

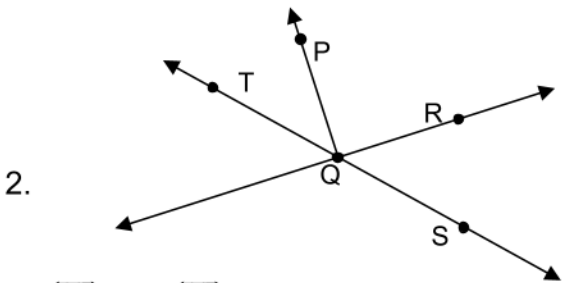
**Geometry Individual Test**  
**January 13, 2024 BC/AHS-PB Statewide Invitational Competition**

The abbreviation "NOTA" found in choice E of each question means "None of the Above [Answers]" and should be chosen if choices A, B, C, and D are not correct.

Diagrams are not drawn to scale.  
 All angle measures are in degrees.

1. The measure of  $\angle P$  is five times the measure of  $\angle Q$ . The measure of the complement of  $\angle Q$  is five times the measure of the complement of  $\angle P$ . Find the measure of  $\angle P$ .

- A.  $75^\circ$                       B.  $50^\circ$   
 C.  $47.5^\circ$                     D.  $15^\circ$   
 E. NOTA



$\overline{QR}$  and  $\overline{QS}$  intersect as shown, with  $\overline{QP} \perp \overline{QR}$ , and Q is between T and S. All points are coplanar.

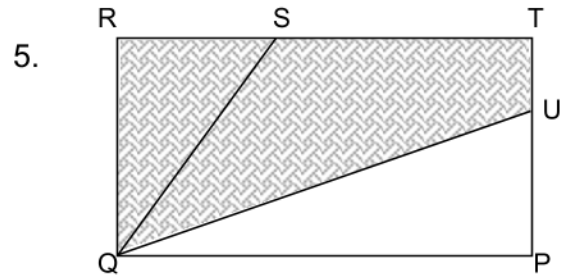
$m\angle TQP = (3x + 12)^\circ$  and  
 $m\angle RQS = (2x + 38)^\circ$ . Find the value of  $x$ .

- A. 65                              B. 26  
 C. 24                              D. 8  
 E. NOTA

3. A regular polygon has one exterior angle that measures 2 degrees. How many sides does the polygon have?

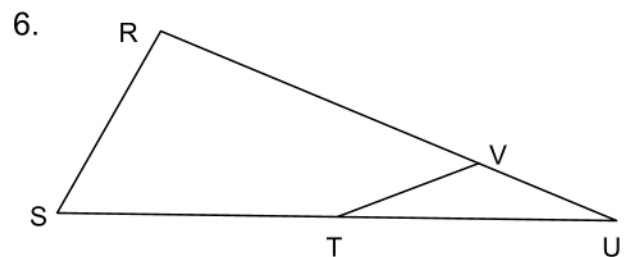
- A. 358                              B. 180  
 C. 178                              D. 176  
 E. NOTA

4. An isosceles trapezoid has bases 12 cm and 22 cm, and height 12 cm. Find the perimeter of the trapezoid in cm.
- A. 60                              B. 58  
 C. 56                              D. 50  
 E. NOTA



RTPQ above is a rectangle, and  $\angle Q$  is trisected by  $\overline{QS}$  and  $\overline{QU}$  ( $\angle RQS \cong \angle SQU \cong \angle UQP$ ). S and U are on  $\overline{RT}$  and  $\overline{TP}$  respectively. If  $QS = 12$ , and  $RT = 15$ , then the perimeter of quadrilateral RQUT is  $a + b\sqrt{c}$ , with  $c$  a prime integer and  $a, b$  rational numbers, then find  $a + b + c$ .

- A. 49                              B. 47  
 C. 35                              D. 36  
 E. NOTA

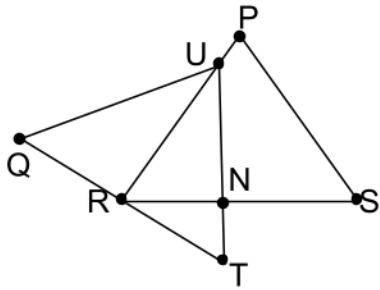


For  $\triangle RUS$  above,  $m\angle S = 55^\circ$ , and  $m\angle R = 92^\circ$ . V is on  $\overline{RU}$  and T is on  $\overline{SU}$ . If  $TV = VU$  then find  $m\angle RVT$ .

- A.  $66^\circ$                               B.  $59^\circ$   
 C.  $55^\circ$                               D.  $33^\circ$   
 E. NOTA

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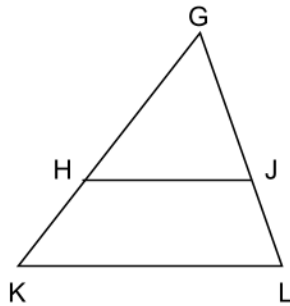
7.



Equilateral triangles  $\triangle PRS$  and  $\triangle QUT$  are shown above.  $R$  is on  $\overline{QT}$  and  $U$  is on  $\overline{RP}$ .  $\overline{UT}$  intersects  $\overline{RS}$  at  $N$  so that  $\overline{UN} \perp \overline{RS}$ . If  $PS = 12$  then find the length of  $\overline{UN}$ .

- A.  $9\sqrt{3}$                       B.  $6\sqrt{3}$   
 C. 9                                D.  $2\sqrt{3}$   
 E. NOTA

8.  $H$  is on side  $\overline{GK}$  of  $\triangle GKL$ , and  $J$  is on side  $\overline{GL}$ .  $GH=8$ ,  $HK=(x-2)$ ,



$HJ = x$  and  $KL = (x+3)$ . If  $\overline{HJ} \parallel \overline{KL}$  then find the value of  $x$ .

- A. 3                                B. 4  
 C. 5                                D. 6  
 E. NOTA

9. A rectangle  $RSTU$  has  $RT=(2x-12)$ ,  $RS=12$  and  $ST=16$ . Find the value of  $x$ .

- A. 18                                B. 16  
 C. 8                                D. 4  
 E. NOTA

10.  $\triangle RST$  has two sides of lengths 12 and 16. The included angle is bisected by  $\overline{RU}$  which divides  $\overline{ST}$  into lengths of  $(x+2)$  and  $(x+5)$ . Give the perimeter of  $\triangle RST$ .

- A. 27                                B. 49  
 C. 52                                D. 54  
 E. NOTA

11. The converse of the inverse of the statement "If  $p$ , then  $q$ " is which statement?

- A. If  $\sim p$  then  $\sim q$ .  
 B. If  $\sim q$  then  $\sim p$ .  
 C. If  $p$  then  $q$ .  
 D. If  $q$  then  $p$ .  
 E. NOTA

12. An isosceles trapezoid with base lengths 4 and 12 has altitude 10. Give the length of one diagonal of the trapezoid.

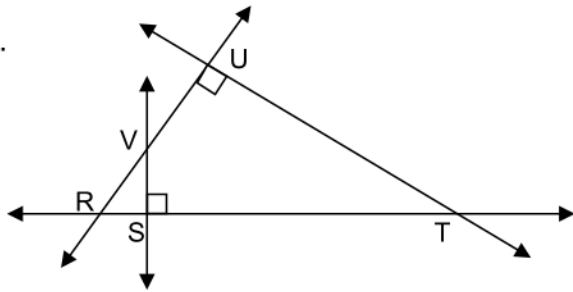
- A.  $2\sqrt{61}$                       B.  $2\sqrt{41}$   
 C. 14                                D.  $2\sqrt{29}$   
 E. NOTA

13. Which of the following must be true?

- A. A right triangle can be equilateral.  
 B. A triangle with sides 3, 4 and 5 may have angles of  $30^\circ$ ,  $60^\circ$  and  $90^\circ$ .  
 C. An equilateral triangle cannot have a height of integral length.  
 D. A triangle with side measures 1, 1 and  $k$  can have an integral perimeter.  
 E. NOTA

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14.



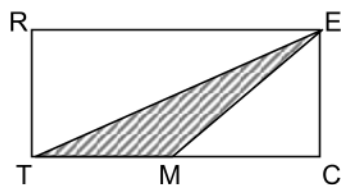
R, V and U are collinear, with  $\overline{RU} \perp \overline{UV}$ .  
 R, S and T are collinear with  $\overline{VS} \perp \overline{RT}$ .  
 $VS = x$ ,  $RS = (x+1)$ ,  $RU = (5x+1)$  and  
 $UT = (3x+3)$ . Find the length of  $\overline{VU}$ .

- A. 13                      B. 12  
 C. 11                      D. 6  
 E. NOTA

15. Planes  $H$  and  $J$  are perpendicular.  
 The set of points  $S$  which is described as "all points which are a distance of 2 units from  $H$  and 4 units from  $J$ ."  
 Which is the graph of  $S$  ?

- A. two parallel lines  
 B. one plane  
 C. two intersecting lines  
 D. four parallel lines  
 E. NOTA

16.



In rectangle  $RECT$ ,  $TC = 2(RT)$  and  $M$  is the midpoint of  $\overline{TC}$ . If  $RT = 4$  then the perimeter of  $\triangle MET$  is  $\sqrt{a} + \sqrt{b} + \sqrt{c}$  for  $a, b, c$  positive integers. Find  $a+b+c$ .

- A. 128                      B. 112  
 C. 108                      D. 104  
 E. NOTA

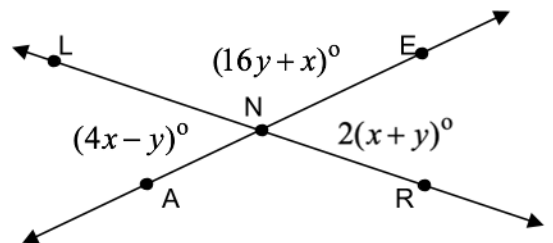
17. The diagonal of a square is 12 cm longer than a side. What is the length of one side of the square, in cm?

- A.  $12 + 12\sqrt{2}$               B.  $12 + 6\sqrt{2}$   
 C.  $12 - \sqrt{2}$                   D.  $6\sqrt{2}$   
 E. NOTA

18.  $\triangle RST$  is isosceles. Two of its angles have measures with an average (arithmetic mean) of  $50^\circ$ . Which could be the measure of one of the angles of the triangle?

- I.  $80^\circ$                       II.  $50^\circ$   
 III.  $30^\circ$                       IV.  $20^\circ$   
 A. I, II only                      B. I, II, IV only  
 C. II, III only                      D. I, II, III, IV  
 E. NOTA

19.

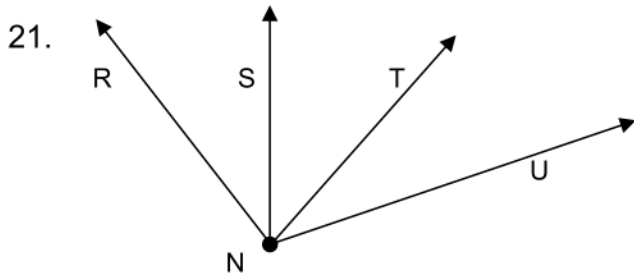


Lines  $\overline{LR}$  and  $\overline{AE}$  intersect at  $N$ .  
 $m\angle ANL = (4x - y)^\circ$ ,  $m\angle ENR = 2(x + y)^\circ$ ,  
 and  $m\angle LNE = (16y + x)^\circ$ . Find the value of  $|x - y|$ .

- A. 10                              B. 8  
 C. 4                                D. 2  
 E. NOTA

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20. In  $\triangle RST$ , exactly two angles are congruent to each other. If the lengths of sides  $RS = (4x + 30)$  cm,  $ST = (2x + 50)^{\circ}$  cm, and  $RT = (3x - 15)^{\circ}$  then which **could** be the length of the shortest side, in cm?
- A. 70                      B. 85  
 C. 90                      D. 180  
 E. NOTA



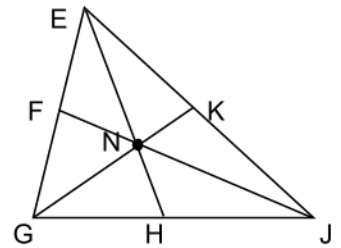
- R, S, T, U and N are coplanar.  
 $m\angle SNT = 4m\angle RNS$ .  $m\angle TNU = \frac{1}{2}m\angle SNT$   
 and  $m\angle RNU = ((m\angle RNS) + 60)^{\circ}$ . Find  $m\angle SNT$ .
- A. 40                      B. 42  
 C. 48                      D. 50  
 E. NOTA

22. The average (arithmetic mean) of the legs of a right triangle is 5. The product of the lengths of the legs is 24. Find the length of the hypotenuse of the triangle.
- A.  $4\sqrt{3}$                       B.  $2\sqrt{13}$   
 C.  $3\sqrt{14}$                       D.  $\sqrt{51}$   
 E. NOTA

23.  $\triangle RST$  has side lengths  $RS=6$  and  $ST=10$ . If  $\triangle RST$  is an acute triangle, then how many integral lengths are possible for  $\overline{RT}$ ?
- A. 9                      B. 5  
 C. 4                      D. 3  
 E. NOTA

24. How many of the following statements are possible, given that all angles involved have **integral** measures?
- I. The square of an angle's measure is equal to the measure of its complement.  
 II. The square of an angle's measure is equal to the measure of its supplement.  
 III. The measures of the angles of a convex hexagon are each divisible by 36.  
 IV. The measures of the angles of a convex pentagon are each divisible by 27.
- A. 1                      B. 2  
 C. 3                      D. 4  
 E. NOTA

25. In  $\triangle EGJ$ , medians  $\overline{JF}$ ,  $\overline{GK}$ , and  $\overline{EH}$  intersect at N.



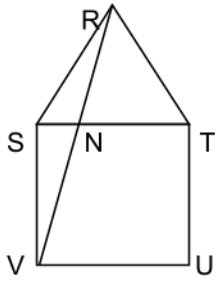
- $EH = (9x - 15)$ ,  $FN = (2x + 30)$ , and  $NH = (x + 15)$  and  $NK = (3x)$ . Give the value of  $GK + FJ$ .

- A. 176                      B. 180  
 C. 210                      D. 240  
 E. NOTA

26.  $\triangle RST$  has  $\overline{SP}$  bisecting  $\angle RST$ , with P on  $\overline{RT}$ .  $RS=12$  and  $ST=15$  and side  $\overline{RT}$  has an integral length. How many **integer** values are possible for  $\overline{RP}$ ?
- A. 0                      B. 1  
 C. 2                      D. 3  
 E. NOTA

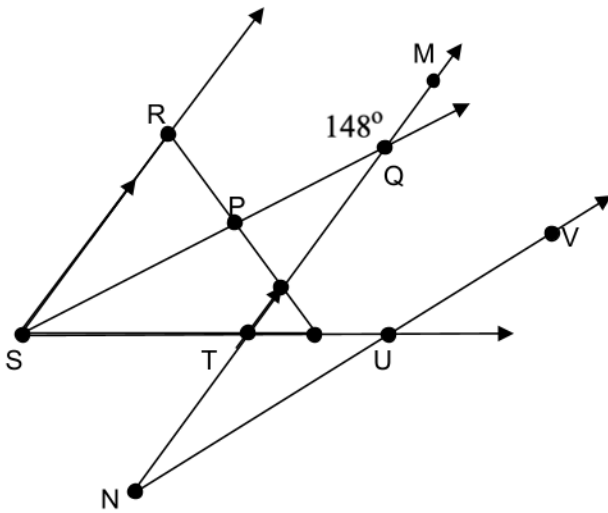
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27.



Square  $STUV$  shares a side with equilateral  $\triangle RST$ . Find  $m\angle RNT$ .

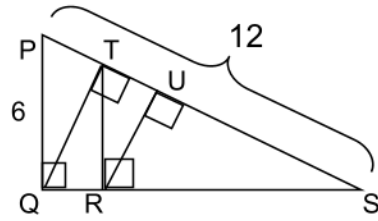
- |                 |               |
|-----------------|---------------|
| A. $75^\circ$   | B. $72^\circ$ |
| C. $67.5^\circ$ | D. $65^\circ$ |
| E. NOTA         |               |



28. In the diagram above,  $\overline{SQ}$  contains  $P$  and bisects  $\angle RST$ .  $\overline{ST}$  contains  $U$ ;  $\overline{NQ} \parallel \overline{SR}$  and  $\overline{NQ}$  contains  $M$  and  $T$  as shown.  $m\angle PQM = 148^\circ$ . Find  $m\angle PST + m\angle QTU$ .

- |                |                |
|----------------|----------------|
| A. $64^\circ$  | B. $96^\circ$  |
| C. $100^\circ$ | D. $116^\circ$ |
| E. NOTA        |                |

29.



In  $\triangle PQS$ ,  $Q$  is a right angle, and  $PS=12$  and  $PQ=6$ .  $T$  and  $U$  are on  $\overline{PS}$ ,  $R$  is on  $\overline{QS}$ ,  $\overline{TQ} \perp \overline{PS}$  and  $\overline{RU} \perp \overline{PS}$ . Find the length of  $\overline{RU}$ .

- |                           |                           |
|---------------------------|---------------------------|
| A. $\frac{9}{4}\sqrt{3}$  | B. $\frac{8}{9}\sqrt{17}$ |
| C. $\frac{8}{3}\sqrt{17}$ | D. $\frac{9}{2}\sqrt{3}$  |
| E. NOTA                   |                           |

30. A convex polygon has  $n$  sides. The sum of the measures of  $(n - 1)$  of the angles of the polygon is 5452 degrees. That is, all but one angle has measures that total 5452 degrees. What is the measure of the one angle not included in the sum?

- |                |                  |
|----------------|------------------|
| A. $132^\circ$ | B. $131.5^\circ$ |
| C. $128^\circ$ | D. $126.5^\circ$ |
| E. NOTA        |                  |