Alpha School Bowl
Test #822
Question #0

Alpha School Bowl
Test #822
Question #0

#0 Alpha School Bowl
MAΘ National Convention 2024

Let A be the period of

$$\cos(x) - \sin(x)$$

Let B be the period of

$$\cos^2(x) - \sin^2(x)$$

Let C be the period of

$$\cos^3(x) - \sin^3(x)$$

Let D be the period of

$$\cos^4(x) - \sin^4(x)$$

Submit $A + B + C + D$.

#0 Alpha School Bowl
MAΘ National Convention 2024

Let A be the period of

$$\cos(x) - \sin(x)$$

Let B be the period of

$$\cos^2(x) - \sin^2(x)$$

Let C be the period of

$$\cos^3(x) - \sin^3(x)$$

Let D be the period of

$$\cos^4(x) - \sin^4(x)$$

Submit $A + B + C + D$.

Alpha School Bowl
Test #822
Question #1

Alpha School Bowl
Test #822
Question #1

#1 Division School Bowl
MAΘ National Convention 2024

Let $A =$

$$\langle 1, -4\ ,3\rangle \cdot \langle 3, -2, 5\rangle$$

Let $B =$

$$\|\langle 2, 5, 0\rangle \times \langle -2, 1, 1\rangle\|^2$$

Let $C =$ square of the area of the triangle formed by the points

$$(1, 3, 4), (0, -2, 3), (4, 0, 5)$$

Volume of the pyramid formed by the points

$$(-1, -4, 1), (2, 0, 3), (3,1,4), (0, 2, 4)$$

is $\frac{m}{n}$ in simplest form.

Let $D = m + n$.

Submit $A + B + C + D$.

#1 Alpha School Bowl
MAΘ National Convention 2024

Let $A =$

$$\langle 1, -4\ ,3\rangle \cdot \langle 3, -2, 5\rangle$$

Let $B =$

$$\|\langle 2, 5, 0\rangle \times \langle -2, 1, 1\rangle\|^2$$

Let $C =$ square of the area of the triangle formed by the points

$$(1, 3, 4), (0, -2, 3), (4, 0, 5)$$

Volume of the pyramid formed by the points

$$(-1, -4, 1), (2, 0, 3), (3,1,4), (0, 2, 4)$$

is $\frac{m}{n}$ in simplest form.

Let $D = m + n$.

Submit $A + B + C + D$.

Alpha School Bowl
Test #822
Question #2

Alpha School Bowl
Test #822
Question #2

#2 Alpha School Bowl
MAΘ National Convention 2024

Triangle XYZ has side lengths 13,14,15.

If cosine of the largest angle is $\frac{m_A}{n_A}$ in simplest form, let $A = m_A + n_A$.

If tangent of the smallest angle is $\frac{m_B}{n_B}$ in simplest form, let $B = m_B + n_B$.

If inradius of XYZ is $\frac{m_C}{n_C}$ in simplest form, let $C = m_C + n_C$.

If circumradius of XYZ is $\frac{m_D}{n_D}$ in simplest form, let $D = m_D + n_D$.

Submit $A + B + C + D$.

#2 Alpha School Bowl
MAΘ National Convention 2024

Triangle XYZ has side lengths 13,14,15.

If cosine of the largest angle is $\frac{m_A}{n_A}$ in simplest form, let $A = m_A + n_A$.

If tangent of the smallest angle is $\frac{m_B}{n_B}$ in simplest form, let $B = m_B + n_B$.

If inradius of XYZ is $\frac{m_C}{n_C}$ in simplest form, let $C = m_C + n_C$.

If circumradius of XYZ is $\frac{m_D}{n_D}$ in simplest form, let $D = m_D + n_D$.

Submit $A + B + C + D$.

Alpha School Bowl
Test #822
Question #3

Alpha School Bowl
Test #822
Question #3

#3 Alpha School Bowl
MAΘ National Convention 2024

Let $N = 2^3 \cdot 3 \cdot 5 \cdot 6$

Let $A =$ the number of positive integral factors of N

Let $B =$ the ratio of the sum of the positive even factors to the sum of the positive odd factors.

Let $C =$ the number of positive integers less than N that are relatively prime to N

If the sum of the reciprocal of positive integral factors of N is $\frac{m}{n}$ in simplest form, let $D = m + n$.

Submit $A + B + C + D$.

#3 Alpha School Bowl
MAΘ National Convention 2024

Let $N = 2^3 \cdot 3 \cdot 5 \cdot 6$

Let $A =$ the number of positive integral factors of N

Let $B =$ the ratio of the sum of the positive even factors to the sum of the positive odd factors.

Let $C =$ the number of positive integers less than N that are relatively prime to N

If the sum of the reciprocal of positive integral factors of N is $\frac{m}{n}$ in simplest form, let $D = m + n$.

Submit $A + B + C + D$.

Alpha School Bowl
Test #822
Question #4

Alpha School Bowl
Test #822
Question #4

#4 Alpha School Bowl
MAΘ National Convention 2024

Let $f(x) = x^3 - 4x^2 + 2x + 1$

Let $A =$ sum of the squares of the roots of $f(x)$

Let $B =$ sum of the cubes of the roots of $f(x)$

Let $g(x) = x^4 - 4x^3 + x^2 + 5x - 1$

Let $C =$ sum of the magnitudes of the roots of $g(x)$

Let $D =$ sum of the magnitudes of the reciprocal of the roots of $g(x)$

Submit $A + B + C + D$.

#4 Alpha School Bowl
MAΘ National Convention 2024

Let $f(x) = x^3 - 4x^2 + 2x + 1$

Let $A =$ sum of the squares of the roots of $f(x)$

Let $B =$ sum of the cubes of the roots of $f(x)$

Let $g(x) = x^4 - 4x^3 + x^2 + 5x - 1$

Let $C =$ sum of the magnitudes of the roots of $g(x)$

Let $D =$ sum of the magnitudes of the reciprocal of the roots of $g(x)$

Submit $A + B + C + D$.

Alpha School Bowl
Test #822
Question #5

Alpha School Bowl
Test #822
Question #5

#5 Alpha School Bowl
MAΘ National Convention 2024

Let A be the number of the solutions to the equation

$$\sin(x) = \frac{|x|}{2024\pi}$$

Let B be the sum of the solutions to the equation

$$3\cos(4x) + \cos(2x) + 2 = 0$$

where $0 \le x \le 2\pi$

Let (C, D) be the interval of x within $\left[0, \frac{\pi}{2}\right]$ for which segments of lengths $\sin(x)$, $\cos(x)$, $\frac{1}{3}$ can be arranged to form a triangle with positive area.

Submit $A\pi + B + 2C + 2D$

#5 Alpha School Bowl
MAΘ National Convention 2024

Let A be the number of the solutions to the equation

$$\sin(x) = \frac{|x|}{2024\pi}$$

Let B be the sum of the solutions to the equation

$$3\cos(4x) + \cos(2x) + 2 = 0$$

where $0 \le x \le 2\pi$

Let (C, D) be the interval of x within $\left[0, \frac{\pi}{2}\right]$ for which segments of lengths $\sin(x)$, $\cos(x)$, $\frac{1}{3}$ can be arranged to form a triangle with positive area.

Submit $A\pi + B + 2C + 2D$

Alpha School Bowl
Test #822
Question #6

Alpha School Bowl
Test #822
Question #6

#6 Alpha School Bowl
MAΘ National Convention 2024

Consider the simplified expansion of

$$(x + 2y - 3z)^8$$

Let A be the sum of the coefficients

Let B be the number of terms

Let C be the number of terms such that the power on x is greater than the power on y

Let D be the number of terms with a negative coefficient

Submit $A + B + C + D$.

#6 Alpha School Bowl
MAΘ National Convention 2024

Consider the simplified expansion of

$$(x + 2y - 3z)^8$$

Let A be the sum of the coefficients

Let B be the number of terms

Let C be the number of terms such that the power on x is greater than the power on y

Let D be the number of terms with a negative coefficient

Submit $A + B + C + D$.

Alpha School Bowl
Test #822
Question #7

Alpha School Bowl
Test #822
Question #7

Let Q be the ellipse formed by the equation

$$x^2 - 2xy + 2y^2 = 4$$

Let $A =$ maximum x-coordinate of Q

Let $B =$ maximum y-coordinate of Q

Let $C =$ area of Q

Let $\theta =$ minimal positive angle (clockwise or counter-clockwise) Q can be rotated by to eliminate the xy term.

Let $D = \tan(\theta)$

Submit $A^2 + B^2 + \frac{C}{\pi} + D$.

#7 Alpha School Bowl
MAΘ National Convention 2024

Let Q be the ellipse formed by the equation

$$x^2 - 2xy + 2y^2 = 4$$

Let $A =$ maximum x-coordinate of Q

Let $B =$ maximum y-coordinate of Q

Let $C =$ area of Q

Let $\theta =$ minimal positive angle (clockwise or counter-clockwise) Q can be rotated by to eliminate the xy term.

Let $D = \tan(\theta)$

Submit $A^2 + B^2 + \frac{C}{\pi} + D$.

Alpha School Bowl
Test #822
Question #8

Alpha School Bowl
Test #822
Question #8

#8 Alpha School Bowl
MAΘ National Convention 2024

$$f(x) = \sqrt{x^2 - 4x + 7} - \sqrt{x^2 - 4x + 1}$$

Let $A =$ product of the solutions to

$$f(x) = 1$$

Let $B =$ absolute value of the difference of the solutions of

$$f(x) = 2$$

Let $C = \lim_{x \to -\infty} x f(x)$

Let $D =$ the value of $a + b + c$ for the system of equations $(a, b, c \neq 0)$

$$ab + bc + ca = 6abc$$
$$ab + 2bc - 3ca = abc$$
$$2ab - bc + ca = abc$$

Submit $ABCD$.

#8 Alpha School Bowl
MAΘ National Convention 2024

$$f(x) = \sqrt{x^2 - 4x + 7} - \sqrt{x^2 - 4x + 1}$$

Let $A =$ product of the solutions to

$$f(x) = 1$$

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$$ab + bc + ca = 6abc$$
$$ab + 2bc - 3ca = abc$$
$$2ab - bc + ca = abc$$

Submit $ABCD$.

Alpha School Bowl
Test #822
Question #9

Alpha School Bowl
Test #822
Question #9

#9 Alpha School Bowl
MAΘ National Convention 2024

Define $sic(x) = \sin(x) + icos(x)$ and $cis(x) = \cos(x) + \mathrm{i}\sin(x)$

Let $A =$

$$sic(60°) + cis(30°)$$

Let $B =$

$$sic(181^o) \cdot cis(1^o)$$

Let $C =$

$$sic(10^o)^3 \cdot cis(10^o)^5$$

Let $D =$

$$\sin(11^o) - i\cos(11^o)$$

$ABCD$ can be written as

$$rcis(\theta_1)\ or\ rsic(\theta_2)$$

where $r > 0$ and $-180^o < \theta_1, \theta_2 \leq 180^o$
Submit

$$\theta_1 + 2\theta_2 + r$$

where θ_1, θ_2 are in degrees. (but disregard the degree signs when adding)

#9 Alpha School Bowl
MAΘ National Convention 2024

Define $sic(x) = \sin(x) + icos(x)$ and $cis(x) = \cos(x) + \mathrm{i}\sin(x)$

Let $A =$

$$sic(60°) + cis(30°)$$

Let $B =$

$$sic(181^o) \cdot cis(1^o)$$

Let $C =$

$$sic(10^o)^3 \cdot cis(10^o)^5$$

Let $D =$

$$\sin(11^o) - i\cos(11^o)$$

$ABCD$ can be written as

$$rcis(\theta_1)\ or\ rsic(\theta_2)$$

where $r > 0$ and $-180^o < \theta_1, \theta_2 \leq 180^o$
Submit

$$\theta_1 + 2\theta_2 + r$$

where θ_1, θ_2 are in degrees. (but disregard the degree signs when adding)

Alpha School Bowl
Test #822
Question #10

Alpha School Bowl
Test #822
Question #10

#10 Alpha School Bowl
MAΘ National Convention 2024

Potato, Edward's dog, is initially standing on the point $(0,0)$. To get a snack, she as to complete the following actions. Submit the coordinates she will be standing at the end in (x, y) form.

Step 1
Potato walks 100 units in the positive x direction, turns counter-clockwise 90^o , walks 50 units, turns counter-clockwise 90^o, walks 25 units and repeats ad infinitum, reducing the distance she walks by half every time

Step 2
Potato walks to the graph $y = |x|$. Because Potato is lazy, she chooses the point on the graph $y = |x|$ that minimizes the distance she walks.

Step 3
Then, Potato walking 1 unit in the positive x direction, turns clockwise by 90°, walks 2 units, turns clockwise by 90°, and repeats, increases the distance she walks by 1 unit every time until she turns for the 2024th times. (Right after she walks 2024 units)

Step 4
Finally, Potato goes to the mid-point between her and Edward, who is waiting for her in the origin.

#10 Alpha School Bowl
MAΘ National Convention 2024

Potato, Edward's dog, is initially standing on the point $(0,0)$. To get a snack, she as to complete the following actions. Submit the coordinates she will be standing at the end in (x, y) form.

Step 1
Potato walks 100 units in the positive x direction, turns counter-clockwise 90^o , walks 50 units, turns counter-clockwise 90^o, walks 25 units and repeats ad infinitum, reducing the distance she walks by half every time

Step 2
Potato walks to the graph $y = |x|$. Because Potato is lazy, she chooses the point on the graph $y = |x|$ that minimizes the distance she walks.

Step 3
Then, Potato walking 1 unit in the positive x direction, turns clockwise by 90°, walks 2 units, turns clockwise by 90°, and repeats, increases the distance she walks by 1 unit every time until she turns for the 2024th times. (Right after she walks 2024 units)

Step 4
Finally, Potato goes to the mid-point between her and Edward, who is waiting for her in the origin.

Alpha School Bowl
Test #822
Question #11

Alpha School Bowl
Test #822
Question #11

#11 Alpha School Bowl
MAΘ National Convention 2024

Potato and Edward are competing in the world chess championship. Assume that there are no draws and both players have equal probability of winning.

The rule of the chess championship match is that the first person to win 3 more games than the opponent wins the match

Let A be the expected number of games until someone wins.

The probability that the Potato wins if Potato won the first game can be written as $\frac{m}{n}$ in simplest form.

Let $B = m + n$.

The probability that the match is still going on after 6 games can be written as $\frac{\mathrm{p}}{\mathrm{q}}$ in simplest form.

Let $C = p + q$.

Submit $A + B + C$.

#11 Alpha School Bowl
MAΘ National Convention 2024

Potato and Edward are competing in the world chess championship. Assume that there are no draws and both players have equal probability of winning.

The rule of the chess championship match is that the first person to win 3 more games than the opponent wins the match

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Let $B = m + n$.

The probability that the match is still going on after 6 games can be written as $\frac{\mathrm{p}}{\mathrm{q}}$ in simplest form.

Let $C = p + q$.

Submit $A + B + C$.

Alpha School Bowl
Test #822
Question #12

Alpha School Bowl
Test #822
Question #12

#12 Alpha School Bowl
MAΘ National Convention 2024

Let

$$A = \lim_{x \to 0} \frac{\sin(x)\sin(2x)\sin(3x)}{x^3}$$

$$B = \lim_{x \to 2} \frac{x^3 - 5x + 2}{x^2 - 4}$$

$$C = \lim_{x \to 0} (1 + 2x)^{\frac{1}{3x}}$$

$$D = \lim_{x \to \infty} \left(\sqrt{x^2 + 2x + 7} - \sqrt{x^2 + 4x + 8}\right)$$

Submit $ABD \ln C$.

#12 Alpha School Bowl
MAΘ National Convention 2024

Let

$$A = \lim_{x \to 0} \frac{\sin(x)\sin(2x)\sin(3x)}{x^3}$$

$$B = \lim_{x \to 2} \frac{x^3 - 5x + 2}{x^2 - 4}$$

$$C = \lim_{x \to 0} (1 + 2x)^{\frac{1}{3x}}$$

$$D = \lim_{x \to \infty} \left(\sqrt{x^2 + 2x + 7} - \sqrt{x^2 + 4x + 8}\right)$$

Submit $ABD \ln C$.

Alpha School Bowl
Test #822
Question #13

Alpha School Bowl
Test #822
Question #13

#13 Alpha School Bowl
MAΘ National Convention 2024

Let $N = 2024^{2024}$

Let $A =$ the remainder when N is divided by 100.

Let $B = i^{N+1} + i^{N+2} + i^{N+3}$

Let $C =$ the number of zeroes at the end of $2024!$

Let $D =$ the number of digits of 2^{2024} (Assume $\log(5) \approx .699$)

Submit $A + B + C + D$.

#13 Alpha School Bowl
MAΘ National Convention 2024

Let $N = 2024^{2024}$

Let $A =$ the remainder when N is divided by 100.

Let $B = i^{N+1} + i^{N+2} + i^{N+3}$

Let $C =$ the number of zeroes at the end of $2024!$

Let $D =$ the number of digits of 2^{2024} (Assume $\log(5) \approx .699$)

Submit $A + B + C + D$.

Alpha School Bowl
Test #822
Question #14

Alpha School Bowl
Test #822
Question #14

#14 Alpha School Bowl
MAΘ National Convention 2024

Potato has infinitely many cards with 3 on it, and infinitely many cards with 2 on it.

Let a_n be the sum of all possible sums Potato can get from n cards. (For example: $a_1 = 2 + 3 = 5$, $a_2 = (2 + 2) + (2 + 3) + (3 + 3) = 4 + 5 + 6 = 15$ ...)

$$\sum_{n=1}^{\infty} \frac{a_n}{6^n} = \frac{p}{q}$$

in simplest form.
Let $A = p + q$.

Let b_n be the sum of all possible products Potato can get from the n cards. (For example: $b_1 = 2 + 3 = 5$, $b_2 = 2 \cdot 2 + 2 \cdot 3 + 3 \cdot 3 = 4 + 6 + 9 = 19$ ...)

$$\sum_{n=1}^{\infty} \frac{b_n}{6^n} = \frac{r}{s}$$

in simplest form.
Let $B = r + s$.

Submit $A + B$.

#14 Alpha School Bowl
MAΘ National Convention 2024

Potato has infinitely many cards with 3 on it, and infinitely many cards with 2 on it.

Let a_n be the sum of all possible sums Potato can get from n cards. (For example: $a_1 = 2 + 3 = 5$, $a_2 = (2 + 2) + (2 + 3) + (3 + 3) = 4 + 5 + 6 = 15$ ...)

$$\sum_{n=1}^{\infty} \frac{a_n}{6^n} = \frac{p}{q}$$

in simplest form.
Let $A = p + q$.

Let b_n be the sum of all possible products Potato can get from the n cards. (For example: $b_1 = 2 + 3 = 5$, $b_2 = 2 \cdot 2 + 2 \cdot 3 + 3 \cdot 3 = 4 + 6 + 9 = 19$ ...)

$$\sum_{n=1}^{\infty} \frac{b_n}{6^n} = \frac{r}{s}$$

in simplest form.
Let $B = r + s$.

Submit $A + B$.