ANSWERS:

1. A
2. D
3. B
4. A
5. C

6. A
7. C - Disputed and Changed to E
8. D
9. B
10. B

11. B
12. B
13. D
14. C
15. D

16. A
17. A
18. B
19. C
20. D - Disputed and Thrown Out

21. A
22. B
23. D
24. C
25. D

26. C
27. A
28. B
29. A
30. C

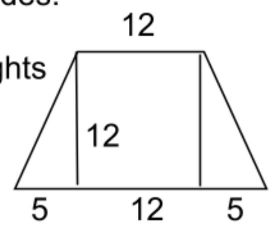
SOLUTIONS:

1. **A.** $P = 5Q$. $90 - Q = 5(90 - P)$. $90 - Q = 5(90 - 5Q)$. $24Q = 360$. $Q = 15$. So, P = 75.

2. **D.** Since $m\angle PQR = 90^o$, $m\angle TQP + m\angle RQS = 90$ and so $5x + 50 = 90$. x=8.

3. **B.** 360/2 = 180 sides.

4. **A.** Drop two heights from the endpoints of the smaller base. This creates a square and two 5-12-13 triangles. Perimeter = 13(2)+12+22 = 60



5. **C.** Since 90/3 = 30, $m\angle RQS = m\angle SQU = m\angle UQP = 30^o$. That gives RQ = $6\sqrt{3}$ and RS=6 and ST=9. TP=$6\sqrt{3}$. Since QP is the long leg of one of the 30-60-90 triangles, UP=$15/\sqrt{3} = 5\sqrt{3}$. That means TU = $6\sqrt{3}$ - $5\sqrt{3} = \sqrt{3}$. The perimeter of RQUT is $6\sqrt{3} + 1\sqrt{3} + 15 + 10\sqrt{3}$ = $15 + 17\sqrt{3}$ = $a + b\sqrt{c}$. So $a + b + c$ = 15+17+3=35.

6. **A.** $m\angle U = 180 - 92 - 55 = 33$. Since TV=VU, $m\angle VTU = 33$ and $m\angle RVT$ =66.

7. **C.** PS is 12 so QU=12. $m\angle Q = 60$ so UR = $6\sqrt{3}$. $m\angle URN = 60$ so, UN=9.
Disputed and Changed to E

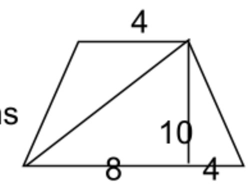
8. **D.** $\Delta GHJ \sim \Delta GKL$ by the AA~ Postulate. $\frac{8}{x} = \frac{x+6}{x+3}$. $8x + 24 = x^2 + 6x$. $x^2 - 2x - 24 = 0$. $(x-6)(x+4) = 0$. x=6.

9. **B.** $12^2 + 16^2 = (2x-12)^2$. $\cancel{144} + 256 = 4x^2 - 48x + \cancel{144}$. Divide by 4: $x^2 - 12x - 64 = 0$. $(x-16)(x+4) = 0$. x=16.

10. **B.** The angle bisector of one angle of a triangle will divide the opposite side into lengths proportional to the other two sides: $\frac{12}{16} = \frac{x+2}{x+5}$. $\frac{3}{4} = \frac{x+2}{x+5}$. $3x + 15 = 4x + 8$. x=7. So, perimeter of ΔRST is 12+15+9+12=49

11. **B.** The inverse of $p \to q$ is $\sim p \to \sim q$. The converse of that is $\sim q \to \sim p$.

12. **B.** If you drop two heights in a manner similar to the solutions to #4, we see the longer base broken by one height, into segments 8 and 4. $10^2 + 8^2 = d^2$ for diagonal d. $d = \sqrt{164} = 2\sqrt{41}$.



13. **D.** The hypotenuse of a right triangle is longer than the legs, so the sides cannot be congruent; A is false. The short leg of a 30-60-90 triangle is half the length of the hypotenuse, so B is false. An equilateral triangle can have an integral length altitude if the sides are irrational. For example, let the side be $2\sqrt{3}$; C is false. D is true, if k=1.

14. **C.** $\Delta VRS \sim \Delta TRU$ due to the reflexive angle R and right angle at U and S. $\frac{VS}{UT} = \frac{RS}{RU}$. $\frac{x}{3x+3} = \frac{x+1}{5x+1}$. $3x^2 + 6x + 3 = 5x^2 + x$. $2x^2 - 5x - 3 = 0$. $(2x+1)(x-3) = 0$. x= 3. In ΔVRS, RS=3, VS=4 so VR=5. Since RU=16, VU=16-5=11.

15. **D.** If the intersection of H and J is line L, then the locus described will be 4 lines parallel to L and to each other, one line in each space quadrant.

16. **A.** Let TC=2x and RT=x. Since M is the midpoint of $\overline{TC}$, TM=MC=x. Since RT=4, TM=MC=4. ΔEMC is isosceles so, ME = $4\sqrt{2}$. TE=$\sqrt{4^2+8^2}=4\sqrt{5}$. The perimeter of ΔMET is $4\sqrt{5}+4\sqrt{2}+4=$ $\sqrt{80}+\sqrt{32}+\sqrt{16}$. This is $\sqrt{a}+\sqrt{b}+\sqrt{c}$, so $a+b+c$=128.

17. **A.** Let the side length be x and the diagonal be x+12. $x+12=x\sqrt{2}$. $x(\sqrt{2}-1)=12$. $x=\dfrac{12}{\sqrt{2}-1}=$ $\dfrac{12(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}=12\sqrt{2}+12$

18. **B.** If the mean of two angles measures is 50 then the sum of the two angle measures is 100, and so the third angle is 80. So, I is possible. The angles could be 50, 50 and 80 degrees, so I is true. The angles could be 80, 80 and 20 so IV is possible. If 30 is one of the two angle measures with a mean of 50, then the other angle is 70. But 30, 70 and 80 is not isosceles, so III is not possible. So true are I, II, IV.

19. **C.** $4x-y=2x+2y$. $2x=3y$. $5x+15y=180$ from $\angle ANL$ and $\angle LNE$. Divide the last equation by 5. $x+3y=36$. $x+2x=36$. x=12. y=8. $|x-y|$ = 4.

20. **D.** If RS=ST then x=10. Side lengths are 70, 70 and 15. If RS=RT, then x is negative. Not possible as RT would be negative. If RT=ST then x=65. Side lengths are 180, 290 and 180. This is possible. So, the shortest side can be 180 or 15. **Disputed and Thrown Out**

21. **A.** Let $m\angle RNS=x$ and $m\angle SNT=4x$ and $m\angle TNU=2x$. $m\angle RNU=((m\angle RNS)+60)^{\circ}$ so $7x=x+60$. $x=10$. $m\angle SNT$ =40.

22. **B.** Let the leg lengths be x and y. $x+y=10$ and $xy=24$. You can guess that the numbers are 4 and 6. But ... $x^2+2xy+y^2=100$ and $x^2+2(24)+y^2=100$. $x^2+y^2=52$. Since the hypotenuse has length $\sqrt{x^2+y^2}=\sqrt{52}=2\sqrt{13}$.

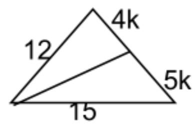
23. **D.** By the Triangle Inequality Theorem $4<RT<16$. To be acute, the third side must be between $\sqrt{100-36}=8$ and $\sqrt{100+36}=\sqrt{136}$. So, the third side can be 9, 10, 11. Three possibilities.

24. **C.** For I, $x^2=90-x$. $x^2+x-90=0$. $(x+10)(x-9)=0$ and if x=9, the angles are 9 and 81 degrees. Possible.
For II, $x^2=180-x$. $x^2+x-180=0$. $x=\dfrac{-1\pm\sqrt{1-4(-180)}}{2}=-\dfrac{1}{2}\pm\dfrac{\sqrt{721}}{2}$. Not integral measures. Not possible.
III. The sum is 720. So, 36(x)=720.and x=20. So, the angle measures can be 3x, 3x, 3x, 3x, 4x and 4x, or 108, 108, 108, 108, 144, 144. Possible. IV. A regular pentagon has angles divisible by 36. Possible.
Three statements are possible.

25. **D.** EN = 2(NH) so $\frac{1}{3}(9x-15)=(x+15)=$ $2(x+15)$. x=10. FN=50 so NJ=100. NK = 30 so

GN = 60. GK = 90. GK + FJ= 90 + 150= 240.

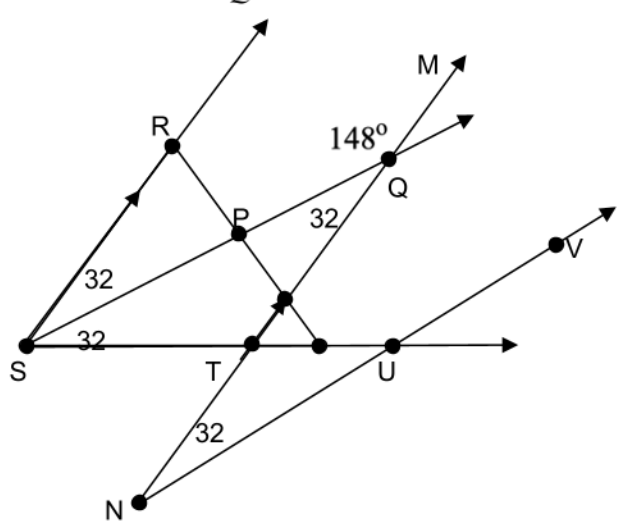
26. **C.** $\frac{12}{15}=\frac{RP}{RT-RP}$ and $3<RT<27$



Since the ratio of the given sides is 4/5, then the third side must also be in that ratio and so let the parts be 3k and 4k. The third side must be divisible by 9 and the part RP must be divisible by 4. So out of the lengths $3<RT<27$, 9 and 18 fit. Verify: RT=9. RP=4. If RT=18, PS=5. Side lengths are 12, 15, 9. OK. If RT=18, RP=8. PS=10. Sides of the triangle are 12, 15, 18. OK. Answer: 2 possible values.

27. **A.** Since SR=SV, and $m\angle RSV=150$, $m\angle SRV=m\angle SVR=15^{o}$. $m\angle RTN=60^{o}$. $m\angle NRT=60-15=45^{o}$. $m\angle RNT=180-45-60=75^{o}$

28. **B.** $m\angle PQT=32^{o}$. $\overleftrightarrow{NQ}\parallel\overleftrightarrow{SR}$ so $m\angle N=32^{o}$. The two angles at S are also 32 degrees. as the sum of the remote interior angles in ΔSTQ. $m\angle PST+m\angle QTU$ = 32+64 = 96.



29. **A.** $(PQ)^2=(PT)(PS)$. $36=PT\bullet 12$ and PT = 3. So, TS=9. Using similar triangles PSQ and TSR, $\frac{12}{9}=\frac{6}{TR}$.

TR=$\frac{9}{2}$. Back to Geometric Mean formulas, $(TR)^2=TS\bullet TU$. $\frac{81}{4}$=(9)(TU) and TU = $\frac{9}{4}$. US = $9-\frac{9}{4}=\frac{36-9}{4}=\frac{27}{4}$. RU = $\sqrt{\frac{27}{4}\bullet\frac{9}{4}}=\frac{9}{4}\sqrt{3}$.

30. **C.** Let $180(n-2)=5452$. Divide by 180 and we get 30 and a remainder. So n-2=30 means 32 sides is not the number of sides, but 33 may be. $180(33-2)=5580$ which is above the sum of 5452. $180(32)=5400$ degrees. The missing angle measure is 5580-5452= 128 degrees.