ANSWERS:

1. C
2. A
3. B
4. B
5. D

6. D
7. C
8. A
9. D
10. A

11. B
12. A
13. B
14. B
15. C

16. D
17. C
18. A
19. A
20. D

21. C
22. D
23. A
24. A
25. D

26. C
27. D
28. C
29. B
30. B

SOLUTIONS:

1. **C.** $2^{2029} \bullet 5^{2024}$ = $\left(2^{2024} \bullet 5^{2024}\right) \bullet 2^5 = 32 \bullet 10^{2024}$ which is 32 followed by 2024 zeros. Sum = 5.

2. **A.** $f(x) = 4 \bullet 8^{x+1}$. $f(2k) = 4 \bullet 2^{3((2k)+1)} = 2^{3(2k)+5}$. To be equal to ½, the exponent must be -1. So $6k + 5 = -1$ and k $= -1$.

3. **B.** $2 = \log_3(x+1) - \log_3(x)$. $\log_3\left(\frac{x+1}{x}\right) = 2$. $\frac{x+1}{x} = 9$. x = $\frac{1}{8}$.

4. **B.** A sketch shows that the graph must open downward, so the equation must be $y = a(x+6)^2 + 2,\ a < 0$. To get from the vertex to a positive x-intercept, the y-intercept must be positive. So c is positive. We are told that one x-intercept is negative. Let's say one root is 1, and that would make the distance from the axis of symmetry (x= -6) to that root is 7. That would make the other root at -6 – 7 = -13. That would make $y = -k(x+13)(x-1)$ for k positive which makes $y = -k(x^2 + 12x - 13)$. That means b is negative. You can do that reasoning with constants: Let the root be n for n a positive number. That makes distance to the axis of symmetry n+6. The other root is -6 – (n+6) = -12-n. The equation is then $y = -k(x-n)(x+12+n), k > 0$. b would be 12+n+n= 12+2n, all times a negative leading coefficient. Since n is positive, b is negative. Only a and b are negative.

5. **D.** Using the intercept form of an equation, L_1 has equation $\frac{x}{4} + \frac{y}{3} = 1$. $3x + 4y = 12$ and slope is $-\frac{3}{4}$. L_2 has slope $\frac{4}{3}$ and equation $4x - 3y = 18$.

3($3x + 4y = 12$)+4($4x - 3y = 18$) gives 25x = 108. So $x = \frac{108}{25}$.

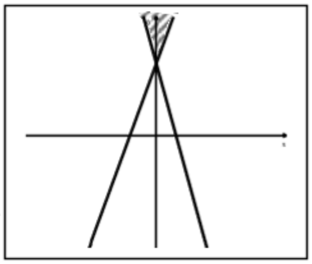
6. **D.** For $x \le 4$, $2 + 4 - x < x$. $-2x < -6$. $x > 3$. That gives (3, 4]. For $x > 4$, $2 - (4 - x) < x$. $-2 < 0$ which is true for all x greater than 4. That gives [4, ∞). That makes the solution $(3, \infty)$.

7. **C.** Let x = $10a + b$ and y = $10b + a$ for a sum of $11a + 11b$ = $11(a+b)$. To be a perfect square, a + b=11 or 11(4) or 11(9)... but since a and b are single digits, a + b =11. C is the only choice possible.

8. **A.** $x + y = 4$ and $x - y = -3$. Add to get 2x = 1. x = ½. y = 7/2. $9^x \bullet 4^y$ = $3 \bullet 128 = 384$.

9. **D.** The discriminant of $4x^2 - x\sqrt{2} + 1$ is $2 - 4(4)(1) = -14$. The discriminant of $2x^2 + kx + k^2$ is $k^2 - 4(2)(k^2) = -7k^2$. So $-14 = 2(-7k^2)$. Since k is positive, k = 1.

10. **A**. For any positive k, the 2nd inequality has a positive x-intercept and a positive y-intercept and the solution set is above the line. The first inequality is also shaded upward for for the solution set. A is not possible. B is possible for k greater than 1. C is possible for some k>9. D is possible for some k greater than 3.



11. **B.** $f^{-1}(x): x = 4\sqrt{y-1}+2$, for x>2.

$f^{-1}(x) = \left(\frac{x-2}{4}\right)^2 + 1$. $f^{-1}(x+1) = 4$ for $\left(\frac{x+1-2}{4}\right)^2 + 1 = 4$. $\left(\frac{x-1}{4}\right)^2 = 3$.

$(x-1)^2 = 48$. x = $1+4\sqrt{3}$ for x > 1.

12. **A.** $y = \left(x-\frac{1}{2}\right)^2 (x-8) =$

$\left(x^2 - x + \frac{1}{4}\right)(x-8)$. We want the coefficient of the x^2 term: $-8x^2 - x^2$. Answer = -9.

13. **B.** To have the two numbers have a small difference, we want them to be close to the square root of 2024.
2024 =
$4(506) = 8(253) = 8(11)(23) = 44(46)$.
So $|x-y|$ = 2.

14. **B.** $y = 4x-3$ so $6x + 4(4x-3) = -1$. $22x - 12 = -1$. x = ½. y = -1. The sum is – ½.

15. **C.**

$$\begin{array}{r l}
 & x^3 \qquad -7x + \frac{c-7}{2} \\
(2x-1) \big) & 2x^4 - x^3 - 14x^2 + cx + d \\
 & -(\ 2x^4 - x^3) \\
 & \qquad -14x^2 + cx + d \\
 & \qquad -(-14x^2 + 7x) \\
 & \qquad\qquad (c-7)x + d \\
 & \qquad\qquad -((c-7)x + \frac{7-c}{2}) \\
 & \qquad\qquad\qquad d - \frac{7}{2} + \frac{c}{2} = 1
\end{array}$$

so $2d + c = 9$

16. **D.** $4x^2 + y^2 = 4$, $\frac{x^2}{1} + \frac{y^2}{4} = 1$. The ends of the major axis are (0, 2) and (0, -2). $a^2+b^2+c^2+d^2 =$ $0+4+0+4=8$.

17. **C.** For $f(x) = x^2 - 4x + P$ to have exactly one real root, it must be $(x-2)^2$ at $x = Q$ so x = 2. If this is lowered 100 units, we have the equation would be $f(x) = (x-2)^2 - 100$ = $x^2 - 4x - 96$ = $(x-12)(x+8)$ so, roots become x = 12 and x = -8. $Q+R+S$ = 2 + 12 - 8 = 6.

18. **A.** $x^2 - x = 5$, has solutions $x = \frac{1 \pm \sqrt{1-4(1)(-5)}}{2}$ = $\frac{1}{2} \pm \frac{\sqrt{21}}{2}$ which has real components $\frac{1}{2} \pm \frac{\sqrt{21}}{2}$. The product is $\frac{1}{4} - \frac{21}{4} = -5$.

19. **A.** $f(x) = x \cdot 6^x$. $f(6) = 6 \cdot 6^6 = 6^7$. $f(-6) = -6 \cdot 6^{-6} = -6^{-5}$. $f(6) = f(-6) \cdot k$, so $6^7 = -k \cdot 6^{-5}$. $k = -\dfrac{6^7}{6^{-5}} = -6^{12}$.

20. **D.** $5^{2x} + 5 = 6 \cdot 5^x$. Let $a = 5^x$. $a^2 - 6a + 5 = 0$. $(a-5)(a-1) = 0$. $5^x = 5$ or $5^x = 1$. So $x = 1$ or $x = 0$. $x^2 + 1$ = 2 or 1.

21. **C.** $4x + 3y = 2, \ -x + 5y = -12$. Add the first equation to four times the second. 23y = -46. y = -2.

22. **D.** $f(0) + f(1) + f(2) + f(3) + f(4)$ = $2 + 4 + 8 + 4 + 8$ = 26

23. **A.** $\dfrac{c-96}{5-c} = c$. $5c - c^2 = c - 96$. $c^2 - 4c - 96 = 0$. $(c-12)(c+8) = 0$ and for $c > 0$, c = 12. The line with slope 12 through (5, 12) and (12, 96) has equation $y = 12x - 48$.

24. **A.** $\dfrac{18^{30}}{36^{15}}$ = $\dfrac{3^{30} \cdot 6^{30}}{6^{30}} = 3^{30}$

25. **D.** $\left(x + \dfrac{1}{x} = \sqrt{3}\right)^2 \rightarrow x^2 + 2 + \dfrac{1}{x^2} = 3$

$x^2 + \dfrac{1}{x^2} = 1$. $\left(x^2 + \dfrac{1}{x^2}\right)^2 = 1$.

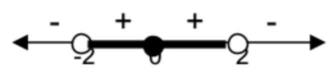
$x^4 + 2 + \dfrac{1}{x^4} = 1$. $x^4 + \dfrac{1}{x^4} = -1$. From that point on, squaring keeps getting values of $x^p + \dfrac{1}{x^p} = -1$ for p in the form 2^n.

Given the information that all equations with powers of x which are multiples of 8 have solutions the same, and verifying that 2024 is divisible by 8, we get the answer of -1.

26. **C.** $f(x) = \dfrac{1}{x+2} - \dfrac{1}{x-2}$ has domain $x \neq \pm 2$. $g(f(x))$ will also have the restriction that $f(x) = \dfrac{1}{x+2} - \dfrac{1}{x-2} \geq 1$.

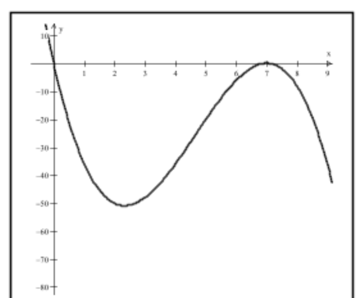
$\dfrac{x-2-(x+2)}{x^2-4} - \dfrac{x^2-4}{x^2-4} \geq 0$.

$\dfrac{-4-(x^2-4)}{x^2-4} \geq 0$. $\dfrac{-x^2}{x^2-4} \geq 0$. So now we check intervals on the number line with endpoints -2, 2, 0. Domain is $(-2, 2)$.



27. **D.** $f(x) = -(x^3 - 14x^2 + 49x)$ = $-x(x-7)^2$.

A quick sketch has the roots at 0 and 7.



and since we have an even power at x=7 the graph does not change sign before and after. So the graph is less than 0 from x=0 to x=7, not inclusive and then to infinity. That gives over the interval $[-10, 10]$, 1, 2, ..., 6, 8, 9, 10 which is 9 integers.

28. **C.** $\log_6\left(\log_2\left(\log_2\left(\log_{60}(N)\right)\right)\right)=2024$,

$\left(\log_2\left(\log_2\left(\log_{60}(N)\right)\right)\right)=6^{2024}$.

$\left(\log_2\left(\log_{60}(N)\right)\right)=2^{6^{2024}}$.

$\left(\log_{60}(N)\right)=2^{2^{6^{2024}}}$.

N = $60^{2^{2^{6^{2024}}}}$. This means we have prime factors 2, 3 and 5 only.

29. **B.** $(6,3)$: $3=-|6+a|+b$ and $3=|6|+c$ gives c = -3, and $b=3+|6+a|$.

$(-3,0)$: $0=-|-3+a|+b$. $b=|a-3|$.

so $|a-3|=3+|6+a|$. For $a>3$, $a-3=3+6+a$. No solutions. For $-6<a<3$. $-a+3=3+6+a$. a = -3. For $a<-6$, $-a+3=3+-6-a$. No solutions. So, a = -3 and b = 6. $y=-|x-3|+6$ and $y=|x|+c$ do contain the given points. $a+b-c$ = -3 + 6 – (-3) = 6.

30. **B.** $(2-2i)^8=2^8\bullet(1-i)^8=2^8(-2i)^4$
$=2^8(16)$ = $2^8 2^4=2^{12}$. p = 12.