1. B
2. C
3. C
4. A
5. B
6. C
7. A
8. B
9. B
10. D
11. B
12. A
13. C
14. B
15. E
16. C
17. C
18. B
19. A
20. D
21. D
22. A
23. A
24. C
25. C
26. B
27. D
28. D
29. A
30. B

1. B This is an infinite geometric series with first term 1 and common ratio $\frac{1}{3}$. Therefore, the sum can be represented as $\frac{a}{1-r} = \frac{1}{1-1/3} = \frac{3}{2}$.
2. C There are 11 days in the given period during which he creates music. During this time, he created $\frac{1}{4} + \frac{1}{2} + 1 + 2 + 4 + \cdots + 256$ songs. Ignore the first two terms; they do not create a full song. Therefore, the sum is $1 + 2 + 4 + \cdots + 256 = 511$ complete songs.
3. C The number of days it would take him to complete 2024 folder tests can be calculated using the formula for the sum of the first n integers. $\frac{n(n+1)}{2} \geq 2024$. The lowest value of n that will accomplish this is 64. This is 63 days from January 1, so since 63 is a multiple of 7, this will also be a Monday.
4. A With partial fractions, the series breaks down into $\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{150} - \frac{1}{151}\right)$. All terms will cancel out except the first and last, leaving the sum $1 - \frac{1}{151} = \frac{150}{151}$.
5. B Adding 0 in a special way, $\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = \sum_{n=1}^{\infty} \left(\frac{n+1}{(n+1)!} - \frac{1}{(n+1)!}\right) = \sum_{n=1}^{\infty} \frac{1}{n!} - \left(-1 + \sum_{n=1}^{\infty} \frac{1}{n!}\right) = 1$.
6. C The total distance the ball falls down is $\frac{600}{1-2/5} = 1000$. However, the ball also travels up. The first movement up is $600 \cdot \frac{2}{5} = 240$, and the total distance moving up is $\frac{240}{1-2/5} = 400$. $1000 + 400 = 1400$.
7. A k will be an even multiple of 3 three times, and it will be an odd multiple of 3 three times, so the sum over k divisible by 3 is 0. For the other terms, let $\theta = \frac{k\pi}{3}$. Rewrite the expression as $\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin 2\theta} = \frac{2\cos^2\theta - 1}{2\sin\theta\cos\theta} = \frac{\cos 2\theta}{\sin 2\theta} = \cot 2\theta = \cot\frac{2k\pi}{3}$. Note that $\cot\frac{2\pi}{3} + \cot\frac{4\pi}{3} = 0$, and since the largest value of k is 20, the total sum is 0.
8. B The roots are all of the powers of 2 between 2 and 1024 (inclusive). The term containing x^9 is the second one. By Vieta's, this coefficient will be the negative of the sum of the roots. Adding 1 to the roots would be a sum of $2^{11} - 1$, so the sum of the roots is $2^{11} - 2 = 2046$ and the coefficient is -2046.
9. B By inspection, Casie watches $59 + (n-1)^2$ seconds on day n. Plugging in $n = 30$ gives $59 + 29^2 = 59 + 841 = 900$ seconds watched. This equals 15 minutes.
10. D This is an infinite geometric series with first term $2\tan\frac{\pi}{12}$ and common ratio $\tan^2\frac{\pi}{12}$. The sum of the sequence is therefore $\frac{2\tan\pi/12}{1-\tan^2\pi/12}$. This is the tangent double-angle formula and evaluates to $\tan\left(\frac{2\pi}{12}\right) = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$.
11. B Let the expression equal x. Then $x = 3 + \frac{1}{x}$. Multiplying by x, $x^2 - 3x - 1 = 0$, which has a positive root of $\frac{3+\sqrt{13}}{2}$.

12. A The sum of the sequence is $\frac{a/b}{1-1/b} = \frac{a}{b-1} = \frac{5}{33}$. Cross-multiplying, $33a - 5b = -5$. a must be a multiple of 5, and b must be 1 more than a multiple of 33. Letting k be a positive integer, if $a = 5k$, then $b = 33k + 1$, so $a + b$ is 1 more than a multiple of 38. $2014 = 38 \cdot 53$, so $a + b$ can equal 2015.

13. C $\lim_{n\to\infty}\left(1 + \frac{a}{n}\right)^{bn} = e^{ab}$, so the limit equals $e^{3i\pi/3} = e^{i\pi} = -1$ by Euler's identity.

14. B The well-known for the sum of the first n cubes is $\left(\frac{n(n+1)}{2}\right)^2$. $\frac{50\cdot 51}{2} = 1275$, and $1275^2 = 1625625$.

15. E Counterexamples for each answer choice include $a_n = \frac{(-1)^n}{n}$, $a_n = \frac{1}{n}$, $a_n = \left(\frac{2+(-1)^n}{n}\right)^2$, and $a_n = \frac{i^n}{\sqrt[5]{n}}$, respectively.

16. C $a_n = \frac{n(n+1)}{2}$, so a_n is divisible by 3 if either n or $n+1$ is; n is either 0 or 2 mod 3. $\frac{2}{3}$ of the positive integers up to and including a number divisible by 3 follow this property, so for $n = 2025$ that is 1350. However, $n = 2025$ needs to be removed because it is not in the given sequence, so there are 1349 valid n.

17. C The harmonic mean is the reciprocal of the mean of the reciprocals of some numbers. Here, $\frac{1}{4}\left(\frac{1}{40} + \frac{1}{30} + \frac{1}{20} + \frac{1}{10}\right) = \frac{3+4+6+12}{4\cdot 120} = \frac{25}{480} = \frac{5}{96}$, so the harmonic mean is $\frac{96}{5}$.

18. B Using sum of powers formulas, the sum is $\sum_{n=1}^{2024} \frac{2}{n(n+1)}$. This telescoping sum is equal to $2\left(\frac{1}{1} - \frac{1}{2025}\right) = \frac{4048}{2025}$. $Z - W = 2023$.

19. A The value inside the parentheses is $\operatorname{cis}\frac{\pi}{4}$, so by De Moivre's, the argument is $\operatorname{cis}\frac{n\pi}{4}$. Further, the product is equal to $\operatorname{cis}\left(\frac{280\cdot 281\pi}{2\cdot 4}\right)$. The argument of this is an odd multiple of π, so the product equals -1.

20. D $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ is the power series for e^x, so plugging in $x = e$ gives a sum of e^e.

21. D The sum of the reciprocals of the factors of 2024 will equal the sum of the factors of 2024 divided by 2024. $2024 = 2^3 \cdot 11 \cdot 23$, which has a sum of factors of $(1+2+4+8)(1+11)(1+23) = 15 \cdot 12 \cdot 24 = 4320$. $\frac{4320}{2024} = \frac{540}{253}$. $540 + 253 = 793$.

22. A The sum of the first five pentagonal numbers is $1 + 5 + 12 + 22 + 35 = 75$. 75 hours from the start is 3 days and 3 hours, so the sixth charging occurs at $3{:}00$ AM.

23. A $\cos(n\pi/4) = \Re\left(e^{in\pi/4}\right)$, so the series is equal to $\Re\left(\sum_{n=0}^{\infty} \frac{e^{in\pi/4}}{n!}\right) = \Re\left(e^{e^{i\pi/4}}\right)$. Simplifying the exponents gives $\Re\left(e^{\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}}\right) = \Re\left(\sqrt[\sqrt{2}]{e}\operatorname{cis}\frac{1}{\sqrt{2}}\right) = \sqrt[\sqrt{2}]{e}\cos\frac{1}{\sqrt{2}}$.

24. C The areas of the rings created form an infinite geometric series with first term $3\pi r^2$ and common ratio $\frac{1}{3}$, which has a sum of $\frac{3\pi r^2}{1-1/3} = \frac{9\pi r^2}{2}$. The first circle has area πr^2 for a total area of $\frac{11\pi r^2}{2}$ and a radius of $\frac{\sqrt{22}}{2}r$.

25. C The characteristic equation of $F_n = F_{n-1} + F_{n-2}$ is $x^2 - x - 1 = 0$, which has roots L and U. F_n is a linear combination of L^n and U^n. Solving a system of equations with the first two terms of the sequence gives $F_n = \frac{L-U}{\sqrt{5}}$.

26. B There are $\binom{10}{3} = \frac{10\cdot 9\cdot 8}{3\cdot 2} = 120$ ways to pick three members of the set. If the common difference is n, then the first term can be between 1 and $10 - 2n$ (inclusive) which is $10 - 2n$ cases. Summing, $\sum_{n=1}^{4}(10 - 2n) = 40 - 2 \cdot 10 = 20$. $\frac{20}{120} = \frac{1}{6}$.

27. D The first term is $\frac{3}{r}$, so the sum of the infinite geometric series is $\frac{3/r}{1-r} = \frac{3}{-r^2+r}$. The sum is minimized when the denominator is maximized, which occurs when $r = \frac{1}{2}$.

28. D The roots form a hexagon with side length 1 and area $\frac{3\sqrt{3}}{2}$. The second hexagon can be formed by subtracting out six triangles, each with area $\frac{1}{2}\left(\frac{1}{2} \cdot \frac{1}{2} \sin\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{16}$ for a total of $\frac{3\sqrt{3}}{8}$. $\frac{1}{4}$ of the original hexagon is removed, so $\frac{3}{4}$ of it remains. This will be an infinite geometric series with first term $\frac{3\sqrt{3}}{2}$ and common ratio $\frac{3}{4}$, so the total area of all the hexagons is $\frac{3\sqrt{3}/2}{1-3/4} = 6\sqrt{3}$.

29. A Note that $M^2 = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 9-6 & -18+12 \\ 3-2 & -6+4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$, so M is idempotent. Each of $M, M^2, \ldots, M^{2024}$ is identical, and each of their traces is 1. The sum of all of their traces is 2024.

30. B $\frac{1}{7} = 0.\overline{142857}$. $142857 \equiv 3 \bmod 6$, so the 142857th digit is a 2.