All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means "None of the Above."

~~~~~~ Good luck, and have fun! ~~~~~~

1. Let the distinct complex roots of the equation $x^4 + x^3 - 2x^2 - 5x + 15 = 0$ equal α, β, γ, and δ. Find $(\alpha + \beta)(\gamma + \delta) + (\alpha + \gamma)(\beta + \delta) + (\alpha + \delta)(\beta + \gamma)$.

   A. -10 B. -6 C. -4 D. -2 E. NOTA

2. Vriska is rolling a fair four-sided die. What is the expected value of the number of rolls it would take until Vriska has rolled all four sides on her die?

   A. 4 B. 8 C. $\frac{25}{3}$ D. 16 E. NOTA

3. Aradia's rate of excitement f can be measured t hours before her birthday with the equation $f = \frac{1}{\lceil 1/t \rceil}$ for positive real t. Find the total amount of excitement Aradia builds up in the hour before her birthday.

   A. $2 - \frac{\pi^2}{6}$ B. $\frac{1}{e}$ C. $\frac{\pi^2}{6} - 1$ D. $e - 2$ E. NOTA

4. Consider two positive real numbers m and n. Let a, g, and h represent their arithmetic, geometric, and harmonic means respectively. Which of these statements must be true?

   A. $a > g > h$
   B. $2g = ah$
   C. It is possible for at least one of a, g, and h to not exist.
   D. If $m \neq n$, then $a \neq g$, $g \neq h$, and $h \neq a$.
   E. NOTA

5. Kris, Ralsei, and Susie pick three integers k, r, and s in the range $[1,10]$. Find the number of ordered triplets (k, r, s) with the property $kr + rs + sk = krs$.

   A. 3 B. 6 C. 8 D. 10 E. NOTA

6. Eridan's house is 4 miles east and 2 miles north of him. To get to his house, Eridan must run some distance east over land, then swim in a straight line to his house. Eridan can run at 10 miles per hour and swim at 8 miles per hour, and there is no current. Find the minimum number of minutes it would take for Eridan to reach his house.

A. 30 B. 33 C. 36 D. 39 E. NOTA

7. Kanaya, Porrim, and Rosa are standing at the points $(4,6,9)$, $(3,5,5)$, and $(6,8,10)$ in space. Find the area of the triangle whose vertices are where the trio is standing.

A. $\frac{7}{2}$ B. $\frac{7}{\sqrt{2}}$ C. 7 D. $7\sqrt{2}$ E. NOTA

8. Captain Mindfang and her crew of 7 other pirates are divvying up their treasure, which is a positive integer number of gold coins. The following dialogue takes place:
Pirate 1: "If we split the coins with Captain Mindfang getting twice as many as each of us, there will be 3 coins left over."
Pirate 2: "Aye, and if we split the coins with Captain Mindfang getting four times as many as each of us, there will be 9 coins left over."
Mindfang: "Or, I will get eight times as many coins as each of you like we agreed, and there will be no coins left over!"
Find the second-smallest number of coins that the pirates plundered, modulo 19.

A. 0 B. 6 C. 9 D. 18 E. NOTA

9. Dave, standing on the curve $y = \sec^2 x$, says to Dirk, standing on the curve $y = 1 + x^2$, says "Hey, we're really close to each other! What's $\lim_{x\to 0} \frac{\sec^2 x - 1 - x^2}{x^4}$?" What would be the correct answer that Dirk gives?

A. $\frac{2}{3}$ B. 1 C. $\frac{3}{2}$ D. 2 E. NOTA

10. Mettaton has four cards, each with a number on one side and a letter on the other. When you look at the cards, you see 8, 7, A, and 8. Mettaton then flips some cards over, and you see T, B, 7, and 4. What number is on the opposite side of the T?

A. 2 B. 4 C. 7 D. 8 E. NOTA

11. Feferi spots a sequence of some very pretty snowflakes. The first snowflake in the sequence is an equilateral triangle with side length 1. The second snowflake, and every snowflake thereafter, is constructed by trisecting each segment and replacing the center segment by two sides of a smaller equilateral triangle projecting outward. If Feferi stays an infinite amount of time, then the area of each snowflake Feferi sees approaches $\frac{C\sqrt{3}}{U}$. Find $C + U$.

A. 5 B. 7 C. 31 D. 107 E. NOTA

12. Which of the values $\aleph = \frac{1}{1-\frac{1}{1-\frac{1}{\ddots}}}$ and $\beth = \sqrt{-\frac{1}{8} - \sqrt{-\frac{1}{8} - \sqrt{-\frac{1}{8} - \cdots}}}$ exist and is real?

A. Neither B. Only $\aleph$ C. Only $\beth$ D. Both E. NOTA

13. Terezi randomly draws 4 marbles out of a bag that contains 5 red and 5 blue marbles without replacement. If the probability that she draws 2 red and 2 blue marbles is $\frac{R}{G}$, find $R + G$.

A. 17 B. 31 C. 68 D. 77 E. NOTA

14. Jade has 5 liters of a 25% acid solution and 3 liters of a 60% acid solution. She combines them to form a 30% acid solution and a 50% acid solution. Given that none of the original solutions remain, if the volume of the 50% acid solution is $\frac{B}{Q}$, find $B + Q$.

A. 17 B. 19 C. 23 D. 27 E. NOTA

15. Muffet's donut can be modeled by the rotation of the closed region in the first quadrant bounded by $y = \sin x + \frac{1}{2}$ and $x = \frac{\pi}{2}$ over the line $y = -1$. The volume of the donut is equal to $A\pi + \frac{B\pi^2}{C}$. Find $A + B + C$.

A. 12 B. 16 C. 18 D. 22 E. NOTA

16. Latula and Aranea are painting their house. Working alone, they could paint the house in 6 and 8 hours respectively. Mituna joins them for the first hour of their work and paints twice as fast as Latula, but then needs to leave to recharge. If the number of hours the group takes to paint the house is equal to $\frac{H}{S}$, find $H + S$.

A. 13 B. 23 C. 25 D. 37 E. NOTA

17. Meenah's Beforan Quantumequation takes the form $x^2 + zx + \text{cis}\frac{\pi}{3} = 0$, where z is a complex number $a + bi$ for nonnegative reals a and b. This quadratic has exactly one real root. Find the minimum possible value of a.

A. $\frac{1}{\sqrt{3}}$ B. $\frac{1}{\sqrt{2}}$ C. $\sqrt{2}$ D. $\sqrt{3}$ E. NOTA

18. From the graph of $y = ax^2 + bx + c$, Undyne waves at her girlfriend Alphys, who is on the graph of $y = 3cx^2 - 2ax + 4b$. Undyne says, "You know, if my curve is only defined for $x < 1$ and yours is only defined for $x \geq 1$, we'd be together on one curve that's differentiable over its whole domain!" "You're a nerd!" replies Alphys. "If a, b, and c are all positive integers, what's the minimum value of $a + b + c$?"

A. 6 B. 9 C. 12 D. 15 E. NOTA

19. Karkat rolls a fair pair of 4-sided dice (each numbered 1 through 4) and notes down their sum. If a die shows a 3, there is a 50% chance that Nepeta eats it, and Karkat will not include its value in the sum he writes down (if the die is not eaten, Karkat will include 3 in the sum). If Nepeta eats both dice, Karkat will note down a 0. Find the expected value of the number Karkat notes down.

A. 4 B. $\frac{33}{8}$ C. $\frac{17}{4}$ D. $\frac{9}{2}$ E. NOTA

20. The permutations of the letters in the word "ASRIEL" are written in alphabetical order in a list. Let the nth element of this list spell "RALSEI." Find n mod 11.

A. 2 B. 3 C. 5 D. 7 E. NOTA

21. A guard dog will let you pass if and only if you give him a number of pets equal to e^k, where $k = \int_{\pi/6}^{\pi/2} \frac{1+\cos x}{x+\sin x}\, dx$. This number of pets is equal to $\frac{A\pi+B}{C\pi+D}$. Find $A + B + C + D$.

A. 7 B. 10 C. 12 D. 13 E. NOTA

22. The distance Sans walks in day d is equal to $\frac{16}{(2d+1)(2d+3)(2d+5)}$. As d increases without bound, how far will Sans end up walking if he starts today on day 1?

A. $\frac{2}{15}$ B. $\frac{1}{5}$ C. $\frac{4}{15}$ D. $\frac{1}{3}$ E. NOTA

23. Read the instructions, then find the number of distinct values of $\frac{A}{B}$ such that $AB = 12!$.

A. 72 B. 128 C. 180 D. 396 E. NOTA

24. Toriel wants to deliver a butterscotch pie to you. If you are standing at the point $\left(\frac{\pi}{4}, \ln\sqrt{2}\right)$, Toriel is at the origin, and Toriel can only travel along the curve $y = \ln\sec x$, find e to the power of the distance Toriel must travel.

A. $\sqrt{2}+1$ B. $\sqrt{3}+1$ C. $\sqrt{2}+2$ D. $\sqrt{3}+1$ E. NOTA

25. Lanque throws a ball in the air, which travels along the parabola $y = 32 - \frac{x^2}{8}$ starting at its lesser root, crossing the y-axis, and stopping when the parabola crosses its latus rectum in the first quadrant. Find the difference between the minimum and maximum x-values the ball attains.

A. $8\sqrt{15}$ B. 24 C. 20 D. 16 E. NOTA

26. To qualify for the Alternian Integration Bee, Damara must solve $\int_0^\infty x^5 e^{-x^4}\, dx$. What is the answer to this integral divided by $\int_0^\infty e^{-x^2}\, dx$?

A. $\frac{1}{6}$ B. $\frac{1}{5}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. NOTA

27. MOO!! I'm Waffles the Cow. Why am I showing up here, on Question 27 of your Applications test? Why, because the test is in the Moo division! And you know me – as a cow, I love to moo! I know it for a fact, take a look at this chart that shows how many times I mooed over the course of the last three weeks!

Week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1	42	69	13	12	41	36	12
2	10	25	94	55	48	27	28
3	68	78	32	16	57	49	43

Find the moodian – sorry, the median – of the number of times I mooed per day.

A. 32 B. 34 C. 36 D. 41 E. NOTA

28. "I wish I was just integrating the numerator of the integrand – it would be so much easier!" remarks a dejected Napstablook. "Maybe if I imagine it being that way I'll get the right answer… maybe not. Can you help me find $\int_0^{\pi/4} \frac{\cos x - \sin x}{1+\sin(2x)}\, dx$?"

A. $\sqrt{2}$ B. $\frac{1}{\sqrt{2}}$ C. $\sqrt{2}-1$ D. $1-\frac{1}{\sqrt{2}}$ E. NOTA

29. Temmie is "OMG!!! So exCITED!!!" to notice that $y = x$ is a solution to the differential equation $xy' = \sqrt{x^2 - y^2} + y$. Unfortunately, Mr. Fox reminds her that she is looking for a solution that contains the point $\left(e^{\pi/2}, \frac{e^{\pi/2}}{2}\right)$ and has $y(1) > 0$. Find the smallest value of $k > 1$ such that $y(k) = 0$.

A. $e^{\pi/6}$ B. $e^{\pi/4}$ C. $e^{\pi/3}$ D. $e^{2\pi/3}$ E. NOTA

30. Laughing evilly, Flowey greets you as you arrive at the final question of this Mu Applications test. "I'm going to give you a problem so hard that you'll never be able to get a perfect score on this test! You'll rue the day you ever tried to fight against me!" Flowey turns on a computer to show you the integral $\int_0^1 2v\ du$. "Solve this!"

A. 0 B. $\frac{1}{2}$ C. 1 D. 2 E. NOTA