For all questions, E. NOTA stands for none of the above. Good luck and have fun!

1. Evaluate $\sin 15° \sin 75° \sin 105°$.

   A. $\frac{\sqrt{6}-\sqrt{2}}{8}$ B. $\frac{\sqrt{6}+\sqrt{2}}{8}$ C. $\frac{\sqrt{6}-\sqrt{2}}{16}$ D. $\frac{\sqrt{6}+\sqrt{2}}{16}$ E. NOTA

2. Spots the dog is leashed to a vertex of a barn that is shaped as a regular hexagonal. Each side of the barn is 5 meters long. Spots' leash is 6 meters long and he cannot enter inside the barn but can roam around in the yard surrounding the barn. What is the area in which Spots can roam?

   A. 24π B. $\frac{73}{3}\pi$ C. $\frac{76}{3}\pi$ D. 36π E. NOTA

3. Compute $\cot\left(\sin^{-1}\frac{5}{\sqrt{x^2+9}}\right)$.

   A. $\frac{\sqrt{x^2+9}}{5}$ B. $\frac{\sqrt{x^2-16}}{5}$ C. $\frac{5}{\sqrt{x^2+9}}$ D. $\frac{x}{\sqrt{x^2+9}}$ E. NOTA

4. Angela likes making pies. Initially, she makes 9 pies and sells them at \$1.50 each. She finds that for each 10 cent decrease in the price she sells 3 more pies. What price should she sell her pies at to maximize her revenue?

   A. \$0.90 B. \$1.20 C. \$1.70 D. \$2.10 E. NOTA

5. A bakery sells unique cakes. One of their cakes is shaped like a triangular pyramid formed by the vectors $\langle 4,-2,5\rangle$, $\langle 3,10,-7\rangle$, and $\langle -2,3,9\rangle$. What is the volume of the cake?

   A. 205 B. $\frac{205}{2}$ C. 432 D. 615 E. NOTA

6. If M is the rotation matrix needed in order to rotate the point $(3,4)$ sixty degrees counterclockwise with respect to the origin, what is $M_{1,2}$?

   A. $-\frac{\sqrt{3}}{2}$ B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. $\frac{\sqrt{3}}{2}$ E. NOTA

7. Give the product of the real solutions for x in the equation $4^x + 3 \cdot 2^x = 4$.

A. $\frac{1}{16}$ B. $\frac{1}{8}$ C. 1 D. 8 E. NOTA

8. Find the minimum value of $\frac{25x^2 (\sin x)^2 + 16}{x \sin x}$ if $0 < x < \frac{\pi}{2}$.

A. 9 B. 16 C. 20 D. 40 E. NOTA

9. Hannah has a favorite function which is defined by $f(x) = \sin(2x)$. Tucker has a favorite function which is defined by $g(x) = 6x \cos^2 \frac{x}{2} - 3x$. Nia has a favorite function which is defined by Hannah's favorite function over Tucker's favorite function (namely $\frac{f(x)}{g(x)}$). What number will Nia's favorite function approach as x goes to 0?

A. 0 B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. $\frac{3}{2}$ E. NOTA

10. A cube with space diagonal length 4 has the same surface area as a sphere. What is the volume of the sphere?

A. $\frac{256\sqrt{3}}{27\sqrt{\pi}}$ B. $\frac{32\sqrt{3}}{3\sqrt{\pi}}$ C. $\frac{64\sqrt{2}}{3\sqrt{\pi}}$ D. $\frac{64\sqrt{6}}{\sqrt{\pi}}$ E. NOTA

11. A bridge is shaped like a parabola. Both ends of the bridge are at ground level. The maximum height of the bridge is 15 feet. Five feet (horizontally) from the middle of the bridge, the bridge is 5 feet tall. How wide is the bridge, in feet?

A. $\frac{5\sqrt{6}}{2}$ B. 15 C. $5\sqrt{6}$ D. 30 E. NOTA

12. A bug travels in an erratic path to reach its home. It travels 2 meters to the right, turns left 90 degrees, then travels 1 meter, turns left 90 degrees, then travels ½ meters, and so on. After an infinite amount of time, how many meters will the bug be from its starting point?

A. $\frac{4\sqrt{5}}{5}$ B. $\frac{8\sqrt{5}}{7}$ C. $\frac{4\sqrt{2}}{3}$ D. 4 E. NOTA

13. A tugboat is pulling a barge with a taut cable. The point of connection on the barge is 20 yards above the point of connection on the tugboat. If the angle of elevation on the cable is 30°, how long, in feet, is the cable?

A. $\frac{20\sqrt{3}}{3}$ B. 20 C. $20\sqrt{3}$ D. 40 E. NOTA

14. A population of bacteria grows exponentially. At time t = 0, there are 350 bacteria. If it takes 7 hours for the bacteria to triple, at what time will there be 700 bacteria in the population?

A. $\frac{14}{3}$ B. $\frac{7\ln 2}{\ln 3}$ C. $\frac{7\ln 3}{2\ln 2}$ D. $\frac{3\ln 7}{2}$ E. NOTA

15. Ms. Frizzle is grading math tests. After grading 23 math tests, she grades Wanda's test. Wanda's test score raises the class average by 2 points. The class average before Wanda's score was added was an integer that was two more than a multiple of 7, and all of the math test scores are integers from 1 to 100. Which of the following could be Wanda's test score?

A. 93 B. 96 C. 97 D. 99 E. NOTA

16. A fair is selling cotton candy in the shape of the polar graph $r = 2 - 5\cos\theta$ from $0 \leq \theta < 2\pi$ (yes, the cotton candy has an inner loop). A fly is traveling near the cotton candy in the shape of the polar graph $r = \cos 2\theta$ from $0 \leq \theta < 2\pi$. What is the smallest θ for which the fly intersects the graph of the cotton candy?

A. 0 B. $\frac{\pi}{6}$ C. $\frac{\pi}{3}$ D. $\frac{5\pi}{3}$ E. NOTA

17. What is the expected value of the number of times Nicole must roll a fair six-sided die until she gets a five if her first four rolls were all twos? (Including the first four rolls)

A. 6 B. 7 C. 10 D. 12 E. NOTA

18. Publix is selling ice cream in the shape of an inverted cone. Initially, the radius of the cone is 4 inches and the height is 6 inches. However, it is very hot and the ice cream beings to melt at a rate of π cubic inches per second out of the bottom of the cone. The remaining ice cream still remains in a smaller inverted cone shape. After 5 seconds, what is the height of the remaining ice cream?

A. $3\sqrt[3]{2}$ B. $\frac{9\sqrt[3]{2}}{2}$ C. $\frac{3}{2}\sqrt[3]{36}$ D. $\sqrt[3]{36}$ E. NOTA

19. Nathan can hit a baseball a certain distance which is inversely proportional to his height and directly proportional to the score he earns on his math test. If Nathan can hit a baseball 35 feet when he is 6 feet tall and scores an 84 on his math test, how far can he hit a baseball, in feet, when he is 5 feet 3 inches tall and scores a 91 on his math test?

A. $\frac{130}{3}$ B. $\frac{735}{26}$ C. $\frac{455}{12}$ D. $\frac{104}{15}$ E. NOTA

20. In the future, MAO tests have changed to have 32 questions each with 8 answer choices, one of them being correct. A correct question earns 3 points, an incorrect earns -4, and a blank earns 1. If a student takes a MAO test and guesses randomly on all of the questions, what is the expected score of the student?

A. -120 B. -100 C. -80 D. -20 E. NOTA

21. If $\sin x = \frac{2}{3}$ where $\frac{\pi}{2} \leq x \leq \pi$ and $\tan y = \frac{9}{40}$ where $\pi \leq y \leq \frac{3\pi}{2}$, what is $\cos(x+y)$?

A. $\frac{-120-41\sqrt{5}}{123}$ B. $\frac{-18+40\sqrt{5}}{123}$ C. $\frac{18+40\sqrt{5}}{123}$ D. $\frac{80+9\sqrt{5}}{123}$ E. NOTA

22. Dana is traveling on a path marked out by the vector $\langle 5,3,-4\rangle$. Hannah is traveling on the vector $\langle 1,2,-7\rangle$.What vector results from the orthogonal projection of Dana's path onto Hannah's?

A. $\left\langle \frac{39}{10}, \frac{117}{50}, -\frac{78}{25} \right\rangle$ B. $\left\langle \frac{\sqrt{2}}{2}, \frac{3\sqrt{2}}{5}, \frac{14\sqrt{2}}{5} \right\rangle$

C. $\left\langle \frac{13}{18}, \frac{13}{9}, -\frac{91}{18} \right\rangle$ D. $\left\langle \frac{5\sqrt{6}}{18}, \frac{\sqrt{6}}{3}, \frac{14\sqrt{6}}{9} \right\rangle$ E. NOTA

23. Himal invests \$P in her bank account. The bank has a 5% interest rate annually and compounded continuously. After 1 year, Himal has \$861 in her bank account. How many years after Himal initially invests in her bank account will she have \$943 in her bank account?

A. $20 \ln\left(\frac{23}{21}\right)$ B. $\frac{\ln\left(\frac{21}{23}\right)}{1.05} + 1$ C. $\frac{\ln(0.05)}{\ln\left(\frac{23}{21}\right)}$ D. $\frac{\ln\left(\frac{23}{21}\right)}{0.05} + 1$ E. NOTA

24. What smallest acute angle of rotation (in either direction) to eliminates the xy term in $2x^2 + xy + y^2 + 4x - 7y = 1$?

A. $\frac{\pi}{8}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{3\pi}{8}$ E. NOTA

25. McKayla has fifteen different markers which are all different colors. She picks three of them and remembers the colors that she chose. If she puts those three back, mixes up the markers, and picks three random markers again, what is the probability that exactly one of them is the same as one of the three original markers that she picked?

A. $\frac{48}{125}$ B. $\frac{66}{455}$ C. $\frac{198}{455}$ D. $\frac{44}{91}$ E. NOTA

26. There is a geometric sequence with first term 2, seventh term 128, and common ratio r, which is a complex number. When plotting the possibilities for r on the complex plane, what is the shortest distance between two distinct possibilities of r?

A. 2 B. $2\sqrt{2}$ C. $2\sqrt{3}$ D. 4 E. NOTA

27. Two birds start at the same point. Bird A flies at a rate of 16 mph from the point. Bird B flies at a rate of 8 mph at a flight path with an angle of 60° to Bird A's flight path but starts flying 30 minutes after Bird A starts flying. How far apart are the two birds 75 minutes after Bird A starts flying?

A. $2\sqrt{31}$ B. $2\sqrt{79}$ C. $10\sqrt{3}$ D. $2\sqrt{151}$ E. NOTA

28. How many times does the line $y = \frac{x}{20\pi}$ intersect the graph of the function $y = \sin x$?

A. 19 B. 21 C. 39 D. 41 E. NOTA

29. On Valentine's Day, Barney writes a personalized love letter to each of the six girls in his science class. Barney asks Joey to hand out each card to its designated recipient. However, Joey is a saboteur. How many ways can Joey hand out the love letters to the six girls such that no girl receives the love letter that was originally written for her?

A. 66 B. 96 C. 120 D. 265 E. NOTA

30. Nicholas has a fair 6-sided dice that has $\{0,1,2,3,4,5\}$ on its faces. He rolls the die infinite times to create the sequence $\{a_1, a_2, a_3, \dots, a_n, \dots\}$ where a_n corresponds to his n^{th} roll. Let k be the smallest positive integer such that $\prod_{n=1}^{k} a_n = m^2$ for some integer m. The expected value of k can be written as $\frac{p}{q}$ in simplest form. Find $p + q$.

A. 29 B. 43 C. 47 D. 55 E. NOTA