Algebra 2 Individual Test

January 2024 BC / AHS-PB Statewide Invitational Competition

The abbreviation NOTA, found in choice E of each question, means "None of The Above [Answers]" and should be chosen if choices A, B, C and D are not correct.

$i = \sqrt{-1}$.

$f^{-1}(x)$ is the inverse function for $f(x)$.

1. Find the sum of the digits of the decimal form of $2^{2029} \cdot 5^{2024}$.

   A. 0 B. 2
   C. 5 D. 6
   E. NOTA

2. Let $f(x) = 4 \cdot 8^{x+1}$. Find the value of k so that $f(2k) = \frac{1}{2}$.

   A. -1 B. 1
   C. 2 D. 4
   E. NOTA

3. Solve $2 + \log_3(x) = \log_3(x+1)$ over the real numbers.

   A. $\frac{1}{16}$ B. $\frac{1}{8}$
   C. $\frac{1}{4}$ D. $\frac{1}{2}$
   E. NOTA

4. A parabola has vertex $(-6, 2)$ and two x-intercepts, one positive and one negative. If the equation of the parabola is $y = ax^2 + bx + c$, then which of the coefficients (a, b, and c) are negative?

   A. a only B. a, b only
   C. b, c only D. a, b, c
   E. NOTA

5. The graph of L_1 is a line with x-intercept 4 and y-intercept 3. The line L_2 has a graph perpendicular to L_1 and which contains the point (6, 2). Find the $x-$coordinate of the point of intersection of L_1 and L_2.

   A. $\frac{102}{25}$ B. $\frac{72}{25}$
   C. $\frac{22}{5}$ D. $\frac{108}{25}$
   E. NOTA

6. Solve $2 + |4 - x| < x$ over the real numbers. Answers are in interval form.

   A. $(-3, 3)$ B. $(-3, 4)$
   C. $(4, \infty)$ D. $(3, \infty)$
   E. NOTA

7. A two-digit number x has the value of y when its digits are reversed and $x > y$. If $(x + y)$ is a perfect square integer, which of the following could be x ?

   A. 99 B. 96
   C. 92 D. 63
   E. NOTA

8. For (x, y) the solution to the system of equations $2^{x+y} = 16$ and $2^{x-y} = \frac{1}{8}$, evaluate $9^x \cdot 4^y$.

   A. 384 B. $18\sqrt{2}$
   C. $16\sqrt{2}$ D. 49.5
   E. NOTA

9. The discriminant of $4x^2 - x\sqrt{2} + 1$ is twice the discriminant of $2x^2 + kx + k^2$. Give the positive value of k.

A. 14 B. 7
C. $\sqrt{7}$ D. 1
E. NOTA

10. The system $\begin{cases} y \geq 3x + 6 \\ 2x + 3y > k \end{cases}$ for k being a positive real constant has a solution graphed on the xy-plane. Which of the following points cannot be in the solution set of the system?

A. $(0, -k)$ B. $(0, 3k)$
C. $(1, k)$ D. $(-1, 4k)$
E. NOTA

11. $f(x) = 4\sqrt{x-1} + 2$. Find the real value of x for which $f^{-1}(x+1) = 4$.

A. -1 B. $1 + 4\sqrt{3}$
C. 7 D. 9
E. NOTA

12. The graph of $y = x^3 + bx^2 + cx + d$ has two distinct real roots. There is a root of multiplicity 2 at $x = \frac{1}{2}$ and a root at $x = 8$. Give the value of b.

A. -9 B. -8
C. 4 D. 33
E. NOTA

13. The number 2024 can be written as the product of two positive integers, x and y, whose difference is as small as possible. What is $|x - y|$?

A. 1 B. 2
C. 3 D. 4
E. NOTA

14. The system S, defined as $4x - y = 3$ and $6x + 4y = -1$, has solution (a, b). Give the value of $(a + b)$.

A. $-\frac{3}{2}$ B. $-\frac{1}{2}$
C. $\frac{1}{2}$ D. $\frac{3}{2}$
E. NOTA

15. When the polynomial P $2x^4 - x^3 - 14x^2 + cx + d$ is divided by $(2x - 1)$, the remainder is 1 or $\frac{1}{2x-1}$. Give the value of $c + 2d$.

A. 11 B. 10
C. 9 D. -5
E. NOTA

16. An ellipse has equation $4x^2 + y^2 = 4$, the endpoints of the major axis are at points (a, b) and (c, d). Find the value of $a^2 + b^2 + c^2 + d^2$.

A. 2 B. 4
C. $2\sqrt{2}$ D. 8
E. NOTA

17. The function $f(x) = x^2 - 4x + P$ has exactly one real root, at $x = Q$. The graph of f is lowered 100 units so that the new graph has two real roots, at $x = R$ and $x = S$. Find $Q + R + S$.

A. 18 B. 14
C. 6 D. 2
E. NOTA

18. For the equation $x^2 - x = 5$, with complex solutions $a_1 + b_1 i$ and $a_2 - b_2 i$, find the product of the real components of the solutions, a_1 and a_2.

A. -5 B. -4
C. $\frac{1}{4}$ D. 21
E. NOTA

19. $f(x) = x \bullet 6^x$. If $f(6) = f(-6) \bullet k$, find the value of k.

A. -6^{12} B. -6^2
C. 6^2 D. 6^{12}
E. NOTA

20. If $5^{2x} + 5 = 6 \bullet 5^x$ then find the value of $x^2 + 1$.

A. 0 B. 1
C. 0 or 1 D. 1 or 2
E. NOTA

21. The solution to the system $a_1 x + b_1 y = c_1$ and $a_2 x + b_2 y = c_2$ has $x-$coordinate given by $\dfrac{\begin{vmatrix} 2 & 3 \\ -12 & 5 \end{vmatrix}}{\begin{vmatrix} 4 & 3 \\ -1 & 5 \end{vmatrix}}$ when Cramer's Rule is used. Find the $y-$coordinate of the system.

A. 0 B. -1
C. -2 D. -4
E. NOTA

22. For a domain of $x \geq 0$, suppose

$$f(x) = \begin{cases} 2^x \div \frac{1}{2} & for \quad x < 3 \\ 2^x \bullet \frac{1}{2} & for \quad x \geq 3 \end{cases}$$

Find the value of $f(0) + f(1) + f(2) + f(3) + f(4)$.

A. 0 B. 2
C. 15.5 D. 26
E. NOTA

23. For $c > 0$, the points $(c, 96)$ and $(5, c)$ lie on a line with slope c. Which of the following is an equation of that line?

A. $y = 12x - 48$ B. $y = 8x - 56$
C. $y = 12x + 63$ D. $y = 8x + 56$
E. NOTA

24. Which of the following is equal to $\frac{18^{30}}{36^{15}}$?

A. 3^{30} B. 6^{15}
C. 3^{15} D. 6^{12}
E. NOTA

25. Suppose p is any positive integral multiple of 8 such that $p \geq 8$ and the expression $x^p + \frac{1}{x^p}$ has the same numerical result regardless of p. Consider the solution to the equation $x + \frac{1}{x} = \sqrt{3}$ solved over the complex numbers; then, find the value of $\left(x^{2024} + \frac{1}{x^{2024}} \right)$.

A. 3^{1012} B. 1
C. $3i$ D. -1
E. NOTA

26. For $f(x) = \frac{1}{x+2} - \frac{1}{x-2}$ and $g(x) = \sqrt{x-1}$, the domain of $g(f(x))$ is all real numbers over which of the following interval(s)?

A. $(-\infty, -2] \cup [2, \infty)$
B. $(-2, -1] \cup [1, 2)$
C. $(-2, 2)$
D. $[1, 2)$
E. NOTA

27. For $f(x) = -x^3 + 14x^2 - 49x$ over the interval $[-10, 10]$, for how many integer values of x is $f(x) < 0$?

A. 15 B. 14
C. 10 D. 9
E. NOTA

28. For the equation $\log_6\left(\log_2\left(\log_2\left(\log_{60}(N)\right)\right)\right) = 2024$, how many distinct prime factors does N have?

A. 1 B. 2
C. 3 D. 6
E. NOTA

29. The graphs of $y = -|x + a| + b$ and $y = |x| + c$ intersect at $(6, 3)$ and $(-3, 0)$. Find the value of $a + b - c$.

A. 9 B. 6
C. 3 D. 0
E. NOTA

30. Given that $(2 - 2i)^8 = 2^p$, find the value of p.

A. 16 B. 12
C. 8 D. 4
E. NOTA