1. D
2. E
3. A
4. A
5. B
6. E
7. C
8. D
9. A
10. D
11. C
12. A
13. B
14. C
15. D
16. C
17. C
18. A
19. C
20. B
21. C
22. B
23. B
24. A
25. E
26. A
27. C
28. B
29. D
30. A

1. D The probability of both events X and Y occurring is $0.3 \times 0.1 = 0.03$. The probability both X and Y don't occur is $(1-0.6) \times (1-0.3) = 0.28$. Since it's clear these cases can't overlap, we add the probabilities to get 0.31.

2. E The probability of flipping 0 heads is
$$\frac{\binom{4}{0}}{2^4} = \frac{1}{16}.$$
So, our conditional probabilities are out of $1 - \frac{1}{16} = \frac{15}{16}.$ As every sequence of coin flips is equally likely, this effectively makes our denominator 15. Then the expected number of tails is
$$\frac{\binom{4}{0} \cdot 0 + \binom{4}{1} \cdot 1 + \binom{4}{2} \cdot 2 + \binom{4}{3} \cdot 3}{15} = \frac{1(0) + 4(1) + 6(2) + 4(3)}{15} = \frac{28}{15}.$$

3. A There are 8 positions for the characters. We first place the space down, which can go in 6 possible spots. Then, there are 7! ways to place the letters. However, there are two C's, A's, and R's. So, we must divide by $(2!)^3$ to account for overcounting. This gives us a final answer of $\frac{6 \cdot 7!}{(2!)^3} = 3780$.

4. A If b represents the number of boys initially at the party and g represents the number of girls initially at the party, then $5b = 2g$. After people leave, we get the equation
$$4(b-3) = 3(g-11).$$
Solving this system gives us $b = 6$ and $g = 15$. Adding them, we get 21.

5. B This is standard stars and bars. There are 11 pieces of candy, so there are 10 spots between them to put our "bars." Our answer is thus $\binom{10}{2} = 45$.

6. E The only options for person A's telephone number are 505,500,5505 and 505,055,0505. The total number of options is $\binom{7}{3}$ so the probability of a palindrome is $\frac{2}{35}$. The only option for person B's telephone number is 336,666,6633 and the total number of options is $\binom{7}{2}$, so the probability of a palindrome is $\frac{1}{21}$. Our final answer is $\frac{1}{21} \times \frac{2}{35} = \frac{2}{735}$.

7. C There is a $\frac{3}{5}$ chance that the hat chooses a green ball and turns it into a red one. Then Fred has a $\frac{3}{5}$ chance to pull a red ball out. There is also a $\frac{2}{5}$ chance the hat chooses a red ball. Then Fred only has a $\frac{1}{5}$ chance to pull out a red ball afterward. The total probability is $\left(\frac{3}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{2}{5}\right)\left(\frac{1}{5}\right) = \frac{11}{25}$.

8. D No matter what 5 numbers are chosen, there are only two orders that will work, one being ascending and one being descending. The number of ways to order the five number is 5!, giving us a probability of $\frac{2}{5!} = \frac{1}{60}$.

9. A Applying the divisibility rule for 11, we know that the sum of the first and third digit must be equal to or 11 greater than the middle digit. So, either the middle digit is double the other two (121, 242, 363, 484) or the first and third digits are greater than 5 and the second digit is 11 less than their sum (616, 737, 858, 979). There are 8 values that work.

10. D The total number of ways to put the three dots on the board is $\binom{9}{3}$. The total number of collinear ways to arrange them is 8 (3 columns, 3 rows, and 2 diagonals). Hence the probability is
$$\frac{8}{9 \times 8 \times \frac{7}{6}} = \frac{2}{21}.$$

11. C The frog has several options to get to the 8th lily pad: A single jump and two triple jumps, one triple jump and four single jumps, or seven single jumps. Their probabilities are $\binom{3}{2}\cdot\frac{1}{2^3}$, $\binom{5}{4}\cdot\frac{1}{2^5}$, and $\binom{7}{7}\cdot\frac{1}{2^7}$ respectively. They come out to be $\frac{3}{8}$, $\frac{5}{32}$, and $\frac{1}{128}$. Added up, it's $\frac{69}{128}$.

12. A $\sec(x)$ is positive between $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. $\csc(x)$ is positive between $(0,\pi)$. $\tan(x)$ and $\cot(x)$ are positive between $\left(0,\frac{\pi}{2}\right)$ and $\left(\pi,\frac{3\pi}{2}\right)$. $\sec(x)$ contains the most in the interval $\left(-\frac{2\pi}{9},\frac{5\pi}{7}\right)$, so it has the highest probability.

13. B The first die doesn't matter. The second has a $\frac{5}{6}$ chance of being different. The third has a $\frac{4}{6}$ chance of being different from the first two. Similarly, there's a $\frac{3}{6}$ chance for the fourth and $\frac{2}{6}$ chance for the fifth. Multiplying everything together, we get $\frac{5}{54}$.

14. C The divisibility rule for 4 states that if the last two digits are a multiple of 4, then the whole number is. Then if B is 8, the number is divisible by 4. There are 5 combinations that have B as 8. If all the digits added together are a multiple of 3 then the number is divisible by 3. The sum of the digits is $19+A+B$. If $A+B=5$, 8, or 11 then $19+A+B$ is a multiple of 3. The values of A and B that satisfy this are: (0,5), (0,8), (1,7), (2,6), (3,5), (2,9), (3,8), and (4,7). This is 8 different combinations, giving us an intermediate sum of $5+8=13$. However, we've overcounted the six-digit numbers that are multiples of 12, so we subtract those two combinations: $13-2=11$. There are 25 possibilities for (A,B), so our final answer is $1-\frac{11}{25}=\frac{14}{25}$.

15. D The three equally likely cases are the first one is a different suit and the last two are the same, the middle one is different and the other two are the same, and the last suit is different and the first two cards have the same suit. We can look at one of these and multiply its probability by 3. If the first two are the same and the last one is different, then the probability is $\frac{12}{51}\times\frac{39}{50}=\frac{78}{425}$. This times 3 is $\frac{234}{425}$.

16. C A normal board has an inner ring area of 9π and a total area of 49π. The expected value of this is $\left(\frac{9}{49}\right)\times 4+\left(\frac{40}{49}\right)\times 1=\frac{76}{49}$. Tom's board has the same total area but an inner ring area of 25π. The expected value for his board is $\left(\frac{25}{49}\right)\times 4+\left(\frac{24}{49}\right)\times 1=\frac{124}{49}$. The positive difference in expected values is $\frac{124-76}{49}=\frac{48}{49}$.

17. C Any two numbers that have one of the numbers as 6 will work. That is 5 possibilities. Also 3 with either 2 or 4 will work. That is 2 possibilities. Any two numbers with a 4 will work, but we already counted (4,3) and (4,6). That is 3 extra possibilities, giving us a final answer of

$$\frac{5+2+3}{\binom{6}{2}}=\frac{2}{3}.$$

18. A The probability Nathan misses one ball is

$$\left(1-\frac{7}{10}\right)\times\frac{6}{10}+\left(1-\frac{5}{10}\right)\times\frac{4}{10}=\frac{18}{100}+\frac{20}{100}=\frac{19}{50}.$$

19. C There are 39 out of 52 cards to take for the first card. There are 26 out of 51 cards to take for the second card. $\frac{39}{52}\times\frac{26}{51}=\frac{13}{34}$.

20. B $\frac{x^2-10x+31}{6}=\frac{(x-5)^2}{6}+1$. Plugging x values into $f(x)$, we get the necessary points: (1,3), (2,2), (3,1), (4,1), (5,1), (6,1), (7,1), (8,2), (9,3), (10,5). So, our expected value is

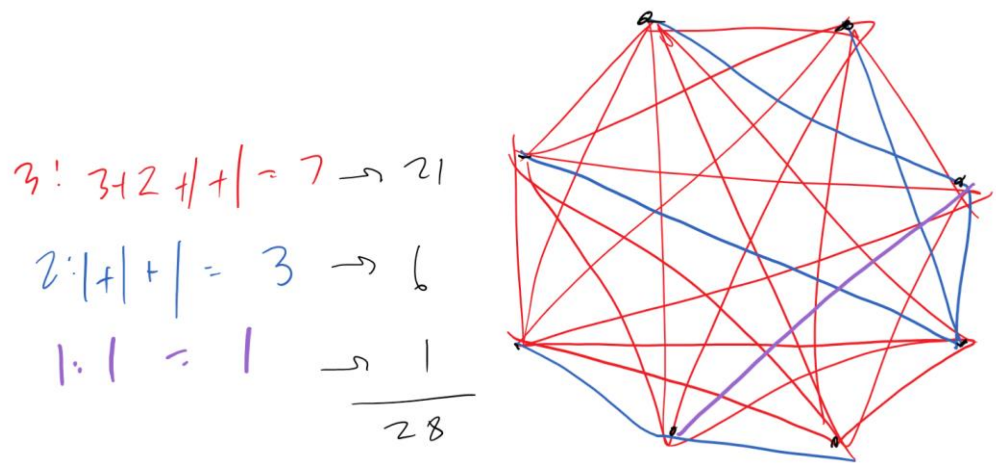
$$\frac{3+2+1+1+1+1+1+2+3+5}{10}=\frac{20}{10}=2.$$

21. C The probability that two people who said, “Mu Alpha Theta” are chosen is $\left(\frac{12}{20}\right)\times\left(\frac{11}{19}\right)$. The probability that two people who said, “Mao Zedong” are chosen is $\left(\frac{6}{20}\right)\times\left(\frac{5}{19}\right)$. These sum to $\left(\frac{66}{190}\right)+\left(\frac{15}{190}\right)=\frac{81}{190}$. This is the probability they said the same phrase so the probability that they are different is $1-\frac{81}{190}=\frac{109}{190}$.

22. B Prime factor 75600 to $2^4\cdot 3^3\cdot 5^2\cdot 7$. The total number of positive factors is $2\times 3\times 4\times 5=120$. The total number of factors without a 3 or 5 is $2\times 5=10$. We subtract the factors without 3 or 5 from the total number of factors to get 110.

23. B The problem can be viewed with graph theory. Let G be an empty graph on 8 nodes (the nodes are analogous to the students). Then every day, a set of 3 nodes from G is chosen (the Dum Dum team) and all the undrawn edges between these 3 nodes are drawn. Hence, we want to find the minimum number of sets of 3 nodes needed to turn G into K_8.

This is done using the greedy algorithm on the number of edges drawn. We get 7 sets where 3 edges are drawn, 3 sets where 2 edges are drawn, and 1 set where 1 edge is drawn.



Thus, our desired answer is $7+3+1=11$.

24. A All the three-digit numbers that start with a 1 only need a 2 in the last two digits. This is 19 three-digit numbers. For numbers that start with 2, they need a 1 in the last two digits. This is also 19 numbers. For any other starting number only two options exist for the last two numbers: 12 and 21. $7\times 2+19+19=52$.

25. E The only ways Tom can get two points before Nolan are he wins the first two rounds (TT), Nolan wins and then Tom wins twice (NTT) or Tom wins then Nolan wins and finally Tom wins (TNT). Their probabilities are $(0.4)(0.4)$, $(0.6)(0.4)(0.4)$, and $(0.4)(0.6)(0.4)$. Adding them together gives $\frac{44}{125}$.

26. A The total number of ways for the rat to get back is $\binom{6}{3}$ and the total number of different paths it can take is 2^6. So, our answer is $\left(6\times 5\times\frac{4}{6}\right)\cdot\frac{1}{64}=\frac{5}{16}$.

27. C The probability that it only rains on the third day is $(0.4)(0.5)(0.3)$. The probability that it only rains on the second day is $(0.4)(0.5)(0.7)$. The probability that it only rains on the first day is $(0.6)(0.5)(0.7)$. The probability it doesn't rain is $(0.4)(0.5)(0.7)$. Add them together to get 0.55.

28. B Bill has to eat at least 5 out of the 6 krill given at random. The probability for 6 is $\frac{\binom{6}{6}}{64}=\frac{1}{64}$ and the probability of 5 is $\frac{\binom{6}{5}}{64}=\frac{3}{32}$. Will only needs at least 4 of the random krill since he is

given an extra one at the end. The probability for 4 is $\frac{\binom{6}{4}}{64} = \frac{15}{64}$ and the probability for 5 and 6 are the same. $\frac{1}{64} + \frac{3}{32} + \frac{1}{64} + \frac{3}{32} + \frac{15}{64} = \frac{29}{64}$.

29. D It's not guaranteed that the events are independent, so there isn't enough information to conclude a probability.

30. A The number of six-digit palindromes is 900. Seven-digit and eight-digit are both 9000. Ten-digit is 90000. Added together is 108900, giving us a digital sum of 18.