0	6π	A: $$lcm(2\pi, 2\pi) = 2\pi$$ B: $$\cos^2(x) - \sin^2(x) = \cos(2x) \to \pi$$ C: $$lcm(2\pi, 2\pi) = 2\pi$$ D: $$\cos^4(x) - \sin^4(x) = \cos^2(x) - \sin^2(x) \to \pi$$ Final: $$2\pi + \pi + 2\pi + \pi = 6\pi$$
1	310	A: $$3 + 8 + 15 = 26$$ B: $$\begin{matrix} i & j & k \\ 2 & 5 & 0 \\ -2 & 1 & 1 \end{matrix} = 5i - 2j + 12k,$$ $$5^2 + 2^2 + 12^2 = 173$$ C: Making the point $(0, -2, 3)$ the origin, $$\begin{matrix} i & j & k \\ 1 & 5 & 1 \\ 4 & 2 & 2 \end{matrix} = 8i + 2j - 18k$$ $$\frac{\sqrt{8^2 + 2^2 + 18^2}}{2} = 7\sqrt{2}, \left(7\sqrt{2}\right)^2 = 98$$ D: Making $(-1, 4, 1)$ the origin, $$\begin{matrix} 3 & 4 & 2 \\ 4 & 5 & 3 \\ 1 & 6 & 3 \end{matrix} = -7$$ $$\frac{\lvert -7 \rvert}{6} = \frac{7}{6}, 7 + 6 = 13$$ Final: $$26 + 173 + 98 + 13 = 310$$
2	103	Let $AB = 13, BC = 14, CA = 15$. If H is the foot of the altitude from A to $BC, BH = 5, CH = 9, AH = 12$ A: $$\frac{5}{13} \to 18$$ B: $$\frac{12}{9} = \frac{4}{3} \to 7$$

<table>
<tr><td></td><td></td><td>C:
$$\frac{[ABC]}{sp}=\frac{84}{21}=\frac{4}{1}\to 5$$
D:
$$\frac{abc}{4[ABC]}=\frac{13\cdot 14\cdot 15}{4\cdot 84}=\frac{65}{8}\to 73$$
Final:
$$18+7+5+73=103$$</td></tr>
<tr><td>3</td><td>775</td><td>$$N=2^3\cdot 3\cdot 5\cdot 6=2^4\cdot 3^2\cdot 5$$
A:
$$(4+1)(2+1)(1+1)=30$$
B: Let K be the sum of the odd factors
$$2K+4K+8K+16K:K=30:1\to 30$$
C:
$$2^4\cdot 3^2\cdot 5\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{4}{5}=192$$
D:
$$\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\right)\left(1+\frac{1}{3}+\frac{1}{9}\right)\left(1+\frac{1}{5}\right)=\frac{403}{120}\to 523$$
Final:
$$30+30+192+523=775$$</td></tr>
<tr><td>4</td><td>62</td><td>A:
$$r_1^2+r_2^2+r_3^2=(r_1+r_2+r_3)^2-2(r_1r_2+r_1r_3+r_2r_3)=16-4=12$$
B:
$$r_1^3=4r_1^2-2r_1-1$$
$$r_2^3=4r_2^2-2r_2-1$$
$$r_3^3=4r_3^2-2r_3-1$$
$$(r_1^3+r_2^3+r_3^3)=4\cdot 12-2\cdot 4-3=37$$

$$g(x)=(x+1)(x^3-5x^2+6x-1)$$
Let
$$h(x)=x^3-5x^2+6x-1$$
Using Descartes rule of signs and
$$h(0)=-1,h(1)=1,h(2)=-1,h(\infty)=\infty$$
Graphing the equation using the points makes it obvious that there must be 3 positive real roots for $h(x)$

C:
$$|-1|+5=6$$
D:
$$|-1|+6=7$$
Final:
$$12+37+6+7=62$$</td></tr>
<tr><td>5</td><td>4057π</td><td>A: There are two solutions per period for $\sin(x)$ and since $\sin(x)\le 1$,
There are $2\cdot\frac{2024\pi}{2\pi}=2024$ non-negative solutions.</td></tr>
</table>

		There are also two solutions per period on the negative side. There are no extra solutions or double-counted solutions, so there are 4048 solutions in total. B: Since $\cos(4x) = 2\cos^2(2x) - 1$, $$6\cos^2(2x) + \cos(2x) - 1 = 0$$ $$\cos(2x) = \frac{1}{3}, -\frac{1}{2}$$ Let $\alpha = \arccos\left(\frac{1}{3}\right), \beta = \arccos\left(-\frac{1}{2}\right)$ The solutions are $2x = \alpha, 2\pi - \alpha, 2\pi + \alpha + 4\pi - \alpha, \beta, 2\pi - \beta, 2\pi + \beta, 4\pi - \beta$ $$\Sigma\, 2x = 16\pi$$ $$\Sigma\, x = 8\pi$$ C: Note that $\sin(x) + \cos(x) > \frac{1}{3}$ for all x in domain. Therefore, C would be when $\cos(x) - \sin(x) = \frac{1}{3}$, and D would be when $\sin(x) - \cos(x) = \frac{1}{3}$. Due to symmetry, these two values add up to $\frac{\pi}{2}$. Final: $$4048\pi + 8\pi + 2\left(\frac{\pi}{2}\right) = 4057\pi$$
6	85	A: $$(1 + 2 - 3)^8 = 0$$ B: $$\binom{10}{2} = 45$$ C: Due to symmetry, if k is the number of terms with $\deg(x) = \deg(y)$, $C = \frac{45-k}{2}$ Since there are 5 terms with $\deg(x) = \deg(y)$, $$C = \frac{45 - 5}{2} = 20$$ D: a negative coefficient means that $\deg(z)$ is odd $$8 + 6 + 4 + 2 = 20$$ Final: $$0 + 45 + 20 + 20 = 85$$
7.	$\frac{31 + \sqrt{5}}{2}$	A: $$D = 0 \to (2x)^2 - 4 \cdot 2 \cdot (x^2 - 4) = 0 \to x = 2\sqrt{2}$$ B: $$D = 0 \to (2y)^2 - 4 \cdot (2y^2 - 4) \to y = 2$$ C: $$\frac{2\pi i}{\sqrt{B^2 - 4AC}} \cdot F = \frac{2\pi i}{\sqrt{-4}} \cdot 4 = 4\pi$$ D:

		$$\tan(2\theta) = \frac{B}{A-C} = 2 = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$$ $$\rightarrow \tan(\theta) = \frac{\sqrt{5}-1}{2}$$ Note that this is minimal since $\tan(\theta) < 1$ Final: $$8+4+4+\frac{\sqrt{5}-1}{2} = \frac{\sqrt{5}+31}{2}$$
8.	$\frac{231\sqrt{13}}{8}$	Let $\sqrt{x^2-4x+7} = a, \sqrt{x^2-4x+1} = b$ A: $$a^2-b^2=6$$ $$a-b=1$$ $$\rightarrow a=\frac{7}{2}, b=\frac{5}{2}$$ $$\rightarrow x^2-4x+7=\frac{49}{4}$$ product of roots = $-\frac{21}{4}$ B: $$a^2-b^2=6$$ $$a-b=2$$ $$\rightarrow a=\frac{5}{2}, b=\frac{3}{2}$$ $$x^2-4x+7=\frac{25}{4}$$ $$\rightarrow (x-2)^2=\frac{13}{4}$$ $$\rightarrow x = 2 \pm \frac{\sqrt{13}}{2}$$ $$B=\sqrt{13}$$ C: $xf(x) = x \cdot \left(\sqrt{x^2-4x+7} - \sqrt{x^2-4x+1}\right) = \frac{6x}{\sqrt{x^2-4x+7}+\sqrt{x^2-4x+1}} \approx \frac{6x}{2\|x\|} = -3$ D: dividing by abc, $$\frac{1}{c}+\frac{1}{a}+\frac{1}{b}=6$$ $$\frac{1}{c}+\frac{2}{a}-\frac{3}{b}=1$$ $$\frac{1}{c}-\frac{1}{a}+\frac{1}{b}=1$$ This is a simple 3 variable 3 equation $$\left(\frac{1}{c},\frac{1}{a},\frac{1}{b}\right) = (1,3,2) \rightarrow 1+\frac{1}{3}+\frac{1}{2}=\frac{11}{6}$$

		Final: $$\frac{-21}{4}\cdot\sqrt{13}\cdot(-3)\cdot\frac{11}{6}=\frac{231}{8}\sqrt{13}$$
9	31	Since $\cos\left(\frac{\pi}{2}-x\right)=\sin(x)$, $\sin\left(\frac{\pi}{2}-x\right)=\cos(x)$ $$cis(x)=sic\left(\frac{\pi}{2}-x\right)$$ A: $$sic\left(\frac{\pi}{3}\right)+cis\left(\frac{\pi}{6}\right)=2cis\left(\frac{\pi}{6}\right)=2cis(30^o)$$ B: $$sic(181^o)\cdot cis(1^o)=cis(-91^o)\cdot cis(1^o)=cis(-90^o)$$ C: $$sic(10^o)^4\cdot cis(10^o)^5=cis(80^o)^3\cdot cis(10^o)^5=cis(290^o)$$ D: $$\cos(79^o)-i\sin(79^o)=cis(-79^o)$$ Final: $$ABCD=2cis(30-90+290-79)=2cis(151)=2sic(-61)$$ $$2+151-122=31$$
10	$(-476,536)$	Step 1 $$x: 100-\frac{100}{4}+\frac{100}{16}\ldots=\frac{100}{1+\frac{1}{4}}=80$$ $$y: \frac{100}{2}-\frac{100}{8}+\frac{100}{32}\ldots=\frac{50}{1+\frac{1}{4}}=40$$ Final $(80,40)$ Step 2 Let the closest point be (a,b) Since $(80,40)$ is in Q1, $a=b$. $$\frac{80-a}{40-a}=-1\rightarrow a=60$$ Final $(60,60)$ Step 3 $$x: 1-3+5\ldots-2023=-1012$$ $$y: -2+4-6+8\ldots+2024=1012$$ Final: $(60-1012,60+1012)=(-952,1072)$ Step 4 (Final) $$x\rightarrow\frac{x}{2}, y\rightarrow\frac{y}{2}$$ $$\left(-\frac{952}{2},\frac{1072}{2}\right)=(-476,536)$$
11	39	A: Let E_n be the expected number of games until someone wins if one player is ahead by n

		It follows that $E_3 = 0$ $E_0 = E_1 + 1$ $E_1 = \frac{1}{2}E_0 + \frac{1}{2}E_2 + 1$ $E_2 = \frac{1}{2}E_1 + \frac{1}{2}E_3 + 1$ $\to E_0 = 9, E_1 = 8, E_2 = 5$ $A = E_0 = 9$ B: Let $P_n$be the probability that Potato wins if shes ahead by n games It follows that $P_0 = \frac{1}{2}, P_3 = 1$ $P_1 = \frac{1}{2}P_0 + \frac{1}{2}P_2$ $P_2 = \frac{1}{2}P_1 + \frac{1}{2}P_3$ $\to P_1 = \frac{2}{3}, P_2 = \frac{5}{6}$ $B = P_1 = \frac{2}{3} \to 5$ C: $i)\ 3-3$ $\binom{6}{3} - 2 = 18$ ways for the game to not end (2 is for $WWWLLL, LLLWWW$) $ii)\ 4-2$ This is counting how to go from $(0,0)$ to $(4,2)$ without reaching $(3,0),(4,1)$ $\to 9$ $\frac{18 + 9 \cdot 2}{64} = \frac{36}{64} = \frac{9}{16} \to 25$ Final: $9 + 5 + 25 = 39$
12	-14	A: $\frac{\sin(x)}{x} \cdot \frac{\sin(2x)}{2x} \cdot \frac{\sin(3x)}{3x} \cdot 6 = 6$ B: $\frac{(x-2)(x^2+2x-1)}{(x-2)(x+2)} = \frac{7}{4}$ $\ln C$: $\ln(1+2x)^{\frac{1}{3x}} = \ln\left(e^{\frac{2}{3}}\right) = \frac{2}{3}$ D: $\sqrt{\infty^2 + 2\infty + 7} - \sqrt{\infty^2 + 4\infty + 8} = (\infty + 1) - (\infty + 2) = -1$ $6 \cdot \frac{7}{4} \cdot \frac{2}{3} \cdot (-1) = -14$
13	1188	A: $N = 0\ (mod\ 4)$

		$$N=(-1)^{2024}=1\ (mod\ 25)$$ $$\rightarrow N=76\ (mod\ 100)$$ B: $$N=0\ (mod\ 4)$$ $$\rightarrow i^{N+1}+i^{N+2}+i^{N+3}=i-1-i=-1$$ C: $$404+80+16+3=503$$ D: $$S=2^{2024}\rightarrow \log(S)=2024\log(2)=2024*(.301)=609.224$$ $$\rightarrow S=10^{609.224}\rightarrow 610$$ $$76-1+503+610=1188$$
14	164	$$a_n=\frac{5(n^2+n)}{2}$$ $$5\sum_{n=1}^{\infty}\frac{n(n+1)}{2\cdot 6^n}=5\left(\frac{1}{6}+\frac{3}{36}+\frac{6}{216}\dots\right)$$ Let $$S=\frac{1}{6}+\frac{3}{36}+\frac{6}{216}\dots$$ $$6S=1+\frac{3}{6}+\frac{6}{36}+\frac{10}{216}$$ $$5S=1+\frac{2}{6}+\frac{3}{36}\dots=\frac{36}{25}$$ $$S=\frac{36}{125}\rightarrow 161$$ $$b_n=2^n+2^{n-1}\cdot 3+2^{n-2}\cdot 3^2+\cdots+2\cdot 3^{n-1}+3^n=3^{n+1}-2^{n+1}$$ $$\sum_{n=1}^{\infty}\frac{3\cdot 3^n-2\cdot 2^n}{6^n}=\frac{\frac{3}{2}}{1-\frac{1}{2}}-\frac{\frac{2}{3}}{1-\frac{1}{3}}=\frac{2}{1}\rightarrow 3$$ Final: $$161+3=164$$