1. What is the determinant of $\begin{bmatrix} 20 & 19 \\ 19 & 20 \end{bmatrix}$

A. 39 B. 40 C. 380 D. 861 E. NOTA

2. A matrix is both idempotent and nilpotent if and only if

A. The trace is 0
B. The determinant is 0
C. 0 is the only eigenvalue
D. All the elements are 0
E. NOTA

3. If $\langle 5x, 3y, z\rangle \cdot \langle 5x, 3y, z\rangle = 35$, what is the maximum possible value of $\langle 5,4,3\rangle \cdot \langle x, y, z\rangle$?

A. $\frac{\sqrt{105}}{3}$ B. $\frac{\sqrt{3710}}{3}$ C. $\frac{\sqrt{805}}{6}$ D. $\frac{\sqrt{470}}{2}$ E. NOTA

For questions 4-6, $M = \begin{bmatrix} -7 & 0 & 4 \\ -12 & 1 & 6 \\ -20 & 0 & 11 \end{bmatrix}$

4. What are the distinct eigenvalues of M?

A. $-2,3,4$ B. $1,2$ C. $1,3$ D. $0,2,3$ E. NOTA

5. Which of the following is not an eigenvector of M?

A. $\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$ B. $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ C. $\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ D. $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ E. NOTA

6. What is the sum of the geometric multiplicity of the eigenvalue 1 and the algebraic multiplicity of the eigenvalue 2? Note that if a is not an eigenvalue, both the algebraic and geometric multiplicities of a are 0.

A. 0 B. 1 C. 2 D. 3 E. NOTA

7. Find the trace of M if M is a 100×100 matrix such that $M_{i,j} = i + j$.

A. 100 B. 200 C. 5050 D. 10100 E. NOTA

8.
$$\begin{aligned} w + 2x + 12y + 4z &= 4 \\ 3w + 7x + 18y + 5z &= 6 \\ 4w + 3x + 3y + 9z &= 1 \\ 11w + 4x + z &= 0 \end{aligned}$$

Solve for x. (Hint: The system is independent.)

A. -4 B. 0 C. 2 D. 6 E. NOTA

9. Compute the area of the triangle with vertices at $(1,1,1), (2,3,4), (-4,5,7)$.

A. $7\sqrt{13}$ B. $7\sqrt{14}$ C. $\frac{7\sqrt{13}}{2}$ D. $\frac{7\sqrt{14}}{2}$ E. NOTA

10. The determinant of an $n \times n$ matrix M is equal to the trace. What is the ratio of the determinant of matrix 2M to the trace of matrix 2M?

A. 1 B. 2 C. 2^{n-1} D. 2^n E. NOTA

11. Which of the following are false regarding reduced row-echelon form?
I. The reduced row-echelon form of a matrix is unique
II. A matrix must be square in order to have a reduced row-echelon form
III. The rank of a square matrix is equal to the number of columns where exactly one element is 1 and all other elements are 0 in the reduced row-echelon form

A. I, II, III B. I, III only C. II, III only D. I, II only E. NOTA

12. For which of the following values of k is the system $ax + by = k, cx + dy = 2$ guaranteed to have at least one solution? a, b, c, d are positive integers.

A. $\frac{2a}{d}$ B. $\frac{2b}{d}$ C. $\frac{2c}{d}$ D. 2 E. NOTA

13. Find the value of x such that the following matrix is singular: $\begin{bmatrix} x & 6 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 8 \end{bmatrix}$

A. 1 B. $\frac{6}{5}$ C. $\frac{11}{5}$ D. 3 E. NOTA

14. Two integers a, b are selected from the set $\{1, 2, 3\}$ with replacement. For a 3×3 matrix M, row a is interpreted as a vector v. Row b is interpreted as a vector w. If M is orthogonal, which of the following are possible values of the dot product $v \cdot w$?

I. -1 II. 0 III. 1

A. II, III only B. II only C. III only D. None E. NOTA

15. Find the sum of the roots of the characteristic polynomial of $\begin{bmatrix} 4 & -2 & 5 \\ -3 & 6 & 3 \\ 9 & 2 & -7 \end{bmatrix}$

A. -504 B. -4 C. 0 D. 3 E. NOTA

For questions 16-17: A Pascal matrix is a matrix form of Pascal's triangle. That is, a matrix P where non-edge elements are the sum of adjacent elements. For this question, consider only square Pascal matrices that have the first column filled with 1s, the first row filled with 1s, and that satisfy $P_{i+1,j+1} = P_{i+1,j} + P_{i,j+1}$ for all other elements. Let P_n denote the Pascal matrix in this form of size n. For example, $P_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$.

16. Let $M = P_5$. Find $M_{5,5}$.

A. 20 B. 70 C. 126 D. 252 E. NOTA

17. Let $M = P_9$. Find the sum of the elements in the third column of M.

A. 55 B. 126 C. 165 D. 336 E. NOTA

18. Which of the following vectors has the greatest magnitude?

A. $\langle 8, -8, 8 \rangle$ B. $\langle 11, 5, 7 \rangle$ C. $\langle 12, 3, -6 \rangle$ D. $\langle 3, 8, 10 \rangle$ E. $\langle 13, 4, -3 \rangle$

19. What is the inverse of $\begin{bmatrix} 2 & -3 & 4 \\ 6 & -1 & 7 \\ 4 & 2 & 3 \end{bmatrix}$?

A. $\begin{bmatrix} -17 & 17 & -17 \\ 10 & -10 & 10 \\ 16 & -16 & 16 \end{bmatrix}$
B. $\begin{bmatrix} -17 & 10 & 16 \\ 17 & -10 & -16 \\ -17 & 10 & 16 \end{bmatrix}$
C. $\begin{bmatrix} -17 & -17 & -17 \\ -10 & -10 & -10 \\ -16 & -16 & -16 \end{bmatrix}$
D. $\begin{bmatrix} -17 & -10 & -16 \\ -17 & -10 & -16 \\ -17 & -10 & -16 \end{bmatrix}$
E. NOTA

20. Characterize the relationship between the following 2 lines as specifically as possible

$$x = \frac{y-1}{2} = \frac{z+25}{6} \qquad \frac{x}{4} = y - 8 = -z$$

A. Skew B. Intersecting C. Parallel D. Perpendicular E. NOTA

21. What is the scalar projection of $\langle 2, -3, 4 \rangle$ onto $\langle -4, 2, 5 \rangle$?

A. $-\frac{2\sqrt{5}}{5}$ B. $-\frac{2}{15}$ C. $\frac{2}{15}$ D. $\frac{2\sqrt{5}}{5}$ E. NOTA

22. How many of the following are true about triangular matrices?
I. The determinant is the product of elements of the main diagonal
II. It is possible for the matrix to be both upper triangular and lower triangular
III. The transpose of a triangular matrix is a triangular matrix
IV. The inverse of an invertible triangular matrix is a triangular matrix.

A. 4 B. 3 C. 2 D. 1 E. 0

23. Let $M = \begin{bmatrix} 1 & -3 & 4 & 2 \\ -2 & 1 & 3 & 5 \\ -4 & 3 & 0 & 1 \\ 0 & 9 & 2 & 3 \end{bmatrix}$. $M^{-1} \cdot \det(M) = \begin{bmatrix} 28 & -18 & x & -29 \\ 0 & 30 & -15 & -45 \\ -168 & 108 & -96 & -36 \\ 112 & -162 & 109 & 19 \end{bmatrix}$. Solve for x.

A. -121 B. 121 C. 201 D. -31 E. NOTA

24. Which of the following pairs (a, b) makes $\begin{bmatrix} a & b \\ 2 & 3 \end{bmatrix}$ invertible?

A. $(0,0)$ B. $(8,12)$ C. $(-12,8)$ D. $(-18,-27)$ E. NOTA

25. What is the minimum distance between the lines $\{x = 2t, y = 3t - 1, z = -t + 3\}$ and $\{x = s - 7, y = -2s + 4, z = s + 1\}$.

A. $\frac{2\sqrt{37}}{37}$ B. $\frac{4\sqrt{37}}{37}$ C. $\frac{6\sqrt{59}}{59}$ D. $\frac{8\sqrt{59}}{59}$ E. NOTA

26. Find the volume of the parallelepiped that has edges defined by $\langle 1,4,5\rangle, \langle 2,0,6\rangle$, and $\langle -4,3,7\rangle$.

A. 20 B. $\frac{70}{3}$ C. 120 D. 140 E. NOTA

27. Consider 2 chess players, A and B. Player A wins a game with 60% probability after winning a previous game, whereas Player B wins a game with 50% probability after winning a previous game. After a draw or for the first game of a match, both players are equally likely to win. The probability of a draw is 20% in all games. If A and B played 21000 consecutive games, which of the following is the closest to the number of games A be expected to win?

A. 9600 B. 10050 C. 10500 D. 10950 E. 11400

28. If $\vec{u} = \langle 0,7,-24\rangle$, and $\vec{v} = \langle 6,8,-24\rangle$, compute the cosine of the angle between $\vec{u}$ and $\vec{v}$.

A. $\frac{12}{13}$ B. $\frac{316}{325}$ C. $\frac{64}{65}$ D. 1 E. NOTA

29. Find the projection of $\langle 1,4,-3\rangle$ onto $x + y - z = 2$.

A. $\langle 1,4,-3\rangle$ B. $\left\langle \frac{11}{3}, \frac{20}{3}, -\frac{17}{3}\right\rangle$ C. $\left\langle -\frac{5}{3}, \frac{4}{3}, -\frac{1}{3}\right\rangle$ D. $\langle 0,3,-2\rangle$ E. NOTA

30. Which of the following are true regarding $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, where each lowercase letter represents a real number.

I. $tr(M) = a + e + i$ II. $\det(M^2) = (\det(M))^2$

III. $\det(M) = a(ei - fh) + b(fg - di) + c(dh - ge)$

A. I, III only B. II, III only C. III only D. I, II, III E. NOTA