1. Find the smallest root of $f(x) = 8x^4 + 30x^3 - 15x^2 - 65x + 42$.

    A. -3 B. $\frac{-5}{2}$ C. $\frac{-1}{2}$ D. $\frac{1}{2}$ E. NOTA

2. Find a solution to $x^2 = 84 - 80i$

    A. $4\sqrt{2-i}$ B. $2\sqrt{4+i}$ C. $10 - 4i$ D. $10 + 4i$ E. NOTA

3. How many petals are on the rose curve $r = 3\sin 4\theta$?

    A. 3   B. 4   C. 6   D. 8   E. NOTA

4. Given that $a^2 + 2ab = 42$, $b^2 + 2bc = 61$, and $c^2 + 2ac = 41$. What is a possible value of $a + b + c$?

    A. 10   B. 12   C. 100   D. 144   E. NOTA

5. $f(x) = (\cos x)^2 + \frac{\sin x}{2} - \frac{1}{2}$. Find the sum of the roots of $f(x)$ on the interval $[0, 2\pi)$.

    A. $\frac{3\pi}{2}$ B. $\frac{5\pi}{2}$ C. $\frac{7\pi}{2}$ D. $\frac{9\pi}{2}$ E. NOTA

6. Find the sum of the reciprocal of the roots of $P(x)$. $P(x) = 4x^3 - 3x^2 + 13x - 29$.

    A. $\frac{3}{4}$ B. $\frac{13}{4}$ C. $\frac{13}{3}$ D. $\frac{13}{29}$ E. NOTA

7. Given that $4^{7x-y} = 256$ and $343^{2x+y} = 16807$. Find x.

    A. $\frac{17}{27}$ B. $\frac{3}{7}$ C. $\frac{3}{28}$ D. $\frac{17}{28}$ E. NOTA

8. $x = 22!$ What is the last nonzero digit of x?
    A. 2   B. 4   C. 6   D. 8   E. NOTA

9. How many lattice points lie on the graph of $(x - 3)^2 + (y - 6)^2 = 25$?
    A. 4   B. 8   C. 12   D. 24   E. NOTA

10. If x, y are the roots of $f(a) = a^2 - 6a + 6$. What is the value of $x^3 + y^3$?
    A. 12   B. 36   C. 108   D. 432   E. NOTA

11. Given that real numbers x, y satisfy $xy = 4(x + y) - 10$, find the minimum possible value of $x^2 + y^2$.
    A. 0   B. 1   C. 2   D. 4   E. NOTA

12. The two vectors $\langle 2, -6, a \rangle$ and $\langle 3, b, 4 \rangle$ are orthogonal. The two vectors $\langle -2, a, 4 \rangle$ and $\langle 3, 1, -3b \rangle$ are also orthogonal. Find the value of b.
    A. $-\frac{5}{7}$ B. $-\frac{1}{7}$ C. $\frac{3}{7}$ D. 2   E. NOTA

13. What is the value of $A - B$ if $\frac{3x-6}{x^2-5x-6} = \frac{A}{x-6} + \frac{B}{x+1}$
    A. $-\frac{3}{7}$ B. 0   C. $\frac{3}{7}$ D. $\frac{5}{7}$ E. NOTA

14. Devika and Navya want to combine factories that make towels. Devika's factory makes towels in batches of 11, while Navya's factory makes towels in batches of 14. What is the biggest number of towels that they cannot make without splitting a batch?
    A. 84   B. 85   C. 129   D. 142   E. NOTA

15. $x = 0.AB$. If $\frac{1}{450}$ is added to x, the result is $0.\overline{AB}$. What is $\lfloor 10x \rfloor + \lfloor 100x \rfloor$? (Note that A, B are digits. Anything involving A and B is concatenation of digits, not multiplication.)

    A. 3   B. 6   C. 9   D. 12   E. NOTA

16. Given $2 \sin 2x - \sqrt{3} = 2(\sqrt{3} \sin x - \cos x)$. Find the sum of all $x \in [0, 2\pi)$ that satisfy the equation.

    A. 2π B. 3π C. 4π D. 5π E. NOTA

17. Find the remainder when $x^{2024} - 2x^{2023} + 6$ is divided by $x^2 - 3x + 2$

    A. 2   B. 4   C. $x + 2$ D. $x + 4$ E. NOTA

18. Mr. Lu decides to row from Narnia to Hogwarts. If he's against the current his trip will take 4 hours. If he's with the current, his trip will take two hours. If he's traveling 1200 miles, then how fast will he travel if he has no current?

    A. 150   B. 300   C. 360   D. 450   E. NOTA

19. Compute $\frac{6+i}{2-i} + \frac{3-2i}{2+i}$.

    A. $\frac{17-11i}{5}$ B. $\frac{17+11i}{5}$ C. $\frac{15-i}{5}$ D. $\frac{15+i}{5}$ E. NOTA

20. The cubic function $f(x) = ax^3 + bx^2 + cx + d$ satisfies $f(2) = 11, f(3) = 19$, $f(4) = 35, f(5) = 55$. What is the value of $a + b + c + d$?

    A. -9 B. -5 C. 11   D. 15   E. NOTA

21. What is the smallest solution of x for the following equation? $6 = \left| 2x + |x - 2| \right|$

    A. -6 B. $-\frac{4}{3}$ C. $\frac{8}{3}$ D. 3   E. NOTA

22. What is the sum of the solutions on the interval $\theta \in [0 \; 2\pi)$? $2 \sin 2\theta + \tan \theta = \frac{(\cos \theta)^2 + 4}{\cot \theta}$

A. $\frac{\pi}{2}$ B. π C. $\frac{3\pi}{2}$ D. 2π E. NOTA

23. Compute $\sin\left(\tan^{-1}\left(\frac{99}{20}\right)\right)$.

A. $\frac{20}{99}$ B. $\frac{21}{99}$ C. $\frac{20}{101}$ D. $\frac{99}{101}$ E. NOTA

24. The characteristic polynomial of $M = \begin{bmatrix} -1 & 2 & 1 \\ 0 & -2 & 5 \\ 6 & 4 & 3 \end{bmatrix}$ is $-\lambda^3 + a\lambda^2 + b\lambda + c$.

Find $a + b + c$.

A. 37   B. 98   C. 110   D. 131   E. NOTA

25. The 12 roots of the equation $x^{12} + x^6 = 2^2 + 2$ are plotted on the complex plane. They are labeled $A_1, A_2, \dots, A_{12}$, in the order of the arguments of the roots from the least to greatest. The area of the polygon $A_1 A_2 \dots A_{12}$ is $2^p 3^q$ where p, q are rational. $pq = \frac{m}{n}$ in simplest form. Find $|m + n|$. Assume principle arguments on $(-\pi, \pi]$.

A. 25   B. 37   C. 43   D. 47   E. NOTA

26. Let the product of the solutions to the following equation be N: $(\log_2 x)^2 + 3 = 6 \log_2(x^6)$. Find the number of digits of N.

A. 4   B. 5   C. 108   D. 109   E. NOTA

27. Find the number of ordered pairs of positive integers (x, y) that satisfies $5x + 11y = 169$.
    A. 0   B. 1   C. 2   D. 3   E. NOTA

28. Let N be the coefficient of x^{2024} of the polynomial
    $$f(x) = \left((1 + 2x)(1 + 4x^5)(1 + 8x^{25})(1 + 16x^{125})(1 + 32x^{625})\right)^4$$
    Find $\log_2 N$.
    A. 30   B. 31   C. 32   D. 33   E. NOTA

29. Let's take this time to learn about derivatives! A derivative of a function is also known as the instantaneous rate of change of that function. It can be calculated by using limits, namely $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. Compute the derivative of $f(x) = x^2 + 3x - 6$, at $x = 2$.
    A. 7   B. 9   C. 12   D. 13   E. NOTA

30. You've reached the end of the test! Now solve this problem: What is the range of $\sin^{-1} x$?
    A. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ B. $(-1,1)$ C. $\left(0, \frac{\pi}{2}\right)$ D. $(0, \pi)$ E. NOTA