrt{6}$. $2 < \sqrt{6} < 2.5$ so it follows that $5 < 3 + \sqrt{6} < 5.5$ and a^2 rounded to the nearest integer is $5(A)$

12. D It looks like the condition that the height and radius of the cylinder are integers will be useful. We let r be the radius of the cylinder and h be the height of the cylinder where r and h are positive integers. The volume of the cylinder is $\pi r^2 h$ and the total surface area of the cylinder is $2\pi r^2 + 2\pi rh$. These expressions are equal, meaning $\pi r^2 h = 2\pi r^2 + 2\pi rh \to rh = 2r + 2h \to rh - 2r - 2h = 0 \to (r-2)(h-2) = 4$. Considering how r and h are positive integers, there are 3 possibilities:
   1. $r - 2 = 1 \;\&\; h - 2 = 4 \to r = 3 \;\&\; h = 6$.
   2. $r - 2 = 2 \;\&\; h - 2 = 4 \to r = 4 \;\&\; h = 4$.
   3. $r - 2 = 4 \;\&\; h - 2 = 1 \to r = 6 \;\&\; h = 3$.

   Thus, the sum of possible distinct height values is $6 + 4 + 3 = 13(D)$

13. B Let E be the foot of the altitude from A to CD and let F be the foot of the altitude from B to CD. $ABFE$ is a rectangle, so $EF = AB = 2$. Let $DE = FC = x$. Then $DA = 2x$ because ΔAED is a 30-60-90 triangle with $\angle ADE = \angle ADC = 60°$. Since $ABCD$ is an isosceles trapezoid, $BC = DA = 2x$. The perimeter of the trapezoid can then be written in terms of x as $AB + BC + CD + DA = (2) + (2x) + (2x + 2) + (2x) = 6x + 4$. We are given that the perimeter is 28, so $6x + 4 = 28 \to x = 4$. To solve for the area, we need the height of the trapezoid. Using 30-60-90 triangle ΔAED, we get that $AE = x\sqrt{3}$, meaning the height of the trapezoid is $x\sqrt{3}$ because AE is a height of the trapezoid. With $AB = 2$ and $CD = 2x + 2$ as bases of the trapezoid, the area is $\frac{1}{2}\left((2) + (2x+2)\right)\left(x\sqrt{3}\right) = x(x+2)\sqrt{3} = 4(6)\sqrt{3} = 24\sqrt{3}(B)$

14. C Graphing on the interval $0 \le x \le 5$, we get that there's a line segment with endpoints (0,3) and (1,1), a line segment with endpoints (1,1) and (2,1), and a line segment with endpoints (2,1) and (5,7). To find area, we need to sum up a trapezoid $\left(\frac{(3+1)(1)}{2} = 2\right)$, a rectangle $\left((1)(1) = 1\right)$, and another trapezoid $\left(\frac{(1+7)(3)}{2} = 12\right)$ for a total area of $2 + 1 + 12 = 15(C)$

15. D Let the rectangle have length l and width w where $l = \log_3 x$ and $w = \log_2 x$. If the area equals the perimeter, then $wl = 2l + 2w$. Divide both sides of this equation by wl to get $\frac{1}{2} = \frac{1}{l} + \frac{1}{w}$. Substituting, we get $\frac{1}{2} = \frac{1}{\log_3 x} + \frac{1}{\log_2 x} \to \frac{1}{2} = \log_x 3 + \log_x 2 \to \frac{1}{2} = \log_x 6 \to x = 36(D)$

16. B In any triangle, $A = rs$ where A is the area the triangle and s is the semi-perimeter of the triangle. $s = \frac{9+10+11}{2} = 15$ and $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15(6)(5)(4)} = 30\sqrt{2}$ by Heron's Formula where $a, b,$ and c are the side lengths of the triangle. This means that $A = rs \to 30\sqrt{2} = r(15) \to r = 2\sqrt{2}$. The area of the incircle is $\pi r^2 = \pi\left(2\sqrt{2}\right)^2 = 8\pi(B)$

17. C Using the information given to us, we can find out what $p, q,$ and r are. This will then allow us to find the area we need to calculate. Since the graph of $y = f(x)$ is tangent to the x-axis at $x = \frac{2\pi}{3}$, we know that $f\left(\frac{2\pi}{3}\right) = 0$ and $f'\left(\frac{2\pi}{3}\right) = 0$. This will give us two equations:

   $$1.\ f\left(\frac{2\pi}{3}\right) = pcos^2\left(\frac{2\pi}{3}\right) + qcos\left(\frac{2\pi}{3}\right) + r = \frac{p}{4} - \frac{q}{2} + r = 0 \to p - 2q + 4r = 0.$$

$$2. f(x) = pcos^2(x) + qcos(x) + r \rightarrow f'(x) = -psin(2x) - qsin(x) \rightarrow f'\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}p - \frac{\sqrt{3}}{2}q = 0 \rightarrow p = q.$$

Finally, we are given that $f(0) = 6 \rightarrow p + q + r = 6.$ So we have 3 equations and 3 variables:

1. $p = q$
2. $p + q + r = 6$
3. $p - 2q + 4r = 0$

Combining 1 and 3, we get $r = \frac{p}{4}$, so now we know all the variables in terms of p. We plug into equation 2 to get $p + p + \frac{p}{4} = 6 \rightarrow p = \frac{8}{3} \rightarrow q = \frac{8}{3} \rightarrow r = \frac{2}{3} \rightarrow f(x) = \frac{8}{3}\cos^2(x) + \frac{8}{3}\cos(x) + \frac{2}{3}$. Note that we can write this as $f(x) = \frac{8}{3}\left(\cos(x) + \frac{1}{2}\right)^2$, so $f'(x) = \frac{16}{3}\left(\cos(x) + \frac{1}{2}\right)(-\sin(x))$ and $-f'\left(\frac{\pi}{4}\right) = \frac{16}{3} \cdot \left(\frac{\sqrt{2}+1}{2}\right) \cdot \frac{\sqrt{2}}{2} = \frac{8+4\sqrt{2}}{3}(C)$

18. D This polar curve is a lemniscate. Let's find the area of the region in the 1st quadrant and then multiply by 4. The area in the first quadrant is:

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} r^2 d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} 2024\cos(x)\,dx = 1012\sin(x)\Big|_0^{\frac{\pi}{2}} = 1012$$

Multiplying by 4, the total area enclosed is $4 * 1012 = 4048(D)$

19. B Let the side length of the tetrahedron be x. Then we can solve for the surface area and volume in terms of x and then set them equal. The volume of the tetrahedron is $\frac{x^3\sqrt{2}}{12}$ and the surface area of the tetrahedron is $4\left(\frac{x^2\sqrt{3}}{4}\right) = x^2\sqrt{3}$ so we have $\frac{x^3\sqrt{2}}{12} = x^2\sqrt{3} \rightarrow x = 6\sqrt{6}(B)$

20. B Let's turn the equation for the circle into a form that is easier to understand. $x^2 - 12x + y^2 - 18y + 108 = 0 \rightarrow (x-6)^2 + (y-9)^2 = 9$. Thus, this circle has center $(6,9)$ and radius $\sqrt{9} = 3$. We can use Pappus to solve for the volume when this circle is rotated about the line $y = -5x$. The volume is $2\pi Ad$ where A is the area of the circle and d is the distance between the center of the circle and the line. We know that $A = \pi r^2 = 9\pi$. We can then use point to line formula to solve for d. The formula says that the distance between point (a, b) and line $Ax + By = C$ is $\frac{|Aa+Bb-C|}{\sqrt{A^2+B^2}}$. We want the distance between point $(6,9)$ and line $5x + y = 0$, which is $\frac{|(5)(6)+(1)(9)-(0)|}{\sqrt{(5)^2+(1)^2}} = \frac{3\sqrt{26}}{2} = d$. Thus, the volume is $2\pi(9\pi)\left(\frac{3\sqrt{26}}{2}\right) = 27\pi^2\sqrt{26}(B)$

21. B The rhombus is comprised of four right triangles with one leg of length 9 and a hypotenuse of integer length; thus, the other leg must have integer length as well. The only possible Pythagorean triples with 9 are $9-12-15$ and $9-40-41$, which have areas of $\frac{9\cdot 12}{2} = 54$ and $\frac{9\cdot 40}{2} = 180$ respectively. The corresponding rhombi have areas of 216 and 720, which sum to $936(B)$

22. C Call the right angle of the quarter circle point A, the center of the circle point O, and the intersection of line AO with the arc of the quarter circle point B. Let r be the radius of circle X. Let points C and D be the feet of the altitude from O to the two

radii of the quarter circle. We then know that $AO = r\sqrt{2}$ and $BO = r$ since $ACOD$ is a square with side length r with AO as a diagonal. We know $AO + BO = AB = 1$ since AB is a radius of the quarter circle. Thus, $r\sqrt{2} + r = 1 \rightarrow r = \sqrt{2} - 1$. The area of circle X is $\pi\left(\sqrt{2}-1\right)^2$ and the area of the quarter circle is $\frac{\pi}{4}$, for a ratio of $\frac{\pi(\sqrt{2}-1)^2}{\frac{\pi}{4}} = 12 - \sqrt{128} = m - \sqrt{n} \rightarrow m = 12 \ \& \ n = 128 \rightarrow m + n = 140(C)$

23. A The graphs intersect at: $2x^2 + 3x + 5 = 7x + 11 \rightarrow 2x^2 - 4x - 6 = 0 \rightarrow 2(x-3)(x+1) = 0 \rightarrow x = -1, 3$. From $x = -1$ to $x = 3$, the difference between the graphs is $7x + 11 - (2x^2 + 3x + 5) = -2x^2 + 4x + 6$, so the area bounded by the two graphs is $\int_{-1}^{3} -2x^2 + 4x + 6\,dx = -\frac{2}{3}x^3 + 2x^2 + 6x \Big|_{-1}^{3} = \frac{64}{3}(A)$

24. B This rectangle has a length of $2c$ where c is the focal radius of the ellipse and a width of the length of the latus rectum. The latus rectum has length $\frac{2b^2}{a}$ where b is half the length of the minor axis and a is half the length of the major axis. Thus, the area of the rectangle is $(2c)\left(\frac{2b^2}{a}\right) = \frac{4b^2c}{a}$. Since the length of the major axis is 8, $a = 4$ and since the length of the minor axis is 6, $b = 3$. We know that $a^2 = b^2 + c^2 \rightarrow 16 = 9 + c^2 \rightarrow c = \sqrt{7}$. Now that we know $a, b,$ and c, we can solve for the area of the rectangle:

$$\frac{4b^2c}{a} = \frac{4(3)^2(\sqrt{7})}{4} = 9\sqrt{7}(B)$$

25. A Let the height of the cone be h and let the radius of the cone be r. Also let the radius of the sphere be R and treat R as some constant. Let point O be the center of the sphere. Imagine a cone inscribed inside a sphere. The distance between O and the vertex of the cone is R, so the distance between O and the base of the cone is $h - R$. There is a right triangle formed by O, the foot of the perpendicular from O to the base of the cone, and some point on the circumference of the base of the cone. This right triangle has hypotenuse that is the radius of the sphere, one leg that is the radius of the cone, and one leg that we found earlier as $h - R$. Thus, $r^2 + (h-R)^2 = R^2 \rightarrow r^2 + h^2 = 2Rh$. The volume of the cone is $V = \frac{\pi}{3}r^2h = \frac{\pi}{3}(2Rh - h^2)h$ after substituting $r^2 + h^2 = 2Rh \rightarrow r^2 = 2Rh - h^2$. To maximize the volume, we take the derivative of the volume with respect to h to get:

$$\frac{dV}{dh} = \frac{\pi}{3}(4Rh - 3h^2) = \frac{\pi}{3}h(4R - 3h).$$

This expression goes from positive to negative at $4R - 3h = 0 \rightarrow h = \frac{4R}{3}$, so $h = \frac{4R}{3}$ maximizes the volume of the cone. Solving for r, $r^2 = 2Rh - h^2 \rightarrow r^2 = 2R\left(\frac{4R}{3}\right) - \left(\frac{4R}{3}\right)^2 = \frac{8R^2}{9} \rightarrow r = \frac{2\sqrt{2}}{3}R$. The ratio of the height to the radius is then $\frac{h}{r} = \frac{\frac{4R}{3}}{\frac{2\sqrt{2}R}{3}} = \sqrt{2}(A)$

26. C We can solve for the area in terms of P and then set it equal to 24. For a parabola with equation $4Py = x^2$, the latus rectum intersects the parabola at points $(2P, P)$ and $(-2P, P)$. Since $P > 0$, we know that point A is $(2P, P)$. The line that passes

through point A and the origin has slope $\frac{P-0}{2P-0}=\frac{1}{2}$ and equation $y=\frac{1}{2}x$, so line L_1 is $y=\frac{1}{2}x$. The area bounded above by line L_1 and below by the parabola is:

$$\int_0^{2P}\left(\frac{1}{2}x\right)-\left(\frac{1}{4P}x^2\right)dx=\frac{1}{4}x^2-\frac{1}{12P}x^3\Big|_0^{2P}=\frac{P^2}{3}$$

This value equals 24, so $\frac{P^2}{3}=24\rightarrow P=6\sqrt{2}(C)$

27. D

$$g(a)=\frac{1}{a}\int_0^a a^2-x^2\,dx=\frac{1}{a}\left(a^2x-\frac{1}{3}x^3\right)\Big|_{x=0}^{x=a}=\frac{2}{3}a^2$$

Point P is the intersection of $y=f(x)$ and $y=g(a)$. Find the x-coordinate of point P:

$$f(x)=g(a)\rightarrow a^2-x^2=\frac{2}{3}a^2\rightarrow x=\frac{a}{\sqrt{3}}$$

So, point P is $\left(\frac{a}{\sqrt{3}},\frac{2}{3}a^2\right)$. This point lies on line $y=2x$, so $\frac{2}{3}a^2=2\left(\frac{a}{\sqrt{3}}\right)\rightarrow a=\sqrt{3}\rightarrow a^2=3(D)$

28. C There is a formula for the circumradius of a triangle that involves the side lengths of the triangle and the area of the triangle. We know the side lengths and we can find the area using Heron's formula. $R=\frac{abc}{4K}$ for $a,b,$ and c as side lengths of the triangle, K as the area of the triangle, and R as the circumradius of the triangle. The side lengths are 7, 8, and 9, so $R=\frac{126}{K}$. $K=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{a+b+c}{2}=12$ is the semi perimeter of the triangle. So $K=\sqrt{12(5)(4)(3)}=12\sqrt{5}$. Then $R=\frac{126}{12\sqrt{5}}=\frac{21}{2\sqrt{5}}$. The area of the circumcircle is $\pi R^2=\pi\left(\frac{21}{2\sqrt{5}}\right)^2=\frac{441}{20}\pi=\frac{m}{n}\pi\rightarrow m=441$ and $n=20\rightarrow m+n=461(C)$

29. C

$$y=x^3\rightarrow y'=3x^2$$

Thus, the slope of the tangent line to $y=x^3$ at $(2,8)$ is $3(2)^2=12$. With point (2,8) on the line and slope of 12, the line (line L_1) has equation $y-8=12(x-2)\rightarrow y=12x-16$. The intercepts of this line are $(0,-16)$ and $\left(\frac{4}{3},0\right)$, so the area of the desired triangle is $\frac{1}{2}\cdot16\cdot\frac{4}{3}=\frac{32}{3}(C)$

30. E Let A_1 be the area bounded by $y=a^x, x=-1$, and the y-axis. Let A_1 be the area bounded by $y=a^x, x=1$, and the y-axis. Since $a>1$, it's clear that it's impossible to have $A_1=2A_2$. Thus, we must have $A_2=2A_1$.

$A_1=\int_{-1}^0 a^x dx=\frac{a^x}{\ln(a)}\Big|_{-1}^{0}=\frac{1}{\ln(a)}\left(1-\frac{1}{a}\right)$

$A_2=\int_0^1 a^x dx=\frac{a^x}{\ln(a)}\Big|_0^1=\frac{1}{\ln(a)}(a-1)$

$$A_2=2A_1\rightarrow\frac{1}{\ln(a)}(a-1)=2\left(\frac{1}{\ln(a)}\right)\left(1-\frac{1}{a}\right)\rightarrow a-1=2-\frac{2}{a}\rightarrow a^2-3a+2=0\rightarrow(a-2)(a-1)=0\rightarrow a=1,2$$

But the problem says that $a>1$, so we know that $a=2(E)$