1. B
2. C
3. D
4. A
5. C
6. A
7. E
8. C
9. D
10. A
11. E
12. B
13. C
14. A
15. B
16. B
17. D
18. B
19. A
20. C
21. C
22. B
23. C
24. C
25. A
26. A
27. C
28. B
29. C
30. A

1. B
$$\sin(25)\cos(25) = \frac{1}{2}\sin(50), \sin(50)\cos(50) = \frac{1}{2}\sin(100), \ldots$$
The numerator becomes
$$\frac{\sin(400)}{16} = \frac{\sin(40)}{16}$$
The denominator is $\sin 40$, thus the answer is $\frac{1}{16}$.

2. C
$$\sin(2x) = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$
$$\sin^4(x) + \cos^4(x) = (\sin^2(x) + \cos^2(x))^2 - 2\sin^2(x)\cos^2(x) = 1 - \frac{8}{25} = \frac{17}{25}$$

3. D Treat as triangle with sides 2, 5, and angle between of $\frac{11\pi}{18} - \frac{-2\pi}{9} = \frac{5\pi}{6} = 150°$. Use Law of Cosines to find third side (distance between points) → $d^2 = 4 + 25 - 2(2)(5)\cos 150° = 29 + 10\sqrt{3}$. A + B + C = 29 + 10 + 3 = 42.

4. A Rewrite as $5r\cos\theta + 4r\sin\theta = 7 \rightarrow 5x + 4y = 7$. $x - int = \left(\frac{7}{5}, 0\right), y - int = \left(0, \frac{7}{4}\right)$. Shape is triangle, so area = $\frac{1}{2} \cdot \frac{7}{5} \cdot \frac{7}{4} = \frac{49}{40}$.

5. C Since it is a right triangle in a circle, the hypotenuse is the diameter = 10. So, other two side lengths are a, b. $a^2 + b^2 = 100$. Area $\frac{ab}{2}$ is maximized when $a^2 = b^2 = 50$. So, $a = b = 5\sqrt{2}$. Max Area = 25.

6. A $\arccos\frac{1}{5} = \theta. \sin 2\theta = 2\sin\theta\cos\theta$. If $\cos\theta = \frac{1}{5}, \sin\theta = \sqrt{1 - \frac{1}{25}} = \frac{2\sqrt{6}}{5}$. So, $2 \cdot \frac{1}{5} \cdot \frac{2\sqrt{6}}{5} = \frac{4\sqrt{6}}{25}$

7. E $sec^2x - tan^2x = 1. \left(\frac{x-1}{3}\right)^2 - (y-3)^2 = 1$. Hyperbola, so $a = 3, b = 1 \rightarrow c = \sqrt{3^2 + 1^2} = \sqrt{10}$. Distance between foci $= 2c = 2\sqrt{10}$

8. C Cosine = $\frac{u \cdot v}{||u|| \cdot ||v||} = \frac{3(2)+4(5)+12(5)}{\sqrt{9+16+144}\sqrt{4+25+25}} = \frac{86}{13*3\sqrt{6}} = \frac{86\sqrt{6}}{234} = \frac{43\sqrt{6}}{117}$

9. D $\cos\left(x + \frac{\pi}{2}\right) = -\sin x$, so $\sin\frac{\pi}{12} + \cos\frac{7\pi}{12} = 0. \tan\frac{\pi}{8} = \frac{\sin\frac{\pi}{4}}{1+\cos\frac{\pi}{4}} = \sqrt{2} - 1$, by half-angle formula.

10. A $i + \sqrt{3} = 2cis\left(\frac{\pi}{6}\right). \left(2\, cis\left(\frac{\pi}{6}\right)\right)^3 = 8cis\left(\frac{\pi}{2}\right) = 8i$

11. E Note that the form $4\cos^2(x) - 3$ looks a lot like triple angle. ($\cos(3x) = 4\cos^3(x) - 3\cos(x) \rightarrow 4\cos^2(x) - 3 = \frac{\cos(3x)}{\cos(x)}$). So, the equation given transforms to
$$\frac{\cos(27)}{\cos(9)} \cdot \frac{\cos(81)}{\cos(27)} = \frac{\cos(81)}{\cos(9)} = \frac{\sin(9)}{\cos(9)} = \tan(9)$$

12. B By inspection, $\sin(x) + \cos(x) = 1$ has solutions $x = 0, \pi$ if $x \in [0, 2\pi)$
Thus, the solutions to our equation are $x = 0, \pi, 4\pi, 5\pi$. The sum is $10\pi$.

13. C A = 7. B = $\frac{2\pi}{4\pi} = \frac{1}{2}$. C = minimum = -7+13=6 (when cos = 1). D = maximum = 7+13 = 20 (when $cos = -1$). Sum = $\frac{67}{2}$.

14. A Converting this to the form $r = \frac{ep}{1 \pm ecos(\theta)}$ gives $\frac{\frac{7}{4}}{1-\frac{5\cos(\theta)}{4}}$. Thus, $e = \frac{5}{4}$.

15. B Numerator is $\frac{sin^2\theta}{\cos\theta}$. Denominator is $\frac{cos^2\theta}{\sin\theta}$. Fraction becomes $tan^3\theta$. $tan^2\theta$ can be rewritten as $sec^2\theta - 1$.

16. B $(\sin x + \cos x)^2 = 1 + \sin 2x = \frac{1}{2}$. $So, x = -\frac{\pi}{12}, \frac{7\pi}{12} + \pi k$. Same process for second equation, find $y = \frac{3\pi}{4}, \frac{-\pi}{4} + \pi k$. So, $x + y = \frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}, \frac{\pi}{3} + \pi k$. The only possible positive value of $\sin(x+y) = \sin\left(\frac{\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

17. D Period is lcm of periods of two trig functions. So, period of $8\cos 8\pi x$ is $\frac{2\pi}{8\pi} = \frac{1}{4}$. Period of $11 \sin 11\pi x$ is $\frac{2\pi}{11\pi} = \frac{2}{11}$. Lcm = 2.

18. B Rewrite both sides as $(\sin x \cos x)(sin^2x - cos^2x) = \frac{-\sqrt{2}}{4}(sin^2x - cos^2x)$. $sin^2x - cos^2x = -\cos 2x, and \sin x \cos x = \frac{1}{2}\sin 2x$. So, $\cos(2x) = 0 \rightarrow \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$. $\sin(2x) = \frac{-\sqrt{2}}{2} \rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$ . Sum is $7\pi$.

19. A $x^6 = 16. x = 2^{2/3} = \sqrt[3]{4}$. Roots form hexagon with side length of $\sqrt[3]{4}$. Longest distance between two would be the long diagonal, from $\sqrt[3]{4}$ to $-\sqrt[3]{4}$, so length = $2\sqrt[3]{4}$

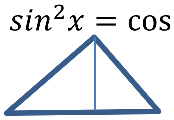
20. C $\tan 225 = 1 = \frac{\tan 102 + \tan 123}{1 - tan 102 \tan 123}$. $So,$ $\tan 102 + \tan 123 = 1 - \tan 102 \tan 123$.

$$\cot 102° \cot 123° - \cot 102° - \cot 123° = \frac{1}{\tan 102 \tan 123} - \frac{1}{\tan 102} - \frac{1}{\tan 123}$$

$$= \frac{1 - \tan 123 - \tan 102}{\tan 102 \tan 123} = \frac{1 - (1 - \tan 102 \tan 123)}{\tan 102 \tan 123} = 1.$$

21. C Creates triangle with side lengths 14, 10 and angle between them of $105°$. $\cos 105 = \frac{\sqrt{2}-\sqrt{6}}{4}$. By law of cosines, distance between them squared is $100 + 196 - 2(140)\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right) = 296 + 70(\sqrt{6} - \sqrt{2})$. $296 + 70 + 6 - 2 = 370$.

22. B Equation can be rewritten as $(cos^2x)^2 - \frac{1}{2} = (sin^2x)^2$. $cos^4x - sin^4x = cos^2x - sin^2x = \cos 2x = \frac{1}{2}$. $x = \frac{\pi}{6}$

23. C 

Angle on bottom left is 60°, on right is 45°. Altitude splits bottom into lenghts of x and 100-x. Altitude can be written as x$\sqrt{3}$ or 100-x, by properties of 30-60-90 and 45-45-90 triangles. Setting x$\sqrt{3} = 100 - x, x = 50\sqrt{3} - 50$. So Height = $\sqrt{3}(50\sqrt{3} - 50) = 150 - 50\sqrt{3}$

24. C Equation is formula for double angle for tan, so it equals tan $\frac{\pi}{6}$, which is cot $\frac{\pi}{3}$

25. A $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$. $\cos\left(x + \frac{\pi}{2}\right) = -\sin x$. $\tan\left(x + \frac{\pi}{2}\right) = -\cot x$. $\cot(x + \pi) = \cot x$, because the period is $\pi$. Sum of all is $-\sin(x) - \cos(x)$

26. A $\lim_{x\to-\infty} \arctan(x) = -\frac{\pi}{2}$
$\lim_{x\to-\infty} \text{arccot}(x) = \pi$
Difference is $-\frac{3\pi}{2}$

27. C $\csc^2\theta - \cot^2\theta = 1$. So, graph is $r = -7$, which is a circle with radius 7. Area = 49π

28. B By Vieta's, $\tan(c) + \tan(d) = -a$, and $\tan(c)\tan(d) = b$ is b. Formula for $\tan(c+d) = \frac{\tan c + \tan d}{1 - \tan c \tan d} = \frac{-a}{1-b} = \frac{a}{b-1}$

29. C

$$w_0^n + w_1^n + w_2^n + \cdots + w_{23}^n = \sum_{k=0}^{23} \text{cis}\left(\frac{kn\pi}{12}\right)$$

Note that these are 24 complex numbers have (not necessarily principal) arguments that are $\frac{n\pi}{12}$ apart.

When $n|k$, those complex numbers make $n$ sets of $\frac{24}{n}$ complex numbers that are equally spaced around the unit circle. They add up to 0, so long $n \neq 24$.

Finally, when $(n, 12) = d$ where $d \neq n$, then those 24 complex numbers can be mapped onto the aforementioned set when $n = d$. So they will also add up to 0.

Therefore, the only $n$ that does not satisfy the condition is $n = 24$, and $1 + 2 + 3 + \cdots + 23 = 276$.

30. A Note that $12 + 82 = 28 + 66 = 94$ and $12 + 74 = 20 + 66 = 86$. We can pair up values in the numerator and denominator and apply sum to product.

$$\frac{\sin 12° + \sin 28° + \sin 66° + \sin 82°}{\cos 12° + \cos 20° + \cos 66° + \cos 74°}$$
$$= \frac{(\sin 12° + \sin 82°) + (\sin 28° + \sin 66°)}{(\cos 12° + \cos 74°) + (\cos 20° + \cos 66°)}$$
$$= \frac{2\sin 47° \cos 35° + 2\sin 47° \cos 19°}{2\cos 43° \cos 23° + 2\cos 43° \cos 31°}$$
$$= \frac{2\sin 47° (\cos 35° + \cos 19°)}{2\cos 43° (\cos 23° + \cos 31°)}$$

$2\sin 47°$ and $2\cos 43°$ cancel. Apply sum to product to the remaining quantities to yield $\frac{2\cos 27° \cos 8°}{2\cos 27° \cos 4°} = \frac{\cos 8°}{\cos 4°}$. Therefore, $x + y = 12$.