-10	$\frac{6-\sqrt{6}}{5}$	15	-508	0
0	20	$-\sqrt{3}$	8	$24\sqrt{3}$
-3	$x=2, y=1$	2	400	$\frac{27\sqrt{3}}{2}$
2	$\frac{2x^2-1}{3x}$	$\frac{7}{24}$	8	$\frac{36}{211}$
$\frac{4}{3}$	121	4	1	$\frac{63}{65}$

1. -10 The projection can be found by plugging in values and simplifying the formula below:
$\langle 5,20,0\rangle \cdot \langle 2,-1,12\rangle = (5)(2)+(20)(-1)+(0)(12) = 10-20 = -10$

2. $\frac{6-\sqrt{6}}{5}$ To rationalize the denominator, we must multiply it by the conjugate and then simplify.
$$N = \frac{\sqrt{6}}{(1+\sqrt{6})}\cdot\frac{(1-\sqrt{6})}{(1-\sqrt{6})} = \frac{\sqrt{6}-6}{1-6} = \frac{6-\sqrt{6}}{5}$$

3. 15
$$\tan(2\theta) = \frac{B}{A-C}$$
$$\tan(2\theta) = \frac{4\sqrt{3}}{7-(-5)} = \frac{4\sqrt{3}}{12}$$
$$\tan(2\theta) = \frac{\sqrt{3}}{3}$$
Since $2\theta = \frac{\pi}{6}$, the angle of rotation is $\frac{\pi}{12}$ or 15 degrees.

4. -508 We can begin by solving for x.
By transitive property, we can say $x = \sqrt{20-x}$
$x^2 = 20 - x$
$x^2 + x - 20 = 0$
$(x-4)(x+5) = 0$
$x \epsilon \{4, -5\}$
Since $x \neq -5$, the only solution is $x = 4$.
Next, we can solve for y.
By factoring, we can get $y = \left[(2\sqrt{2})\left(\frac{i}{2} - \frac{\sqrt{3}}{2}\right)\right]^6$.
$$y = (2\sqrt{2})^6 cis\left(\frac{5\pi}{6}\right)^6$$
$y = (512)cis(5\pi) = (512)cis(\pi) = -512$
$x + y = 4 + (-512) = -508$

5. 0 The bounds of the ranges are:

$\sin^{-1}\theta$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}\theta$	$[0, \pi]$
$\tan^{-1}\theta$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

The sum of the bounds of the ranges can be calculated as: $\left(-\frac{\pi}{2}\right) + \frac{\pi}{2} = 0$

6. 0

The first step of finding the inverse of the matrix would be to find the determinant, as so:

$$\begin{matrix} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \end{matrix}$$

$$= (1)(5)(9) + (2)(6)(7) + (3)(4)(8) - (3)(5)(7) - (8)(6)(1) - (2)(4)(9) = 0$$

7. 20

To find the determinant of a 4x4 matrix, we must use the cofactor expansion.

$$\begin{vmatrix} 3 & 0 & 2 & -1 \\ 1 & 2 & 0 & -2 \\ 4 & 0 & 6 & -3 \\ 5 & 0 & 2 & 0 \end{vmatrix}$$

$$0\begin{vmatrix} 1 & 0 & -2 \\ 4 & 6 & -3 \\ 5 & 2 & 0 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 & -1 \\ 4 & 6 & -3 \\ 5 & 2 & 0 \end{vmatrix} + 0\begin{vmatrix} 3 & 2 & -1 \\ 1 & 0 & -3 \\ 5 & 2 & 0 \end{vmatrix} + 0\begin{vmatrix} 3 & 2 & -1 \\ 1 & 0 & -2 \\ 4 & 6 & -3 \end{vmatrix}$$

$$= 2\begin{vmatrix} 3 & 2 & -1 \\ 4 & 6 & -3 \\ 5 & 2 & 0 \end{vmatrix} = 20$$

8. $-\sqrt{3}$

$\tan(\cos^{-1}(\sin\left(-\frac{\pi}{6}\right)))$

$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

9. 8

Since we know $\log_6 y = x$, we know that $y = 6^x$.
We can then re-write $6^x + 36^x = 72$ as $y + y^2 = 72$.
$y^2 + y - 72 = 0$
$(y - 8)(y + 9) = 0$
The solutions are $y = 8$ and $y = -9$. The second solution does not work, so $y = 8$.

10. $24\sqrt{3}$

The tricky part of this question was realizing that $4^6 = 4096$.
When graphed, we get a hexagon which can be broken in to equilateral triangles of side length 4.

$Area = 6(\frac{1}{2}ab\sin C) = 6(\frac{1}{2} \cdot 4 \cdot 4 \cdot \sin\frac{\pi}{3})$

$Area = 24\sqrt{3}$

11. -3

A formula of the sum of the reciprocal of the roots is the negation of the linear term over the constant term. For this polynomial, it would be $-\frac{12}{4} = -3$.

12. $x = 2$ & $y = 1$

By factoring our equation we get $y = \frac{(x-3)^2}{(x-2)(x-3)}$. This gives us a vertical asymptote of $x = 2$, a hole at $x = 3$, and a horizontal asymptote of $y = 1$ (since the powers of the numerator and denominator are equal).

13. 2 — Finding the determinant of the 3x3 matrix, we get 6, leaving our equation as $\log_{\sqrt{6}} 6 = x = 2$.

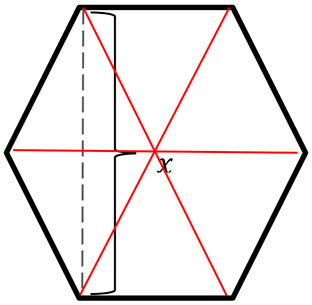
14. 400 — First we must do change of base and simplify with cancellation.

$\log_{11} 625 \cdot \log_7 243 \cdot \log_5 14641 \cdot \log_3 16807$

$= \frac{4 \log 5}{\log 11} \cdot \frac{5 \log 3}{\log 7} \cdot \frac{4 \log 11}{\log 5} \cdot \frac{5 \log 7}{\log 3}$

$= (4)(5)(4)(5) = 400$

15. $\frac{27\sqrt{3}}{2}$ — If we were to re-draw the hexagon to include the 6 equilateral triangles with side length y, we would notice that the line is goes through 2 altitudes.



$$Altitude\ of\ an\ equilateral\ triangle = \frac{y\sqrt{3}}{2}$$

$$x = 3\sqrt{3} = 2\left(\frac{y\sqrt{3}}{2}\right)$$

$$y = 3\ \&\ Area = 6\left(\frac{y^2\sqrt{3}}{4}\right) = \frac{27\sqrt{3}}{2}$$

16. 2 — $\log_2(\log_3(\log_7(\log_{15}(S)))) = 6$

$\log_3(\log_7(\log_{15}(S))) = 2^6$

$\log_7(\log_{15}(S)) = 3^{2^6}$

$\log_{15}(S) = 7^{3^{2^6}}$

$S = 15^{7^{3^{2^6}}}$

15 only has 2 prime factors of 3 and 5.

17. $\frac{2x^2 - 1}{3x}$ — Plugging in $\frac{1}{x}$, we get $2f\left(\frac{1}{x}\right) + f(x) = \frac{1}{x}$. Subtract this from two times the original to get $3f(x) = 2x - \frac{1}{x}$. Therefore, $f(x) = \frac{2x^2-1}{3x}$.

18. $\frac{7}{24}$ — $\sum_{n=1}^{\infty} \frac{1}{n^2 + 6n + 8} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n+2} - \frac{1}{n+4}$

$= \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$

We can see that this is a telescoping series where most terms will cancel out. The only terms that do not cancel leave us with a sum of $\frac{1}{2}\left(\frac{1}{3} + \frac{1}{4}\right) = \frac{7}{24}$.

19. 8 — Using the conversion formulas to Cartesian coordinates, we get $x^2 + y^2 = -6x + 7$. This becomes $(x + 3)^2 + y^2 = 16$The difference between the maximum and minimum is the diameter which is 8.

20. $\frac{36}{211}$ — For Anjana to win in the first round, Aditi and Angela must roll a 4 or below and Anjana must roll a 5 or 6. $P(win\ round\ 1) = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}$. For

Anjana to win in the second round, everyone must have rolled a 4 or below the first round, and Aditi and Angela must the second round.
$P(win\ round\ 2) = (\frac{2}{3})^7 \cdot \frac{1}{3}$. This becomes an infinite geometric series with ratio $\frac{2}{3}^5$ The sum is $\frac{(\frac{2}{3})^2\frac{1}{3}}{1-(\frac{2}{3})^5} = \frac{36}{211}$

21. $\frac{4}{3}$ Factor the bottom by sum of cubes, and simply the expression:

$$\frac{\sin\left(\frac{\pi}{12}\right) + \cos\left(\frac{\pi}{12}\right)}{(\sin(\frac{\pi}{12}))^3 + (\cos(\frac{\pi}{12}))^3}$$
$$= \frac{\sin\left(\frac{\pi}{12}\right) + \cos\left(\frac{\pi}{12}\right)}{(\sin\left(\frac{\pi}{12}\right) + \cos\left(\frac{\pi}{12}\right))((\sin\frac{\pi}{12})^2 - \sin\frac{\pi}{12}\cos\frac{\pi}{12} + (\cos\frac{\pi}{12})^2)}$$
$$= \frac{1}{(\sin\frac{\pi}{12})^2 - \sin\frac{\pi}{12}\cos\frac{\pi}{12} + (\cos\frac{\pi}{12})^2}$$

Using the Pythagorean identity and sine double angle formula we get $\frac{1}{1-\frac{1}{2}\sin\frac{\pi}{6}} = \frac{4}{3}$

22. 121 An odd cubic polynomial means that our function is of the form $f(x) = Ax^3 + Bx$. Factoring an x out gives $f(x) = x(Ax^2 + B)$. This makes 0 one of our solutions, but also the negation of the root $x = 11$, or $x = -11$, must also be a root by difference of squares (A or B must be negative to have a real root). Our value of $n^2 + s^2 = 0 + 121 = 121$.

23. 4 Completing the square on the x terms and the y terms to get $x^2 + 6x + 9 - (y^2 - 4y + 4) = (x+3)^2 - (y-2)^2 = (x+y+1)(x-y+5)$. The y-intercepts of the two lines are at -1 and 5. They add to 4.

24. 1 Expanding out both expressions and noting $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, we obtain $\frac{1}{\sqrt{2}}(3\sin x - 3\cos x + 4\cos x - 4\sin x) = \frac{\cos x - \sin x}{\sqrt{2}}$. Because the arguments of sine and cosine in the numerator are identical, its amplitude is $\sqrt{1^2 + 1^2} = \sqrt{2}$, so the overall amplitude of the function is $\frac{\sqrt{2}}{\sqrt{2}} = 1$.

25. $\frac{63}{65}$ Let $x = \tan^{-1}\left(-\frac{3}{4}\right)$ and $y = \cot^{-1}\left(-\frac{5}{12}\right)$, then x is in quadrant 4 and y is in quadrant 2. Thus $\sin(x + y) = \sin x \cos y + \cos x \sin y = \left(-\frac{3}{5}\right)\left(-\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) = \frac{63}{65}$.