Answers:

1.	A: $-\frac{31}{4}$	B: $\frac{335}{114}$	C: 8, –24	D: $19, -\frac{1}{3}$
2.	A: 125	B: 32	C: $\frac{1}{27}$	D: $\frac{27}{512}$
3.	A: 2	B: 4	C: 1.5 or $\frac{3}{2}$	D: 10:40 am

NOTE: The answer for part D <u>must</u> be in the form of a time, but the "am" is not necessary.

4.	A: 10	B: 11	C: 2	D: 3
5.	A: 2	B: $-\frac{17}{5}$	C: $-\frac{29}{6}$	D: $-\frac{42}{19}$
6.	A: $40	B: 40	C: $225	D: $120

NOTE: The dollar signs ($) are not necessary for parts A, C, and D.

7.	A: 0.003	B: 40,320	C: 140	D: 36
8.	A: 800	B: $42.90	C: 80	D: 42.8%

NOTE: The dollar sign ($) and the percent sign (%) are not necessary for parts B and D, respectively.

9.	A: 1	B: 9	C: 6	D: $0.67

NOTE: The dollar sign ($) is not necessary for part D.

10.	A: $3a^3b^3$	B: $x-3$	C: $225x^5y^4$	D: $12x^2-24x-96$
11.	A: 8	B: 117	C: 27	D: –51
12.	A: 3	B: 4	C: 9	D: 8
13.	A: 1	B: 0	C: 0	D: 1

Solutions:

1. **A:** $-\frac{31}{4}$ **B:** $\frac{335}{114}$ **C:** 8, –24 **D:** $19, -\frac{1}{3}$

A: $\frac{1}{5}(35x-15)-\frac{1}{2}(8x+2)+\frac{1}{3}(27x+45)=\frac{2}{3}(12x-30) \Rightarrow 7x-3-4x-1+9x+15=8x-20$
$\Rightarrow 12x+11=8x-20 \Rightarrow 4x=-31 \Rightarrow x=-\frac{31}{4}$

B: $\frac{5x}{3}+\frac{7}{9}-\frac{2x}{5}=\frac{9}{2}$; multiply both sides by 90 $\Rightarrow$ $150x+70-36x=405 \Rightarrow 114x=335 \Rightarrow$
$x=\frac{335}{114}$

C: $\left|3x+5-\frac{5}{2}x-1\right|=\frac{4}{3}(3+12\div 4) \Rightarrow \left|\frac{1}{2}x+4\right|=8 \Rightarrow \frac{1}{2}x+4=8 \text{ or } \frac{1}{2}x+4=-8 \Rightarrow$
$\frac{1}{2}x=4 \text{ or } \frac{1}{2}x=-12 \Rightarrow x=8 \text{ or } x=-24$

D: $\left|2(3x-\frac{5}{2})-(x+3)\right|=\left|\frac{2}{3}(12x+15)-(4x-1)\right| \Rightarrow |6x-5-x-3|=|8x+10-4x+1| \Rightarrow$
$|5x-8|=|4x+11| \Rightarrow 5x-8=4x+11 \text{ or } 5x-8=-(4x+11) \Rightarrow x=19 \text{ or } x=-\frac{1}{3}$

2. **A:** 125 **B:** 32 **C:** $\frac{1}{27}$ **D:** $\frac{27}{512}$

A: $5^3\cdot 25^{-2}\cdot\left(\frac{1}{5}\right)^{-4} \Rightarrow 5^3\cdot 4^{-4}\cdot 5^4=5^3=125$

B: $16^3\cdot\left(\frac{1}{8}\right)^2\div 32\cdot 4^2 \Rightarrow 2^{12}\cdot 2^{-6}\cdot 2^5\cdot 2^4 \Rightarrow 2^5=32$

C: $27^3\left(\frac{1}{9}\right)^5\left(\frac{1}{243}\right)^{-2}\div 81^3 \Rightarrow 3^9\cdot 3^{-10}\cdot 3^{10}\cdot 3^{-12} \Rightarrow 3^{-3} \Rightarrow \frac{1}{27}$

D: $\left(\frac{4}{81}\right)^2\div\left(\frac{16}{9}\right)^{-2}\cdot\left(\frac{27}{32}\right)^3\div\left(\frac{8}{27}\right)^2 \Rightarrow \frac{2^4}{3^8}\cdot\frac{2^8}{3^4}\cdot\frac{3^9}{2^{15}}\cdot\frac{3^6}{2^6} \Rightarrow \frac{2^{12}}{3^{12}}\cdot\frac{3^{15}}{2^{21}} \Rightarrow \frac{3^3}{2^9} \Rightarrow \frac{27}{512}$

3. **A:** 2 **B:** 4 **C:** 1.5 or $\frac{3}{2}$ **D:** 10:40 am

A:

	Liters	%	Total
A	x	20%	0.2x
B	3	70%	0.7(3)
Mix	x + 3	50%	0.5(x+3)

$0.2(x)+0.7(3)=0.5(x+3) \Rightarrow 2x+21=5x+15 \Rightarrow x=2$

B:

	Rate	Time	Distance
Train A	20	x	20x
Train B	15	x – 3	15(x – 3)

$20(x)+15(x-3)=95 \Rightarrow 20x+15x-45=95 \Rightarrow 35x=140 \Rightarrow x=4$

C:

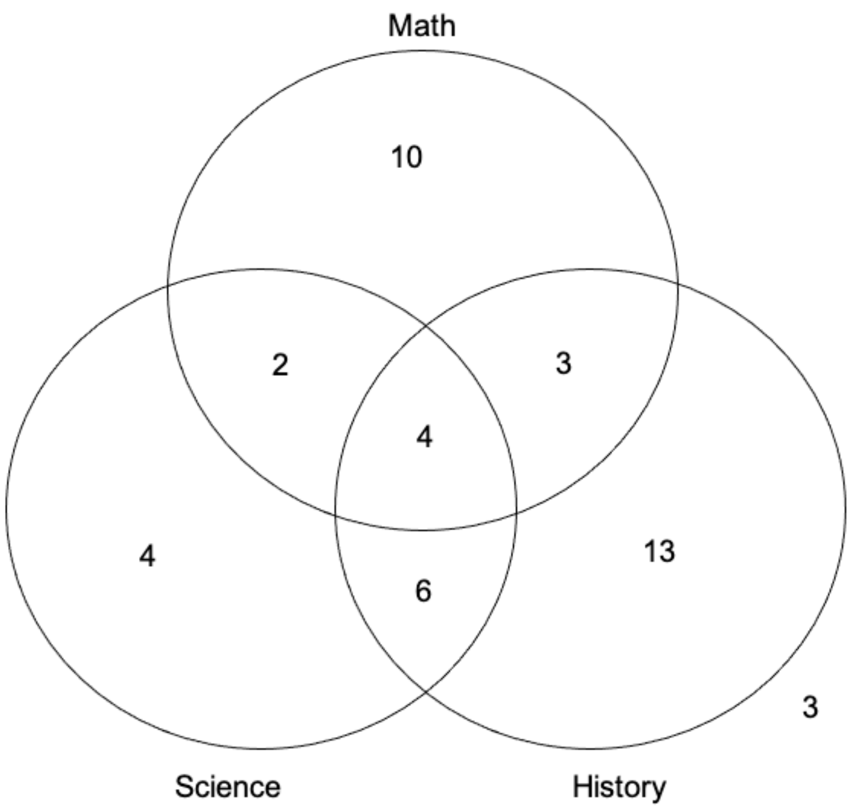
	Rate	Time	Distance
Downstream	3 + x	5	5(3 + x)
Upstream	3 – x	7	7(3 - x)

$5(3+x)+7(3-x)=33 \Rightarrow 15+5x+21-7x=33 \Rightarrow -2x+36=33 \Rightarrow -2x=-3 \Rightarrow x=1.5\ or\ \frac{3}{2}$

D: Moe's rate $=\frac{1}{8}$ per hour, Larry's rate $=\frac{1}{6}$ per hour, Curly's rate $=\frac{1}{3}$ per hour. Since Curly started 2 hours later, we have the following equation, where x is the number of hours since 8 am:
$\frac{1}{8}x+\frac{1}{6}x+\frac{1}{3}(x-2)=1 \Rightarrow$ multiply by the LCD of 24 $\Rightarrow 3x+4x+8x-16=24 \Rightarrow 15x=40 \Rightarrow x=\frac{40}{15}$
$\frac{40}{15}=2\frac{2}{3}$ hours $\Rightarrow$ 2 hours and $\frac{2}{3}(60)$minutes $\Rightarrow$ 2 hours and 40 minutes $\Rightarrow$ 10: 40 am.

4. A: 10 B: 11 C: 2 D: 3

Use the data collected to fill out the Venn diagram as is shown below. Start in the middle and work your way out. Each time you need to subtract from the total of the previous group. For example: if 7 students like math and history, but 4 of them like all three subjects then 3 students like math and history only.



A: Math only: 10 students
B: Exactly two of the subjects: 2 + 3 + 6 = 11 students
C: Math and Science, but not history: 2 students
D: None of the subjects: 45 – 42 = 3 students

5. A: 2 B: $-\frac{17}{5}$ C: $-\frac{29}{6}$ D: $-\frac{42}{19}$

Put all equations into standard form $Ax + By = C$:

1) $5x + 3y = -15$
2) $3x - 2y = -4$
3) $4x + 5y = -2$

A: The slope of each line when the equations are written in standard form is $\frac{-A}{B}$: $\Rightarrow \frac{-5}{3} \cdot \frac{3}{2} \cdot \frac{-4}{5} = 2$

B: The y-intercept of each line when the equations are written in standard form is $\frac{C}{B}$: $\Rightarrow -5 + 2 + \frac{-2}{5} = \frac{-17}{5}$

C: The x-intercept of each line when the equations are written in standard form is $\frac{C}{A}$: $\Rightarrow -3 + \frac{-4}{3} + \frac{-1}{2} = \frac{-29}{6}$

D: Solve the system for the X variable.

$2(5x + 3y = -15) \Rightarrow 10 + 6y = -30$

$3(3x - 2y = -4) \Rightarrow 9x - 6y = -12$

Adding the equations together results in $\Rightarrow 19x = -42 \Rightarrow x = \frac{-42}{19}$

6. A: \$40 B: 40 C: \$225 D: \$120

Let $x =$ the number of batches of cookies that you intend to make.

Cost function: $C(x) = 5x + 150$ Revenue function: $R(x) = 5(4x) = 20x$

A: Each batch of cookies that you make yields 4 boxes of cookies and each box sells for \$5 to the customer. So, it will cost the customer \$20 to purchase 4 boxes (or, equivalently, 1 batch of cookies); and hence, it will cost the customer \$40 to purchase 2 batches of cookies in 8 boxes.

B: To break even means Cost = Revenue. $5x + 150 = 20x \Rightarrow 150 = 15x \Rightarrow x = 10$, it takes selling 10 batches of cookies to break even, which is equal to 40 boxes of cookies.

C: 100 boxes = 25 batches

Cost of making 25 batches: $C(25) = 5(25) + 150 = \$275$

Revenue of selling 25 batches: $R(25) = 20(25) = \$500$

Revenue – Cost = Profit $\Rightarrow 500 - 275 = \$225$

D: Cost of 2 batches: $C(2) = 5(2) + 150 = \$160$

Revenue of selling 2 batches: $R(2) = 20(2) = \$40$

Revenue - Cost = Profit $\Rightarrow 160 - 40 = \$120$

7. A: 0.003 B: 40,320 C: 140 D: 36

A: $300\ cm \cdot \frac{1\ km}{100{,}000\ cm} = 0.003\ km$

B: $4\ weeks \cdot \frac{7\ days}{1\ week} \cdot \frac{24\ hours}{1\ day} \cdot \frac{60\ minutes}{1\ hour} = 40{,}320\ hours$

C: $\frac{7\ feet}{1\ second} \cdot \frac{60\ seconds}{1\ minute} \cdot \frac{1\ yard}{3\ feet} = 140\ yards\ per\ minute$

D: $20\ snaps \cdot \frac{3/2\ pops}{1\ snap} \cdot \frac{1\ crackle}{5/6\ pops} = 20 \cdot \frac{3}{2} \cdot \frac{6}{5} = 36$ crackles

8. A: 800 B: $42.90 C: 80 D: 42.8%

A: $36 = \frac{15}{100} \cdot \frac{30}{100} \cdot x \Rightarrow 36 = \frac{3}{20} \cdot \frac{3}{10} \cdot x \Rightarrow 36 = \frac{9}{200}x \Rightarrow x = 800$

B: To discount 25%, multiply by 75%. $\$52 \cdot \frac{75}{100} \Rightarrow 52 \cdot \frac{3}{4} = \$39$. Then add 10% of $39: $39 + 3.9 = \$42.90$.

C: Let x = the number of gas-powered cars in your neighborhood last year.

So, $x - (0.3)x = 56 \Rightarrow (0.7)x = 56 \Rightarrow x = 56\left(\frac{10}{7}\right) \Rightarrow x = 80$.

D: Since we need to calculate percent change, assume the price of the stock was $100.

Day 1: $100 - (0.2)100 \Rightarrow 100 - 20 = 80$; Day 2: $80 + (0.3)80 \Rightarrow 80 + 24 = 104$;

Day 3: $104 - (0.5)104 \Rightarrow 104 - 52 = 52$; Day 4: $52 + (0.1)52 \Rightarrow 52 + 5.2 = 57.2$

Then, find the percent decrease from 100 to 57.2: $\frac{57.2-100}{100} \Rightarrow \frac{42.8}{100} \Rightarrow 42.8\%$.

9. A: 1 B: 9 C: 6 D: $0.67

A: Put both equations in standard form.

Equation 1: $5(x-1) - 2(y+3) = 8 \Rightarrow 5x - 5 - 2y - 6 = 8 \Rightarrow 5x - 2y = 19$

Equation 2: $4(x-2) + 3(y+4) = 10 \Rightarrow 4x - 8 + 3y + 12 = 10 \Rightarrow 4x + 3y = 6$

Now use the elimination method to solve for x:

Multiply equation 1 by 3 $\Rightarrow 15x - 6y = 57$

Multiply equation 2 by 2 $\Rightarrow 8x + 6y = 12$

Add the equations to get: $23x = 69 \Rightarrow x = 3$

Substitute back in to solve for y: $4(3) + 3y = 6 \Rightarrow 12 + 3y = 6 \Rightarrow y = -2$

The ordered pair solution is (3, –2) and the sum of the coordinates is 1.

B: Let $d =$ the number of dimes and let $n =$ the number of nickels in your pile of coins.

Thus, we have: $d + n = 19$ and $0.1d + 0.05n = 1.20$

You can use substitution to solve this system:

$d = 19 - n \Rightarrow 0.1(19 - n) + 0.05n = 1.20 \Rightarrow 1.9 - 0.1n + 0.05n = 1.20 \Rightarrow$

$-0.05n = -0.7 \Rightarrow n = 14, \mathit{so}\ d = 5.$

The number of $Nickels - Dimes = n - d = 14 - 5 = 9$.

C: Let $c = \mathit{the\ number\ of\ cars}$ and let $v = \mathit{the\ number\ of\ vans}$. So, we have:

$$5c + 8v = 63\ \mathit{and}\ 2c = v$$

Use substitution to solve the system:

$$5c + 8(2c) = 63 \Rightarrow 5c + 16c = 63 \Rightarrow 21c = 63 \Rightarrow c = 3, \mathit{so}\ v = 6.$$

D: Let $g = \mathit{the\ price\ of\ a\ gumball}$ and let $l = \mathit{the\ price\ of\ a\ lollipop.}$ Thus, we have:

$$5g + 12l = 7.20\ \mathit{and}\ 2g + 5l = 2.99$$

Use elimination to solve the system:

First, multiply the first equation by 2 and the second equation by –5 to get:

$$10g + 24l = 14.40\ \ \mathit{and}\ -10g - 25l = -14.95$$

Then add the two above equations together and the result is:

$$-l = -0.55, \mathit{so}\ l = 0.55$$

Plug back in for *g:*

$$2g + 5(0.55) = 2.99 \Rightarrow 2g + 2.75 + 2.99 \Rightarrow 2g = 0.24 \Rightarrow g = 0.12$$

So, 1 gumball and 1 lollipop will cost you a total of $0.12 + $0.55 = $0.67.

10. A: $3a^3b^3$ B: $x-3$ C: $225x^5y^4$ D: $12x^2-24x-96$

A: $(48a^3b^4) \Rightarrow 2^4 \cdot 3 \cdot a^3 \cdot b^4$; $(42a^4b^3) \Rightarrow 2 \cdot 3 \cdot 7 \cdot a^4 \cdot b^3$; $(21a^{10}b^{10}) \Rightarrow 3 \cdot 7 \cdot a^{10} \cdot b^{10}$
The GCF is the product of all common factors: $3a^3b^3$.

B: $(6x^2-12x-18) \Rightarrow 3 \cdot 2 \cdot (x+1)(x-3)$, $(2x^2-2x-12) \Rightarrow 2 \cdot (x+2)(x-3)$,
$(3x^2-27) \Rightarrow 3 \cdot (x+3)(x-3)$
The GCF is the product of all common factors: $(x-3)$.

C: $(15x^2y^3) \Rightarrow 3 \cdot 5 \cdot x^2 \cdot y^3$, $(25xy^4) \Rightarrow 5^2 \cdot x \cdot y^4$, $(9x^5y) \Rightarrow 3^2 \cdot x^5 \cdot y$
The LCM is the product of all factors raised to the highest power: $3^2 \cdot 5^2 \cdot x^5 \cdot y^4 = 225x^5y^4$.

D: $(2x+4) \Rightarrow 2 \cdot (x+2)$, $(6x-24) \Rightarrow 2 \cdot 3 \cdot (x-4)$, $(4x^2-8x-32) \Rightarrow 2^2 \cdot (x+2)(x-4)$
The LCM is the product of all factors raised to the highest power: $2^2 \cdot 3 \cdot (\mathrm{x}+2)(\mathrm{x}-4) \Rightarrow 12\mathrm{x}^2-24\mathrm{x}-96$

11. A: 8 B: 117 C: 27 D: –51

A: $[(1 \# 3) @ 4] \Rightarrow [(1^2+3^2) @ 4] \Rightarrow [10 @ 4] \Rightarrow 2(10)-3(4) \Rightarrow 20-12=8$

B: $(3 @ 5) \# (8 \$ 2) \Rightarrow (2(3)-3(5)) \# \left(\frac{8}{2}+2\right) \Rightarrow (-9) \# (6) \Rightarrow (-9)^2+(6)^2 = 81+36 = 117$

C: $(4 \# 6) \$ [(-4) @ (-3)] \Rightarrow (4^2+6^2) \$ (2(-4)-3(-3)) \Rightarrow (52) \$ (1) \Rightarrow \frac{52}{2}+1 = 27$

D: $[(3 \$ 9) @ 8] @ [((-4) \# (-2)) \$ 5] \Rightarrow \left[\left(\frac{3}{2}+9\right) @ 8\right] @ [((-4)^2+(-2)^2) \$ 5] \Rightarrow$

$$\left[\frac{21}{2} @ 8\right] @ [20 \$ 5] \Rightarrow \left[2\left(\frac{21}{2}\right)-3(8)\right] @ \left[\frac{20}{2}+5\right] \Rightarrow [21-24] @ [10+5] \Rightarrow (-3) @ (15) \Rightarrow$$
$$2(-3)-3(5) \Rightarrow -6-45 = -51$$

12. A: 3 B: 4 C: 9 D: 8

A: If an integer with a unit's digit of 3 is raised to a power, the unit's digit will repeat as follows: 3,9,7,1,3,9,7,1…
The pattern repeats every 4 powers, so 13 times will result in a unit's digit of 3.

B: Any integer with a unit's digit of 5 raised to any power will always have a unit's digit of 5. Any number with a unit's digit of 9 raised to a power will have a unit's digit that will alternate between 9 and 1 with 9 for odd powers and 1 for even powers. So, 5 + 9 = 14 indicating that the unit's digit is 4.

C: If an integer with a unit's digit of 7 is raised to a power, the unit's digits will repeat as follows: 7,9,3,1,7,9,3,1… The pattern repeats every 4 powers; so, $106 \div 4 = 26\ R.\ 2$. Since the remainder is 2 the pattern will end on the second term, so the answer is 9.

D: If a number with a unit's digit of 2 is raised to a power, the unit digits will repeat as follows: 2,4,8,6,2,4,8,6… The pattern repeats itself every 4 powers; so, $355 \div 4 = 88\ R.\ 3$. So, the unit's digit is 8. If a number with a unit's digit of 6 is raised to a power, the unit's digit will always be 6. So, the final product will be a unit's digit of 8 times a unit's digit of 6 which will result in a unit's digit of 8.

13. A: 1 B: 0 C: 0 D: 1

A: The set of whole numbers is a subset of the rational numbers. So, the statement always true.

B: The set of real numbers included natural numbers and non-natural numbers. So, the statement is sometimes true.

C: For most cases, dividing two rational numbers will result in a rational number; however, 0 is a rational number but dividing by 0 does not result in a rational number. So, the statement is sometimes true.

D: When we multiply any two of the numbers in the set {-1, 0, 1}, the result will be a number in the set {-1, 0, 1}. This is the closure property. So, the statement is always true.