1. A
2. B
3. C
4. D
5. E
6. A
7. B
8. C
9. C
10. A
11. A
12. B
13. B
14. D
15. E
16. D
17. D
18. C
19. B
20. D
21. C
22. D
23. A
24. A
25. D
26. D
27. A
28. B
29. A
30. A

For conics in the form $\frac{(x-h)^2}{a^2} \pm \frac{(y-k)^2}{b^2} = 1$, *$a$ is half of the length of the major axis, b is half of the length of the minor axis, and c is the distance between the center and a focus.*

1. A The equation can be written as $\frac{(x-3)^2}{16} + \frac{(y+5)^2}{27} = 1$, so the length of the major axis is $\boldsymbol{6\sqrt{3}}$.
2. B The distance between the foci is $2c$, so $c = 14$. Since eccentricity is $\frac{c}{a}$, $a = 6$ and $b = 4\sqrt{10}$. The length of the latus rectum is $\frac{2b^2}{a}$ and plugging in the values gets $\boldsymbol{\frac{160}{3}}$.
3. C By graphing the equation, we can find the length of the major and minor axes: 16 and $8\sqrt{3}$, respectively. The area of an ellipse is $ab\pi$, so the area is $8 \times 4\sqrt{3} \times \pi = \boldsymbol{32\pi\sqrt{3}}$.
4. D An equilateral triangle is formed about the origin with the distance between a vertex and the origin being 3. Using the Law of Cosines, the side of the triangle is found to be $3\sqrt{3}$ and the area $\boldsymbol{\frac{27\sqrt{3}}{4}}$.
5. E The equation of the hyperbola is $\frac{(x+3)^2}{16} - \frac{(y-2)^2}{84} = 1$, so the asymptotes are $\boldsymbol{y = \pm\frac{\sqrt{21}}{2}(x+3) + 2}$.
6. A The equation of the parabola is $-12(y-5) = (x-3)^2$, so the base of the triangle is 12 and the height 6. The area is then **36**.
7. B From the equation, we can tell that $a = 8$, $b = 15$, and $c = 17$. The length of the latus rectum is $\frac{2b^2}{a} = \frac{225}{4}$. The rectangle's other two sides has length of $2c = 34$, so the area is $34 \times \frac{225}{4} = \boldsymbol{\frac{3825}{2}}$.
8. C The locus of points is a **line**.
9. C There are two cases.
   i) $\angle APB = \frac{\pi}{2}$
   ii) $\angle ABP = \frac{\pi}{2}$

   Let $AP = p, BP = q$.

   In case 1,

$$p^2 + q^2 = 4c^2 = 40$$
$$p + q = 2a = 8$$
$$\rightarrow p = 6, q = 2 \therefore [APB] = \frac{6 \cdot 2}{2} = 6$$

   In case 2,

$$p^2 - q^2 = 40$$
$$p + q = 8$$
$$\rightarrow p = \frac{13}{2}, q = \frac{3}{2} \therefore [APB] = \frac{\frac{3}{2} \cdot 2\sqrt{10}}{2} = \frac{3\sqrt{10}}{2}$$
$$6^2 + \left(\frac{3\sqrt{10}}{2}\right)^2 = \mathbf{58.5}$$

10. A The graph is called **Fermat's spiral** with equation $\boldsymbol{r = \pm a\sqrt{\theta}}$.

11. A Draw two line segments: from E to the origin and from X to the origin. The lengths are 4 and 6 respectively. The angle between the line segments is $\frac{7\pi}{12}$, or $\frac{5\pi}{6} - \frac{\pi}{4}$. ($\cos\left(\frac{5\pi}{6} - \frac{\pi}{4}\right) = \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$) Using the Law of Cosines, we can find the square of the distance.
$4^2 + 6^2 - 2(4)(6)\left(cos\frac{7\pi}{12}\right) = 16 + 36 + 12\sqrt{6} - 12\sqrt{2} = \mathbf{4(13 + 3\sqrt{6} - 3\sqrt{2})}$.

12. B Vectors going from (-1, 2, 7) to (3, 6, -1) and form (-1, 2, 7) to (8, 5, 0) are $< 4, 4, -8 >$ and $< 9, 3, -7 >$ respectively. The cross-product of these two vectors is the normal vector of the plane. $< 4, 4, -8 > \times < 9, 3, -7 > = < -4, -44, -24 >$ or $< 1, 11, 6 >$. The equation is $x + 11y + 6z + D = 0$; to find D, just plug in one of the three given points. $\boldsymbol{x + 11y + 6z = 63}$

13. B
$$y = r\sin(\theta)$$
So, we aim to maximize $f(\theta) = \sin(\theta)\,(1 + \cos(\theta))^2$
$$f(\theta)^2 = \sin^2(\theta)\,(1 + \cos(\theta))^4 = (1 - \cos^2(\theta))(1 + cos(\theta))^4$$
$$= (1 - \cos(\theta))(1 + \cos(\theta))^5$$
To maximize this function, we use AM-GM on
$$5(1 - \cos(\theta)), 1 + \cos(\theta), 1 + \cos(\theta), 1 + \cos(\theta), 1 + \cos(\theta), 1 + \cos(\theta)$$
The maximum is the equality case $5 - 5\cos(\theta) = 1 + \cos(\theta) \rightarrow \cos(\theta) = \frac{2}{3}$
Plugging our value in,
$$f(\theta) = \frac{\sqrt{5}}{3}\left(\frac{5}{3}\right)^2 = \frac{25\sqrt{5}}{27}$$
$$25 + 5 + 27 = \mathbf{57}$$

14. D The point (-1, 3, 2) to the other three points creates three vectors: $< 7, -7, -5 >$, $< 8, 2, -4 >$, and $< 10, -10, 10 >$. The volume of the tetrahedron can be found by dividing the determinant of $\begin{bmatrix} 7 & -7 & -5 \\ 8 & 2 & -4 \\ 10 & -10 & 10 \end{bmatrix}$ by 6. The determinant of the matrix is 1200, so the volume of the tetrahedron is **200**.

15. E The equation is $\frac{(x+3)^2}{9} + \frac{(y-3)^2}{16} = -1$, which is **not a conic**.

16. D Vector $\vec{\boldsymbol{u}}$ can be found by using the formula $\frac{\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{b}}}{\|\vec{\boldsymbol{b}}\|^2}\hat{\boldsymbol{b}}$.
$\frac{13}{6} < 1,2,1 > = < \boldsymbol{\frac{13}{6}, \frac{13}{3}, \frac{13}{6}} >$

17. D Note that B is a focus of the graph and let $F = (5,0)$, the other focus.
By definition
$$BP - FP = 2a = 8$$
$$BP = FP + 8$$
$$AP + BP = AP + FP + 8$$
This will be minimized when APF is a straight line.
$$15 = 8 + \sqrt{q^2 + 25} \rightarrow q^2 = \mathbf{24}$$

18. C Conic A is already in the form of $\mathrm{A}x^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. The discriminant $B^2 - 4AC = 16 - 224 = -208$ is negative, so if Conic A is non-degenerate, then it is an ellipse. To check for degeneracy, calculate the determinant

of $\begin{vmatrix} A & \frac{B}{2} & \frac{D}{2} \\ \frac{B}{2} & C & \frac{E}{2} \\ \frac{D}{2} & \frac{E}{2} & F \end{vmatrix} = \begin{vmatrix} 7 & 2 & -18 \\ 2 & 8 & 8 \\ -18 & 8 & -75 \end{vmatrix} = -7516 \neq 0$. Since Conic A is an ellipse if not degenerate, we also have to check whether or not the determinant is the same sign as $AC = 56$. Since they have different signs, Conic A is an **ellipse**.

19. B Conic A is already in the form of $\mathrm{A}x^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. To find the acute angle θ that the conic has to be rotated counterclockwise is given by the formula $tan2\theta = \frac{B}{A-C} = \frac{4}{7-8} = -4 = \frac{2tan\theta}{1-tan^2\theta}$. Solving for $tan\theta$ gets $\frac{1\pm\sqrt{17}}{4}$; since θ is an acute angle, $tan\theta = \boldsymbol{\frac{1+\sqrt{17}}{4}}$.

20. D By definition, $F_1P + F_2P = 2a = 18$
*Note that $F_1 = (-3,0)$
So,

$$AP + PF_2 = 18 - F_1A$$

So, we aim to maximize and minimize F_1A.
The minimum is when F_1, A, center of C is in a line, $F_1C - r = \sqrt{3^2+4^2} - 3 = 2$
The maximum is when F_1A is tangent to C, $\sqrt{5^2 - 3^2} = 4$

$$\to r = 14, s = 16 \to r + s = \mathbf{30}$$

*Note that 14 is unachievable since if F_1A is tangent, it will only pass through one point.

21. C

$$x = 14\sin(t)\cos(t) = 7\sin(2t)$$
$$y = 12\sin^2(t) + 10 = -6\cos(2t) + 16$$

Since $cos^2\theta + sin^2\theta = 1$, the equation can be written as $\frac{x^2}{7^2} + \frac{(y-16)^2}{6^2} = 1$. The area of an ellipse is $ab\pi$, so the area is $\boldsymbol{42\pi}$.

22. D If we find the plane parallel to line n that contains line m, then we can use the distance from point to plane formula with that plane and a point on line n. To find the plane, we need to find normal vector of the plane by taking the cross-product of the vectors of the two lines.

$$< 2, -1, 4 > \times < 3, -2, 7 > = < 1, -2, -1 >$$

The plane is then $x - 2y - z + D = 0$; plug in a point on line m to find D. The equation of the plane is $x - 2y - z + 6 = 0$. Use the point (2, 1, 1) in the formula:

$$\frac{|1(2) - 2(1) - 1(1) + 6|}{\sqrt{1^2 + (-2)^2 + (-1)^2}} = \frac{5}{\sqrt{6}} = \boldsymbol{\frac{5\sqrt{6}}{6}}$$

23. A The formula for the area of a rotated ellipse with equation in the form $(x-h)^2 + B(x-h)(y-k) + C(y-k)^2 = 1$ is $\boldsymbol{\frac{2\pi i}{\sqrt{B^2-4AC}}}$.

24. A Rotating clockwise by $\frac{5\pi}{4}$ is the same thing as rotating counterclockwise by $\frac{3\pi}{4}$. Use the rotational matrix to find the new coordinates of J.

$\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 14 \\ 8 \end{bmatrix} = \begin{bmatrix} -11\sqrt{2} \\ 3\sqrt{2} \end{bmatrix}$ or $(\boldsymbol{-11\sqrt{2}, 3\sqrt{2}})$

25. D **Three points** do not always define a plane; three collinear points only defines a line.

26. D The equation can be rewritten as $13\left|x+\frac{23}{13}\right|+7\left|y-\frac{8}{7}\right|=91$. Since shifting the graph does not affect area, the area is equivalent to the area of $13|x|+7|y|=91$. This graph creates a rhombus where the vertices are where x or y is 0. So, the vertices are (7, 0), (-7, 0), (0, 13), and (0, -13) and the area is **182**.

27. A Note that the intersection will be a circle.
The radius of the circle will be $\sqrt{r^2-d^2}$ where r is the radius of the sphere, and d is the distance between the center and the plane.

$$r=\sqrt{64}=8$$
$$d=\frac{|1-4+8-14|}{\sqrt{1^2+2^2+2^2}}=\frac{9}{3}=3$$
$$\pi(r^2-d^2)=\boldsymbol{55\pi}$$

28. B The arch can be written as $y=-\frac{1}{506}x^2+2024$ when $y\geq 0$. If the coin is 253 feet away from one of the edges, then it is 759 feet away from the center. Plugging 759 into the equation gets $\mathbf{\frac{1771}{2}}$ feet. (Note that $253=\frac{1012}{4}$, so computations are simpler without simplifying the equation of the parabola first.)

29. A The length of the latus recta is 14. We already know the center of the hyperbola the distance from the center to the foci (same as those of the ellipse). The formula for the length of the latus rectum is $\frac{2b^2}{a}=14$, so we have $2b^2=14a=2(c^2-a^2)=2(144-a^2)$. This gets us the quadratic $a^2+7a-144=0$ and $a=9$ and $b^2=144-81=63$. So, the equation of the hyperbola is $\boldsymbol{\frac{(y-5)^2}{81}-\frac{(x-1)^2}{63}=1}$.

30. A The number of petals has to be either odd or a multiple of 4; only **18** does not meet the requirements.