For the following questions, select E. NOTA if "None of the Above" answers are correct. Good luck and have fun!

1. Evaluate: $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots$.

   A. $\frac{2}{5}$ B. $\frac{3}{2}$ C. $\frac{5}{2}$ D. $\frac{2}{3}$ E. NOTA

2. Jermaine doubles his music output every third day after today. Assume he does not make any music in between these days. If he makes a quarter of a song today, how many *complete songs* will he make in the 31-day period beginning today?

   A. 255 B. 256 C. 511 D. 512 E. NOTA

3. In order to consider himself worthy, Tuoheng must complete a total of 2024 folder tests during this year. If he completed 1 test on Monday (January 1), 2 tests the next day, 3 tests the day after, and so on, on which day of the week did he reach his goal of 2024 tests completed?

   A. Saturday B. Sunday C. Monday D. Tuesday E. NOTA

4. Evaluate the sum: $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{150\cdot 151}$.

   A. $\frac{150}{151}$ B. $\frac{151}{152}$ C. $\frac{150\cdot 151-1}{150\cdot 151}$ D. 1 E. NOTA

5. Given $e=\sum_{n=0}^{\infty}\frac{1}{n!}$, evaluate: $\sum_{n=1}^{\infty}\frac{n}{(n+1)!}$.

   A. $e-2$ B. 1 C. $e-1$ D. $e-0.5$ E. NOTA

6. Vietvanvu was playing with a ball and accidentally dropped it off a 600-meter cliff. The ball retains 40% of the previous height after each bounce. What will be the total distance the ball will travel after being dropped, assuming the ball does not travel horizontally and this process continues infinitely?

   A. 1000 B. 1200 C. 1400 D. 1600 E. NOTA

7. Let $f(x) = \begin{cases} \cot\left(\frac{k\pi}{3}\right) - \csc\left(\frac{2k\pi}{3}\right) & \text{if } k \neq 0 \bmod 3 \\ \cos\left(\frac{k\pi}{3}\right) & \text{if } k = 0 \bmod 3 \end{cases}$, evaluate $\sum_{k=1}^{20} f(k)$

A. 0 B. $\frac{\sqrt{3}}{3}$ C. $-\frac{\sqrt{3}}{3}$ D. $\sqrt{3}$ E. NOTA

8. Chris's favorite monic 10^{th} degree polynomial has roots that are the first 10 terms of a geometric sequence with first term 1024 and common ratio $\frac{1}{2}$. What is the coefficient of the term containing x^9?

A. -2047 B. -2046 C. 2046 D. 2047 E. NOTA

9. Casie is watching anime. If she watches 59 seconds on day 1, 60 seconds on day 2, 63 seconds on day 3, 68 seconds on day 4, and so on (the number of seconds she watches on a particular day is based on a quadratic), how many minutes will she watch on day 30?

A. 5 B. 15 C. 300 D. 900 E. NOTA

10. Evaluate the sum: $2\tan\frac{\pi}{12} + 2\tan^3\frac{\pi}{12} + 2\tan^5\frac{\pi}{12} + 2\tan^7\frac{\pi}{12} + \cdots$.

A. $\frac{1}{2}$ B. $\sqrt{3}$ C. $\frac{\sqrt{3}}{2}$ D. $\frac{\sqrt{3}}{3}$ E. NOTA

11. Evaluate: $3 + \cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{3+\cdots}}}$.

A. $\frac{9}{2}$ B. $\frac{3+\sqrt{13}}{2}$ C. $\frac{3+3\sqrt{3}}{2}$ D. $\frac{3+3\sqrt{2}}{2}$ E. NOTA

12. If the fraction $\frac{5}{33}$ can be represented as an infinite geometric series with first term $\frac{a}{b}$ and common ratio $\frac{1}{b}$ for positive integers a and b, which of the following is a possible value of $a + b$?

A. 2015 B. 2018 C. 2021 D. 2024 E. NOTA

13. A random dude is doing a test and is asked to evaluate $\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{i\pi n/3}$, where $i=\sqrt{-1}$. Help him out, since he's not so far from a perfect score and is using his option to phone-a-friend!

A. 0 B. 1 C. -1 D. ∞ E. NOTA

14. Evaluate the sum of the first 50 positive perfect cubes.

A. 672400 B. 1625625 C. 1500025 D. 216225 E. NOTA

15. Which of the following statements is true for all sequences $\{a_n\}_{n\geq 1}$ of complex numbers?

A) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty}(-1)^n a_n$ also converges.
B) If $\sum_{n=1}^{\infty} a_n^2$ converges, then $\sum_{n=1}^{\infty} a_n$ also converges.
C) If $\sum_{n=1}^{\infty} a_n^2$ converges, then $\sum_{n=1}^{\infty}(-1)^n\sqrt{|a_n|}$ also converges
D) If $\sum_{n=1}^{\infty} a_n^2$ converges, then $\sum_{n=1}^{\infty} a_n^4$ also converges.
E) NOTA

16. For a sequence $\{a_n\}_{n\in\mathbb{N}}$, a_n is the nth triangular number, $a_n=\sum_{i=1}^{n} i$. How many elements of the sequence $\{a_1, a_2, a_3 \ldots, a_{2023}, a_{2024}\}$ are divisible by 3?

A. 674 B. 675 C. 1349 D. 1350 E. NOTA

17. For a while, RKGiant formatted one test a day. He had gotten bored of formatting. On the plus side, as we all know, practice makes perfect. He initially would take 40 minutes to format an entire test. He was able to reduce the amount of time he spends formatting by 10 minutes each day until it only takes him 10 minutes to format that day's test. Eventually, he ran out of tests to format and got worse. His current time spent formatting one test is the harmonic mean of the formatting times for the first 4 days of practice (counting the initial day). What is his current formatting time, in minutes?

A. $\frac{125}{6}$ B. 22 C. $\frac{96}{5}$ D. 25 E. NOTA

18. If $\sum_{n=1}^{2024} \frac{1+2+3+\cdots+n}{1^3+2^3+3^3+\cdots+n^3} = \frac{Z}{W}$ in simplest form for positive integers Z and W, find $Z - W$.

A. 1011 B. 2023 C. 4047 D. 8095 E. NOTA

19. Evaluate: $\prod_{n=0}^{280} \left(\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}\right)^n$.

A. -1 B. $\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}$ C. $\frac{\sqrt{2}}{2} - \frac{i\sqrt{2}}{2}$ D. 1 E. NOTA

20. Evaluate: $\sum_{n=0}^{\infty} \frac{e^n}{n!}$.

A. $\sqrt[e]{e}$ B. e C. e^2 D. e^e E. NOTA

21. The sum of the reciprocals of the positive integer factors of 2024 can be expressed as $\frac{a}{b}$ for relatively prime positive integers a and b. Compute the value of $a + b$.

A. 6091 B. 2917 C. 1009 D. 793 E. NOTA

22. Gamerninja115 charges up his first ultimate at 12:00AM, the second at 1:00AM, the third at 6:00AM, and so on, where the difference (in hours) between the nth time at which he charges and the $(n+1)$th time he charges is the nth pentagonal number. At what time will he charge up his sixth ultimate?

A. 3:00AM B. 5:00AM C. 6:00AM D. 7:00AM E. NOTA

23. Evaluate: $\sum_{n=0}^{\infty} \frac{\cos(n\pi/4)}{n!}$.

A. $e^{\frac{1}{\sqrt{2}}} \cos\frac{1}{\sqrt{2}}$ B. $e^{\frac{1}{\sqrt{2}}}$ C. $e^{\sqrt{2}} \cos\sqrt{2}$ D. $e^{\sqrt{2}}$ E. NOTA

24. Tanjiro creates concentric circles, the first with radius r and the second with radius $2r$. He continues such that starting with $n = 2$, the area of the ring between the nth and $(n+1)$th circles is $\frac{1}{3}$ of the area of the ring formed by the nth and $(n-1)$th circle. Find the outer radius of the figure Tanjiro creates if he continues this process infinitely.

A. $\frac{3\sqrt{2}}{2}r$ B. $\sqrt{5}r$ C. $\frac{\sqrt{22}}{2}r$ D. $\sqrt{6}r$ E. NOTA

25. Beginning with the terms $F_0 = 0$ and $F_1 = 1$, the Fibonacci sequence continues the terms 0,1,1,2,3,5, ... where the sum of the previous two terms determines the next term. Now let $L = \left(\frac{1+\sqrt{5}}{2}\right)^n$ and $U = \left(\frac{1-\sqrt{5}}{2}\right)^n$. Calculate F_n in terms of L and U.

A. $\frac{L+U}{\sqrt{5}}$ B. $\frac{L+U}{\sqrt{3}}$ C. $\frac{L-U}{\sqrt{5}}$ D. $\frac{L-U}{\sqrt{3}}$ E. NOTA

26. Three distinct integers are chosen from the set $\{1, 2, 3, \ldots, 10\}$. Find the probability that they can be arranged to form an arithmetic sequence.

A. $\frac{3}{20}$ B. $\frac{1}{6}$ C. $\frac{1}{5}$ D. $\frac{5}{24}$ E. NOTA

27. Kaguya was taking her math exam and has one question left. The question is as follows. An infinite geometric sequence has sum S and common ratio r with $0 < r < 1$. Given that the second term of the sequence is 3, what value of r minimizes S?

A. $\frac{1}{9}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. NOTA

28. The roots of $x^6 = 1$ in the Argand plane can be connected to form a convex polygon. By connecting the midpoints of each side, another polygon is formed. Connect the midpoints of this polygon to form another. Repeat the process of connecting the midpoints of each side to form new polygons infinitely. Find the sum of the areas of these polygons.

A. $9\sqrt{3}$ B. $3\sqrt{3}$ C. $4\sqrt{3}$ D. $6\sqrt{3}$ E. NOTA

29. Let $M = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$. Find the trace of $\sum_{n=1}^{2024} M^n$.

A. 2024 B. 4028 C. $\frac{3^{2025}+2^{2025}-3}{2}$ D. $\frac{3^{2025}+2^{2026}-3}{2}$ E. NOTA

30. Find the 142857th digit after the decimal point in the expansion of $\frac{1}{7}$.

A. 1 B. 2 C. 4 D. 8 E. NOTA