-1	$-\frac{11}{12}$	$\frac{27}{10}$	$\frac{9}{10}$	$\frac{16}{105}$
$2-e^{3/4}$	$\frac{7}{6}$	$\frac{7}{25}$	$\frac{11}{61}$	-35
3π	$\frac{9}{4}$	$-\frac{211}{30}$	$120\pi^2$	15
$\frac{\pi^2}{12}$	2	270π	0	A
$\frac{64}{3}$	$3\pi^2\sqrt{5}$	$2-\sqrt{e}$	25	51

1. -1 $\lim_{x\to\infty}\left(\sqrt{x^2+6x+8}-\sqrt{x^2+8x+6}\right)=\lim_{x\to\infty}\left(\sqrt{(x+3)^2-1}-\sqrt{(x+4)^2-10}\right)=\lim_{x\to\infty}\left((x+3)-(x+4)\right)=-1$.

2. $-\frac{11}{12}$ $y(1)=\frac{1}{2}-\frac{2}{3}+4-5=-\frac{7}{6}$. The derivative is $y'=2x^3-2x^2+4$, which at $x=1$ equals 4. The slope normal to the function is $-\frac{1}{4}$. Solving $y+\frac{7}{6}=-\frac{1}{4}(x-1)$ gives a y-intercept of $-\frac{11}{12}$.

3. $\frac{27}{10}$ By the Product Rule, $4xy+2x^2\frac{dy}{dx}+9x^2y^2+6x^3y\frac{dy}{dx}-1=0$. Plugging in the point $(1,-2)$ yields $-8+2\frac{dy}{dx}+36-12\frac{dy}{dx}-1=0$ and $\frac{dy}{dx}=\frac{27}{10}$.

4. $\frac{9}{10}$ By partial fractions, $\frac{1}{n^2-n}=\frac{1}{n-1}-\frac{1}{n}$, which telescopes. From $n=2$ to 10, this sum is $\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{9}-\frac{1}{10}\right)=1-\frac{1}{10}=\frac{9}{10}$.

5. $\frac{16}{105}$ $u=1-x$ gives an integral equal to $\int_0^1(1-u)^2\sqrt{u}\,du=\int_0^1\left(u^{5/2}-2u^{3/2}+u^{1/2}\right)du=\frac{2}{7}-\frac{4}{5}+\frac{2}{3}=\frac{16}{105}$.

6. $2-e^{3/4}$ Multiplying, $x^5\frac{dy}{dx}=2-y$. $\frac{dy}{2-y}=\frac{dx}{x^5}$, so integrating gives $-\ln(2-y)=-\frac{1}{4x^4}+C$, or $\ln(2-y)=\frac{1}{4x^4}-\frac{1}{4}$. Setting $x=\frac{1}{\sqrt{2}}$ gives $\ln(2-y)=\frac{3}{4}$, or $y=2-e^{3/4}$.

7. $\frac{7}{6}$ This is a $\frac{0}{0}$ indeterminate form, so l'Hospital's gives the limit as equaling $\lim_{x\to1}\frac{3x^2+10x+1}{4x^3-9x^2+2x+15}=\frac{7}{6}$.

8. $\frac{7}{25}$ $\int_3^4 \frac{dx}{\sqrt{25-x^2}} = \arcsin\frac{4}{5} - \arcsin\frac{3}{5}$. $\sin\left(\arcsin\frac{4}{5} - \arcsin\frac{3}{5}\right) = \frac{4}{5}\cdot\frac{4}{5} - \frac{3}{5}\cdot\frac{3}{5} = \frac{7}{25}$.

9. $\frac{11}{61}$ To find the intersection, set $x^3 - x - 4 = x$, or $x^3 - 2x - 4 = 0$, whose only real root is 2. The slope of the graph is $3x^2 - 1$, which equals 11 at $x = 2$. The angle between the vectors $\langle 1,11\rangle$ and $\langle 11,1\rangle$ can be found with $11 + 11 = \sqrt{122}\sqrt{122}\cos\theta$, or $\cos\theta = \frac{11}{61}$.

10. -35 The radius of convergence is 5, centered about -3, so the integers in the range $[-7,1]$ are valid. When $x = 2$, this is the harmonic sequence, but when $x = -8$, this is the alternating harmonic sequence, so the valid integers are in the range $[-8,1]$, and the sum of them is -35.

11. 3π $\int_0^{2\pi} y\,dx = \int_0^{2\pi}(1-\cos t)^2\,dt = \int_0^{2\pi}(1+\cos^2 t - 2\cos t)\,dt = 2\pi + \pi = 3\pi$.

12. $\frac{9}{4}$ The height is 3 times the radius, so the volume of the cone is $\frac{1}{3}\pi r^2 h = \pi r^3$. When $h = 4$, $r = \frac{4}{3}$. $\frac{dV}{dt} = 3\pi r^2\frac{dr}{dt}$, so $4\pi = 3\pi\cdot\frac{16}{9}\frac{dr}{dt}$ and $\frac{dr}{dt} = \frac{3}{4}$ and $\frac{dh}{dt} = \frac{9}{4}$.

13. $-\frac{211}{30}$ $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ and $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$, so $\frac{7}{x-1} = -7 - 7x - 7x^2 - 7x^3 - \cdots$ and $24\cos\sqrt{x} = 24 - 12x + x^2 - \frac{x^3}{30} + \cdots$ and the coefficient of the x^3 term in their sum is $-7 - \frac{1}{30} = -\frac{211}{30}$.

14. $120\pi^2$ The circle can be rewritten as $(x-2)^2 + (y-2)^2 = 15$, which has area 15π. The centroid is at the center, which is $(2,2)$. The distance from $(2,2)$ to the given line is $\frac{8+6+6}{\sqrt{4^2+3^2}} = 4$. By Theorem of Pappus, the volume of the rotated figure is $2\pi\cdot 4\cdot 15\pi = 120\pi^2$.

15. 15 30 meters of fence must be used for the length, and 30 must be used for the width. This means a length of 15 meters and a height of 5 meters for the rectangles, which is a total area of 75 square meters, or 15 square meters per region.

16. $\frac{\pi^2}{12}$ The sum is equal to $\sum_{n=0}^{\infty}\sqrt{3}\cdot\frac{(-1)^n\left(1/\sqrt{3}\right)^{2n+1}}{2n+1} = \sqrt{3}\arctan\frac{1}{\sqrt{3}} = \frac{\pi}{2\sqrt{3}}$. Answer is $\frac{\pi^2}{12}$.

17.	2	The integral is equal to $\int_3^7 \frac{\ln(x-2)}{\ln(x-2)+\ln(8-x)}\,dx$. By the Bounds Trick, substituting $x \to 10-x$ has this equal to $\int_3^7 \frac{\ln(8-x)}{\ln(x-2)+\ln(8-x)}\,dx$. Summing these together gives $\int_3^7 dx = 4$, so the original integral is equal to half this: 2.
18.	270π	$\pi\int_1^4((x+7)^2-1^2)\,dx = \pi\int_1^4(x^2+14x+48)\,dx = \pi\left(\frac{x^3}{3}+7x^2+48x\Big]_1^4\right) = \pi(21+105+144) = 270\pi$.
19.	0	This function has an even derivative and is odd about $(0,-9)$, so since the range is symmetric, the sum of the values that satisfy MVT is 0.
20.	A	Noting the pattern of the numerators, $A-B = \frac{1}{2026}-\frac{4}{2025}+\frac{6}{2024}-\frac{4}{2023}+\frac{1}{2022} = \int_0^1 \frac{(1-x)^4}{x^{2021}}\,dx$, which is clearly positive. Thus, A is larger (by $\sim 7\cdot 10^{-16}$).
21.	$\frac{64}{3}$	This Riemann sum is equal to $\int_0^4 x\sqrt{16-x^2}\,dx = \frac{1}{2}\int_0^{16}\sqrt{u}\,du = \frac{64}{3}$.
22.	$3\pi^2\sqrt{5}$	$\frac{dr}{d\theta} = 4e^{2\theta}$. The arc length is $\int_{\ln\pi}^{\ln 2\pi}\sqrt{4e^{4\theta}+16e^{4\theta}}\,d\theta = 2\sqrt{5}\int_{\ln\pi}^{\ln 2\pi} e^{2\theta}\,d\theta = \sqrt{5}(4\pi^2-\pi^2) = 3\pi^2\sqrt{5}$.
23.	$2-\sqrt{e}$	$u = \sin(\sin x)$ is very useful here. Utilizing the double-angle formula as well, we have $2\int_0^{\arcsin(\pi/6)} e^{\sin(\sin x)}\sin(\sin x)\cos(\sin x)\cos x\,dx = 2\int_0^{1/2} ue^u\,du$. Integrating by parts, this is $2ue^u - 2e^u]_0^{1/2} = 2-\sqrt{e}$.
24.	25	To confirm continuity, plug in $x=1$ to obtain $a+b+c = 2c-3a+2b$, or $4a-b-c=0$. To confirm differentiability, plug in $x=1$ to the derivative to obtain $2a+b = 4c-3a$, or $5a+b-4c=0$. Adding these together gives $9a-5c=0$, Adding to $-3a+5c$ gives $6a=30$ or $a=5$, following with $c=9$. This results in $b=11$. $a+b+c=25$.
25.	51	$b^2-a^2 = (b-a)(b+a) = 2023$. These must both be positive integer factors of $2023 = 7\cdot 17^2$, and a is minimized when they are closest together. This occurs when $b-a=17$ and $b+a=119$. The solution to this is $b=68$ and $a=51$.