1. C
2. C
3. A
4. D
5. D
6. B
7. B
8. A
9. A
10. D
11. B
12. A
13. B
14. A
15. C
16. B
17. C
18. B
19. C
20. A
21. D
22. C
23. E
24. A
25. C
26. C
27. D
28. D
29. D
30. E

1. C The desired expression evaluates to twice the sum of the roots taken two at a time, which by Vieta's is $2 \cdot (-2) = -4$.
2. C If n values have appeared so far, the time until a new result appears is geometrically distributed with parameter of success $\frac{4-n}{4}$ and thus mean $\frac{4}{4-n}$. Adding $n = 0$ through 3 gives an expected value of $4H_4 = \frac{25}{3}$.
3. A Geometrically, $\int_0^1 \frac{dt}{\lceil 1/t \rceil} = \sum_{t=1}^{\infty} \left(\frac{1}{t} - \frac{1}{t+1}\right) \frac{1}{t+1} = \sum_{t=1}^{\infty} \frac{1}{t(t+1)^2}$. Through partial fractions, $\frac{1}{t(t+1)^2} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{(t+1)^2}$, so $A(t+1)^2 + Bt(t+1) + Ct = 1$. $t = 0$ gives $A = 1$, $t = -1$ gives $C = -1$, and $t = 1$ gives $4A + 2B + C = 1$ so $B = -1$. $\sum_{t=1}^{\infty} \left(\frac{1}{t} - \frac{1}{t+1} - \frac{1}{(t+1)^2}\right)$ is a telescoping series minus the index-shifted Basel problem, and its value is $1 - \left(\frac{\pi^2}{6} - 1\right) = 2 - \frac{\pi^2}{6}$.
4. D By AM-GM-HM, $A \geq G \geq H$, with equality at $m = n$. $AH = \left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right) = ab = G^2$. A always exists, G fails to exist only when m or n is negative, and H fails to exist when m or n equals 0. None of the means can be equal if $m \neq n$.
5. D Dividing, $\frac{1}{k} + \frac{1}{r} + \frac{1}{s} = 1$. WLOG, let $k \leq r \leq s$. Then $k = r = s = 3$ is the only solution where $k > 2$. Solving $\frac{1}{r} + \frac{1}{s} = \frac{1}{2}$ gives the solutions $(r, s) = (3,6)$ or $(4,4)$, so the unordered triplets are $(2,3,6)$, $(2,4,4)$, and $(3,3,3)$ for a total of $6 + 3 + 1 = 10$ solutions.
6. B Let Eridan walk x miles east. Then he swims $\sqrt{(4-x)^2 + 2^2}$ miles, and the amount of time it takes for him to get home is $T = \frac{x}{10} + \frac{\sqrt{x^2 - 8x + 20}}{8}$. Then $T' = \frac{x-4}{8\sqrt{x^2-8x+20}} + \frac{1}{10}$, which has the root $x = \frac{4}{3}$, giving $T = \frac{11}{20}$, or 33 minutes.
7. B The two vectors that represent sides of the triangle from $(4,6,9)$ are $\langle -1, -1, -4 \rangle$ and $\langle 2,2,1 \rangle$. Their cross product is $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -1 & -4 \\ 2 & 2 & 1 \end{vmatrix} = (-1+8)i + (-8+1)j + (-2+2)k = 7\hat{\imath} - 7\hat{\jmath}$. The area of the parallelogram this forms is $\sqrt{7^2 + 7^2 + 0^2} = 7\sqrt{2}$, so the area of the triangle is $\frac{7}{\sqrt{2}}$.
8. A Pirate 1 and Mindfang give the system $x \equiv 3 \bmod 9$ and $x \equiv 0 \bmod 15$, so $x \equiv 30 \bmod 45$. Pirate 2 adds the equation $x \equiv 9 \bmod 11$, giving $x \equiv 75 \bmod 495$ by the Chinese Remainder Theorem. Plugging in $n = 1$ to $x = 495n + 75$ gives $x = 570$, which is equivalent to $0 \bmod 19$.
9. A The Maclaurin series of $\tan x$ is $x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$, so the Maclaurin series of $\sec^2 x$ is $1 + x^2 + \frac{2x^4}{3} + \cdots$. The limit is equal to $\frac{2x^4/3 + \cdots}{x^4} = \frac{2}{3}$.
10. D There is a missing 4 from the first set of cards, so it must be opposite the A. The four numbers are $\{4,7,8,8\}$, so both T and B are opposite an 8.

11. B This is the Koch snowflake. At the kth iteration, $3 \cdot 4^{k-1}$ triangles are added, each with area $\frac{\sqrt{3}}{4}\left(\frac{1}{3^k}\right)^2$, for a total area of $\frac{\sqrt{3}}{4}\left(\frac{3 \cdot 4^{k-1}}{9^k}\right)$. The total area of the snowflake is thus $\frac{\sqrt{3}}{4}\left(1+\sum_{k=1}^{\infty}\frac{3\cdot 4^{k-1}}{9^k}\right)=\frac{\sqrt{3}}{4}\left(1+\frac{3/9}{1-4/9}\right)=\frac{2\sqrt{3}}{5}$. $2+5=7$

12. A Solving $x=\frac{1}{1-x}$ yields $x-x^2=1$, which has non-real solutions; in fact, the series $x_{n+1}=1-\frac{1}{x_n}$ is periodic and only constant at the non-real numbers. Thus, א does not exist. Even though solving $x^2=-\frac{1}{8}-x$ gives $x=\frac{\sqrt{2}-2}{4}$ as a solution, ב is still the square root of a negative number. Neither value exists.

13. B The probability that Terezi picks the sequence $BBRR$ is $\frac{5}{10}\cdot\frac{4}{9}\cdot\frac{5}{8}\cdot\frac{4}{7}=\frac{5}{63}$. There are 6 ways to order these marbles, so the probability is $\frac{10}{21}$. $10+21=31$.

14. A Jade has $\frac{61}{20}$ liters of acid and $\frac{99}{20}$ liters of water. Because the 50% acid solution is equal acid and water, the extra $\frac{19}{10}$ liters of water is in the 30% acid solution, equalling the 40% difference between acid and water. Thus, the 30% solution has volume $\frac{19}{10}\cdot\frac{5}{2}=\frac{19}{4}$, so the 50% solution has volume $8-\frac{19}{4}=\frac{13}{4}$. $13+4=17$.

15. C The volume is $\pi\int_0^{\pi/2}\left(\left(\sin x+\frac{3}{2}\right)^2-1^2\right)\,dx=\pi\int_0^{\pi/2}\left(\sin^2 x+3\sin x+\frac{5}{4}\right)\,dx=\pi\left(\frac{\pi}{4}+3+\frac{5\pi}{8}\right)=3\pi+\frac{7\pi^2}{8}$. $3+7+8=18$.

16. B This is equivalent to Latula and Aranea painting $\frac{2}{3}$ of the house. Solving $\left(\frac{1}{6}+\frac{1}{8}\right)t=\frac{2}{3}$ gives $t=\frac{16}{7}$. $16+7=23$.

17. C By the quadratic formula, the real root is $\frac{-z+\sqrt{z^2-2-2i\sqrt{3}}}{2}$. Let $z=a+bi$. Then the root is $\frac{-a-bi+\sqrt{(a^2-b^2-2)+\left(2ab-2\sqrt{3}\right)i}}{2}$. We know $\sqrt{(a^2-b^2-2)+\left(2ab-2\sqrt{3}\right)i}=c+bi$, so $a^2-b^2-2=c^2-b^2$ and $ab-\sqrt{3}=bc$, so $a^2-c^2=2$ and $b=\frac{\sqrt{3}}{a-c}$. Rearranging gives $a-c=\frac{\sqrt{3}}{b}$, so $a+c=\frac{2b}{\sqrt{3}}$. Solving this system gives $a=\frac{\sqrt{3}}{2b}+\frac{b}{\sqrt{3}}$. By AM-GM, the minimum of this is $\sqrt{2}$.

18. B To assure continuity, $a+b+c=3c-2a+4b$, so $3a-3b-2c=0$. For differentiability, $2a+b=6c-2a$, so $4a+b-6c=0$. Elimination of c gives $5a-10b=0$, so $a=2b$. Then $3b-2c=0$, so $2c=3b$. The minimal solution is $(a,b,c)=(4,2,3)$. $4+2+3=9$.

19. C The dice are independent, so this is twice the expected value of the value of one die. In the $\frac{1}{4}$ probability that a 3 is rolled, there is a $\frac{1}{2}$ chance it will contribute 3 to Karkat's total and a $\frac{1}{2}$ chance it will contribute 0 to Karkat's total. Thus, its expected contribution is 1.5. The expected value of Karkat's number is $2\cdot\frac{1+2+1.5+4}{4}=\frac{17}{4}$.

20. A "ASRIEL" has $6!=720$ permutations, and the fifth sixth of them alphabetically (481 through 600) start with "R". The first fifth of these (481 through 504) have "A" in the right position, the third fourth of these (493 through 498) have "L" in the

right position, the third third of these (497 through 498) have "S" in the right position, and the first half of these (497) spells "RALSEI." $497 \bmod 11 = 2$.

21. D Note that the numerator of the integrand is the derivative of the denominator. Thus, the integral is equal to $\ln(x+\sin x)\Big]_{\pi/6}^{\pi/2} = \ln\left(\frac{\pi}{2}+1\right) - \ln\left(\frac{\pi}{6}+\frac{1}{2}\right) = \ln\frac{3\pi+6}{\pi+3}$. e to the power of this is $\frac{3\pi+6}{\pi+3}$. $3+6+1+3=13$.

22. C Let $\frac{16}{(2d+1)(2d+3)(2d+5)} = \frac{A}{2d+1}+\frac{B}{2d+3}+\frac{C}{2d+5}$. Then $A(2d+3)(2d+5)+B(2d+1)(2d+5)+C(2d+1)(2d+3)=16$. $2d=-1$ gives $8A=16$, so $A=2$. $2d=-3$ gives $-4B=16$, so $B=-4$. $2d=-5$ gives $8C=16$, so $C=2$, so $\frac{16}{(2d+1)(2d+3)(2d+5)} = \frac{2}{2d+1}-\frac{4}{2d+3}+\frac{2}{2d+5} = \left(\frac{2}{2d+1}-\frac{2}{2d+3}\right)+\left(\frac{2}{2d+5}-\frac{2}{2d+3}\right)$. The sum of this as d ranges from 1 to ∞ is telescoping, with the first set equaling $\frac{2}{3}$ and the second set equaling $-\frac{2}{5}$, for a total distance of $\frac{4}{15}$.

23. E The prime factors of 12! are 2, 3, 5, 7, and 11, and since the fraction is in lowest terms (read the instructions), these primes must be allocated to either the numerator or the denominator, which can happen in $2^5 = 32$ ways.

24. A $\frac{dy}{dx} = \tan x$. $\int_0^{\pi/4}\sqrt{1+\tan^2 x}\ dx = \int_0^{\pi/4}\sec x\ dx = \ln(\sec x+\tan x)\Big|_0^{\pi/4} = \ln\left(\sqrt{2}+1\right)$. e to the power of this is $\sqrt{2}+1$.

25. C The lesser root of the parabola is -16. The latus rectum lies on the line $y=30$, which intersects the parabola when $\frac{x^2}{8}=2$, so $x=4$. The difference between the minimum and maximum values of x is 20.

26. C With $u=x^2$, $\int_0^\infty x^5 e^{-x^4}\ dx = \frac{1}{2}\int_0^\infty u\cdot ue^{-u^2}\ du$. Integrating by parts yields $-\frac{1}{4}ue^{-u^2}\Big|_0^\infty + \frac{1}{4}\int_0^\infty e^{-u^2}\ du = \frac{1}{4}\int_0^\infty e^{-u^2}\ du$, so the desired division produces $\frac{1}{4}$.

27. D The median of the set will have ten numbers greater than it and ten numbers less than it. The number that has this property is 41.

28. D The denominator is equal to $\sin^2 x + 2\sin x\cos x + \cos^2 x = (\sin x+\cos x)^2$. With $u=\sin x+\cos x$, the integral becomes $\int_1^{\sqrt{2}}\frac{du}{d^2} = -\frac{1}{u}\Big]_1^{\sqrt{2}} = 1-\frac{1}{\sqrt{2}}$.

29. D $y' = \sqrt{1-\frac{y^2}{x^2}}+\frac{y}{x}$. Let $z=\frac{y}{x}$. Then $\frac{dy}{dx} = \frac{d}{dx}(xz) = z + x\frac{dz}{dx}$, so $\frac{dx}{x} = \frac{dz}{\sqrt{1-z^2}}$. Integrating, $\ln x = \arcsin z + C_1$, so $z=\sin(\ln C_2 x)$ and $y = x\sin(\ln C_2 x)$. Plugging in the given point gives $C_2 = e^{\pi/3+2\pi n}$ or $C_2 = e^{-\pi/3+2\pi n}$. $y(1)>0$ gives the proper solution as $y = x\sin\left(\ln e^{\pi/3}x\right)$, whose smallest root greater than 1 is $e^{2\pi/3}$.

30. E The integrand has a v and the integral is taken du, making it equal to $2v$.