BC/AHS-PB January Invitational Algebra 1 Team Question #1.

Solve each of the following equations for x:

A: $\frac{1}{5}(35x-15)-\frac{1}{2}(8x+2)+\frac{1}{3}(27x+45)=\frac{2}{3}(12x-30)$

B: $\frac{5x}{3}+\frac{7}{9}-\frac{2x}{5}=\frac{9}{2}$

C: $\left|3x+5-\frac{5}{2}x-1\right|=\frac{4}{3}(3+12\div 4)$

D: $\left|2(3x-\frac{5}{2})-(x+3)\right|=\left|\frac{2}{3}(12x+15)-(4x-1)\right|$

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BC/AHS-PB January Invitational Algebra 1 Team Question #2.

Evaluate each of the following expressions. Express each final answer is in the form of a single numerical value without using any exponents.

A: $5^3 \cdot 25^{-2} \cdot \left(\frac{1}{5}\right)^{-4}$

B: $16^3 \cdot \left(\frac{1}{8}\right)^2 \div 32 \cdot 4^2$

C: $27^3 \left(\frac{1}{9}\right)^5 \left(\frac{1}{243}\right)^{-2} \div 81^3$

D: $\left(\frac{4}{81}\right)^2 \div \left(\frac{16}{9}\right)^{-2} \cdot \left(\frac{27}{32}\right)^3 \div \left(\frac{8}{27}\right)^2$

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BC/AHS-PB January Invitational Algebra 1 Team Question #3.

Solve each of the following problems:

A: How many liters of a 20% acid solution must be mixed with 3 liters of a 70% acid solution in order to produce a 50% acid solution?

B: Train A and Train B both left from the same station traveling in opposite directions. Train A travels at a speed of 20 mph while Train B can only travel at 15 mph. If Train B left the station 3 hours after Train A, how long was Train A traveling before the trains were 95 miles apart?

C: Nicholas is going on a canoe trip. After practicing for over a month he is now able to paddle his canoe 3 mph in still water. However, this trip is on a fast-moving river with a current. He plans to make a round trip to a spot downstream and back. If Nicholas paddled for 5 hours downstream and 7 hours back up stream and the whole trip was a distance of 33 miles how fast was the current of the river?

D: Three friends (Moe, Larry, and Curly) have decided to start a house painting business. This Saturday, they have been hired to paint the big house at the end of the street. Moe is the slowest worker; it would take him 8 hours to paint the entire house by himself. Larry is a little bit faster; it would take him 6 hours to paint the entire house by himself. However, Curly has a special paint spraying machine and could paint the entire house in 3 hours by himself. Saturday morning, Moe and Larry show up at 8 am to start painting the house, but Curly does not show up with his paint spraying machine until 10am. By what time will they finish painting the house? NOTE: You must express your final answer in the form of a time! For example: 11:58 pm.

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BC/AHS-PB January Invitational Algebra 1 Team Question #4.

A survey was sent out to a group of 45 middle school students asking them what their favorite subjects in school were. The following data was collected. Four students like math, science, and history. Seven students like math and history, 10 students like science and history, and 6 students like science and math. 26 students like history, 16 students like science, and 19 students like math. Use this survey data to answer the following questions.

A: How many students like only math?

B: How many students like exactly 2 of the three subjects?

C: How many students like math and science, but not history?

D: How many students don't like any of these three subjects?

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BC/AHS-PB January Invitational Algebra 1 Team Question #5.

Use the following linear equations to answer the questions that follow:

1) $5x + 3y = -15$ 2) $3(x + 1) - 2(y - 3) = 5$ 3) $\frac{x+3}{5} + \frac{y+2}{4} = 1$

A: Determine the slope of the line defined by each the three equations. What is the product of these three slopes?

B: Determine the y-coordinate of the y-intercept of the line defined by each of the three equations. What is the sum of these three y-coordinates of these three y-intercepts? Express your final answer in the form of an improper fraction, if necessary.

C: Determine the x-coordinate of the x-intercept of the line defined by each of the three equations. What is the sum of these three x-coordinates of these three x-intercepts? Express your final answer in the form of an improper fraction, if necessary.

D: What is the x-coordinate of the point of intersection of the two lines defined by equations 1 and 2?

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BC/AHS-PB January Invitational Algebra 1 Team Question #6.

You have decided to start a business by selling homemade cookies. To start the business, you will need to purchase $100 worth of materials and pay $50 in permits. In addition to that, each batch of cookies that you make will cost you $5 worth of ingredients to make it. Once the cookies are made, you plan on selling them for $5 per box. Each batch of cookies that you make can fill exactly 4 boxes of cookies.

A: How much money will it cost a single customer to purchase 2 batches of cookies that you made and filled into in boxes?

B: How many boxes of cookies would you need to sell in order to break even?

C: If you sold 100 boxes of cookies, how much profit would you make?

D: If you were only able to make 2 batches of cookies, and sold them all in boxes. How much money would you lose?

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BC/AHS-PB January Invitational Algebra 1 Team Question #7.

Solve each of the following unit conversion problems:

A: How many kilometers are there in 300 centimeters?

B: How many minutes are there in 4 weeks?

C: How many yards per minute are equivalent to 7 feet per second?

D: If $1\ snap = \frac{3}{2}\ pops$ and $\frac{5}{6}\ pops\ =\ 1\ crackle$, then how many crackles are equivalent to 20 snaps?

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BC/AHS-PB January Invitational Algebra 1 Team Question #8.

Solve each of the following percent problems:

A: 36 is 15% of 30% of what number?

B: GameStop is going out of business and is having a big sale. Everything in the store is 25% off. You want to buy a new video game that originally costs $52. How much will you end up paying for the game with the discount and after paying 10% sales tax on the discounted price?

C: As more people start to drive electric cars, the number of gas-powered cars on the roads will decrease. Suppose that this year in your neighborhood, there was a 30% decrease in the number of gas-powered cars from last year. If there are 56 gas-powered cars in your neighborhood this year, how many gas-powered cars were there in your neighborhood last year?

D: Suppose you want to invest some money into the stock market. So, you decide to buy a single share of AMC stock. Over the next few days, you track the percent gains and losses. On the first day, the stock dropped by 20%. On the next day, the stock increased by 30%. On the third day, the stock dropped again by 50%. On the fourth day, the stock increased by 10%. What was the overall percent decrease of the stock over the 4-day period?

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BC/AHS-PB January Invitational Algebra 1 Team Question #9.

Solve each of the following:

A: Find the sum of the *x* and *y* coordinates of the point of intersection of these two linear equations:

$$5(x-1)-2(y+3)=8 \quad and \quad 4(x-2)+3(y+4)=10$$

B: Suppose you have a small pile of coins and all of the coins are either dimes or nickels. If there is a total of 19 coins and they are worth a total of $1.20, how many more nickels than dimes are there in the pile?

C: Your school's football team has made it to the state championship game. To help the fans (students and teachers) go to the game, the school has reserved a number of cars and vans. Each car can hold 5 people and each van can hold 8 people. If the school must reserve twice the number of vans as cars for 63 people in total who want to travel to the game and assuming every seat in a car or every seat in a van is filled, how many vans are needed?

D: After school you and your friends go to the candy store around the corner to get some sweets. Your friend Kyle decides to buy 5 gumballs and 12 lollipops which costs him $7.20. Next, your friend Sam buys 2 gumballs and 5 lollipops which costs her $2.99. If you just wanted to buy 1 gumball and 1 lollipop from this store, how much would it cost you? Give your final answer in the form of dollars and cents; for example: $1.23.

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BC/AHS-PB January Invitational Algebra 1 Team Question #10.

Find the GCF (greatest common factor) or LCM (least common multiple) for each set of expressions below. All answers must be in standard form.

A: Find the GCF of the following terms: $(48a^3b^4)$, $(42a^4b^3)$, $(21a^{10}b^{10})$

B: Find the GCF of the following polynomials: $(6x^2 - 12x - 18)$, $(2x^2 - 2x - 12)$, $(3x^2 - 27)$

C: Find the LCM of the following terms: $(15x^2y^3)$, $(25xy^4)$, $(9x^5y)$

D: Find the LCM of the following polynomials: $(2x + 4)$, $(6x - 24)$, $(4x^2 - 8x - 32)$

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BC/AHS-PB January Invitational Algebra 1 Team Question #11.

Suppose we define the symbols #, @, and $ to represent operations on real numbers as defined below:

$$a \# b = a^2 + b^2$$

$$x @ y = 2x - 3y$$

$$m \$ n = \frac{m}{2} + n$$

Find each of the following:

A: $[(1 \# 3) @ 4]$

B: $(3 @ 5) \# (8 \$ 2)$

C: $(4 \# 6) \$ [(-4) @ (-3)]$

D: $[(3 \$ 9) @ 8] @ [((-4) \# (-2)) \$ 5]$

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BC/AHS-PB January Invitational Algebra 1 Team Question #12.

Find the unit's digit for each of the following numeric expressions when they are evaluated into a single integer without any exponents:

A: 13^{13}

B: $15^{41} + 29^{23}$

C: 107^{106}

D: $22^{355} \cdot 66^{428}$

BC/AHS-PB January Invitational Algebra 1 Team Question #12.

Find the unit's digit for each of the following numeric expressions when they are evaluated into a single integer without any exponents:

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BC/AHS-PB January Invitational Algebra 1 Team Question #13.

Determine whether each of the following statements are **always true**, **sometimes true**, or **never true**.
Answer each part with either a "1" for "always true," "0" for "sometimes true," or "–1" for "never true."

A: A whole number is also a rational number.

B: A real number is also a natural number.

C: The quotient of two rational numbers is a rational number.

D: The set of {-1, 0, 1} is closed under multiplication.

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