BC January Invitational — Algebra 2 Team — Question #1.

A: Three more than a number is equal to five less than twice that number. What is the number?

B: Simplify the following expression:

$$\frac{(6-10)^3 - (-4^2)}{2 + 8(2) \div 4}$$

C: Solve the following linear inequality and express your final answer in interval notation:

$$\frac{1}{15} \leq \frac{8-3x}{15} < \frac{4}{5}$$

D: For all positive even integers, n, with $n \geq 2$, let s represent the sum of the roots of $f(x) = x^n - 1$ and let p represent the product of the roots of $f(x) = x^n - 1$. What is $s + p$?

BC January Invitational — Algebra 2 Team — Question #1.

A: Three more than a number is equal to five less than twice that number. What is the number?

B: Simplify the following expression:

$$\frac{(6-10)^3 - (-4^2)}{2 + 8(2) \div 4}$$

C: Solve the following linear inequality and express your final answer in interval notation:

$$\frac{1}{15} \leq \frac{8-3x}{15} < \frac{4}{5}$$

D: For all positive even integers, n, with $n \geq 2$, let s represent the sum of the roots of $f(x) = x^n - 1$ and let p represent the product of the roots of $f(x) = x^n - 1$. What is $s + p$?

BC January Invitational **Algebra 2 Team** **Question #2.**

Consider the following polynomial function:

$$f(x) = x^2(4x^2 - 9)(9x^2 + 4)(x^2 - 7)(x^2 - 9)^3$$

Answer each of the following:

A: How many <u>total</u> complex roots does $f(x)$ have? Make sure to count the complex roots with multiplicities greater than 1 as contributing accordingly to the total.

B: How many <u>distinct</u> complex roots does $f(x)$ have?

C: How many distinct <u>non-real</u> complex roots does $f(x)$ have?

D: How many distinct <u>real</u> roots does $f(x)$ have?

BC January Invitational **Algebra 2 Team** **Question #2.**

Consider the following polynomial function:

$$f(x) = x^2(4x^2 - 9)(9x^2 + 4)(x^2 - 7)(x^2 - 9)^3$$

Answer each of the following:

A: How many <u>total</u> complex roots does $f(x)$ have? Make sure to count the complex roots with multiplicities greater than 1 as contributing accordingly to the total.

B: How many <u>distinct</u> complex roots does $f(x)$ have?

C: How many distinct <u>non-real</u> complex roots does $f(x)$ have?

D: How many distinct <u>real</u> roots does $f(x)$ have?

BC January Invitational — Algebra 2 Team — Question #3.

Solve each of the following systems of equations. For any system that has a unique solution, express your final answer in the form of either an ordered pair (x, y) or an ordered triple (x, y, z). If the system has no solution, use the symbol for the empty set, $\emptyset$, as your response. If the system has infinitely many solutions, use the infinity symbol, ∞, as your response.

A: $\begin{cases} 2x + 6y = 8 \\ 3x + 9y = 12 \end{cases}$

B: $\begin{cases} x - y + 2z = 3 \\ 4x + y - z = 8 \\ 3x - y + z = 6 \end{cases}$

C: $\begin{cases} -\frac{1}{2}x - \frac{1}{3}y = 3 \\ 0.125x + 0.25y = 1 \end{cases}$

D: $\begin{cases} 2\sqrt{3}x + 3\sqrt{2}y = \pi\sqrt{3} \\ 3\sqrt{2}x + 3\sqrt{3}y = \pi\sqrt{2} \end{cases}$

BC January Invitational — Algebra 2 Team — Question #3.

Solve each of the following systems of equations. For any system that has a unique solution, express your final answer in the form of either an ordered pair (x, y) or an ordered triple (x, y, z). If the system has no solution, use the symbol for the empty set, $\emptyset$, as your response. If the system has infinitely many solutions, use the infinity symbol, ∞, as your response.

A: $\begin{cases} 2x + 6y = 8 \\ 3x + 9y = 12 \end{cases}$

B: $\begin{cases} x - y + 2z = 3 \\ 4x + y - z = 8 \\ 3x - y + z = 6 \end{cases}$

C: $\begin{cases} -\frac{1}{2}x - \frac{1}{3}y = 3 \\ 0.125x + 0.25y = 1 \end{cases}$

D: $\begin{cases} 2\sqrt{3}x + 3\sqrt{2}y = \pi\sqrt{3} \\ 3\sqrt{2}x + 3\sqrt{3}y = \pi\sqrt{2} \end{cases}$

BC January Invitational — Algebra 2 Team — Question #4.

Evaluate the determinant of each of the following matrices. If the determinant is not defined for the matrix, then respond with the letters "DNE" for "Does Not Exist."

A: $\begin{pmatrix} \frac{4}{5} & \frac{3}{4} \\ \frac{2}{3} & \frac{5}{3} \end{pmatrix}$

B: $\begin{pmatrix} 2 & -2 & 1 \\ 4 & 1 & 3 \\ 3 & 1 & 2 \end{pmatrix}$

C: $\begin{pmatrix} 2 & 4 & 2 & 9 & 1 & 3 \\ 3 & 8 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 5 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 7 & 1 & 9 & 2 & 1 \\ 5 & 3 & 8 & 6 & 4 & 7 \end{pmatrix}$

D: $\begin{pmatrix} 1 & 2 & 7 & 8 \\ 3 & 4 & 9 & 10 \\ 5 & 6 & 11 & 12 \end{pmatrix}$

BC January Invitational — Algebra 2 Team — Question #4.

Evaluate the determinant of each of the following matrices. If the determinant is not defined for the matrix, then respond with the letters "DNE" for "Does Not Exist."

A: $\begin{pmatrix} \frac{4}{5} & \frac{3}{4} \\ \frac{2}{3} & \frac{5}{3} \end{pmatrix}$

B: $\begin{pmatrix} 2 & -2 & 1 \\ 4 & 1 & 3 \\ 3 & 1 & 2 \end{pmatrix}$

C: $\begin{pmatrix} 2 & 4 & 2 & 9 & 1 & 3 \\ 3 & 8 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 5 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 7 & 1 & 9 & 2 & 1 \\ 5 & 3 & 8 & 6 & 4 & 7 \end{pmatrix}$

D: $\begin{pmatrix} 1 & 2 & 7 & 8 \\ 3 & 4 & 9 & 10 \\ 5 & 6 & 11 & 12 \end{pmatrix}$

BC January Invitational — Algebra 2 Team — Question #5.

A: Simplify the following logarithmic expression into a single rational number:

$$\log_5\left(\log_2 32 + \log_{32} 4 - \log_5 \sqrt[5]{25}\right)^{2/5}$$

B: Solve the following exponential equation for all real values of x:

$$4^{3x-12} = \left(\frac{1}{64}\right)^{-2x}$$

C: Solve the following logarithmic equation for all real values of x:

$$\ln(x^3 + 1) - \ln(x + 1) = \ln(-x + 2)$$

D: Find the product of the real number solutions to the following equation:

$$(x^2 - 8)^{x^2-4} = 1$$

BC January Invitational — Algebra 2 Team — Question #5.

A: Simplify the following logarithmic expression into a single rational number:

$$\log_5\left(\log_2 32 + \log_{32} 4 - \log_5 \sqrt[5]{25}\right)^{2/5}$$

B: Solve the following exponential equation for all real values of x:

$$4^{3x-12} = \left(\frac{1}{64}\right)^{-2x}$$

C: Solve the following logarithmic equation for all real values of x:

$$\ln(x^3 + 1) - \ln(x + 1) = \ln(-x + 2)$$

D: Find the product of the real number solutions to the following equation:

$$(x^2 - 8)^{x^2-4} = 1$$

BC January Invitational Algebra 2 Team Question #6.

A: The graph of the following equation in the Cartesian coordinate plane is that of a parabola. What is the vertex of the parabola as an ordered pair in the form (x, y)?

$$x = -2y^2 + 4y + 6$$

B: Let a equal the x-coordinate of the center of the graph of the following circle, let b equal the y-coordinate of the center of the graph of the following circle, and let r equal the radius of the following circle's graph. What is abr?

$$x^2 + y^2 + 10y + 6x - 6 = 0$$

C: What is the product of the length of the minor axis of the graph times the length of the major axis of the graph of the following ellipse?

$$5x^2 + 4y^2 = 80$$

D: What is the distance between the two foci of the following hyperbola?

$$5x^2 - 4y^2 = 100$$

BC January Invitational Algebra 2 Team Question #6.

A: The graph of the following equation in the Cartesian coordinate plane is that of a parabola. What is the vertex of the parabola as an ordered pair in the form (x, y)?

$$x = -2y^2 + 4y + 6$$

B: Let a equal the x-coordinate of the center of the graph of the following circle, let b equal the y-coordinate of the center of the graph of the following circle, and let r equal the radius of the following circle's graph. What is abr?

$$x^2 + y^2 + 10y + 6x - 6 = 0$$

C: What is the product of the length of the minor axis of the graph times the length of the major axis of the graph of the following ellipse?

$$5x^2 + 4y^2 = 80$$

D: What is the distance between the two foci of the following hyperbola?

$$5x^2 - 4y^2 = 100$$

BC January Invitational **Algebra 2 Team** **Question #7.**

A: Determine the solution set to the following inequality: $2x + 35 \geq x^2$. Express your final answer in interval notation.

B: Determine the solution set to the following inequality: $x - 2 \leq \frac{35}{x}$. Express your final answer in interval notation.

C: Determine the solution set to the following inequality: $\left|\frac{3x-5}{x}\right| \leq 2$. Express your final answer in interval notation.

D: Find the area of the region in the coordinate plane satisfying the following system of linear inequalities:

$$\begin{cases} y \leq 1 \\ 2x - y \leq 4 \\ x + 2y \geq 2 \end{cases}$$

BC January Invitational **Algebra 2 Team** **Question #7.**

A: Determine the solution set to the following inequality: $2x + 35 \geq x^2$. Express your final answer in interval notation.

B: Determine the solution set to the following inequality: $x - 2 \leq \frac{35}{x}$. Express your final answer in interval notation.

C: Determine the solution set to the following inequality: $\left|\frac{3x-5}{x}\right| \leq 2$. Express your final answer in interval notation.

D: Find the area of the region in the coordinate plane satisfying the following system of linear inequalities:

$$\begin{cases} y \leq 1 \\ 2x - y \leq 4 \\ x + 2y \geq 2 \end{cases}$$

BC January Invitational — Algebra 2 Team — Question #8.

A: Suppose the sum of x and y is 10 and the square of their difference, $(x-y)^2$, is 64. Find the product of x and y.

B: Solve the following equation for x: $x = \dfrac{-x}{1-\frac{1}{1-\frac{1}{x}}}$

Use the following rational function for parts C and D:

$$f(x) = \frac{x+5}{x^2-2x-3} + \frac{x}{x-3} - \frac{x}{x+1}$$

C: The graph of $f(x)$ has a vertical asymptote located at $x = a$ and a horizontal asymptote located at $y = b$. What is $a + b$?

D: The graph of $f(x)$ has a hole (a.k.a. a removable discontinuity) in its graph located at the point (c, d). What is the point (c, d) as an ordered pair?

BC January Invitational — Algebra 2 Team — Question #8.

A: Suppose the sum of x and y is 10 and the square of their difference, $(x-y)^2$, is 64. Find the product of x and y.

B: Solve the following equation for x: $x = \dfrac{-x}{1-\frac{1}{1-\frac{1}{x}}}$

Use the following rational function for parts C and D:

$$f(x) = \frac{x+5}{x^2-2x-3} + \frac{x}{x-3} - \frac{x}{x+1}$$

C: The graph of $f(x)$ has a vertical asymptote located at $x = a$ and a horizontal asymptote located at $y = b$. What is $a + b$?

D: The graph of $f(x)$ has a hole (a.k.a. a removable discontinuity) in its graph located at the point (c, d). What is the point (c, d) as an ordered pair?

BC January Invitational **Algebra 2 Team** **Question #9.**

A: What is the sum of the integral solutions to: $|x-2|<5$?

B: What is the sum of the positive integral solutions to: $|x^2-25|<24$?

C: How many integers satisfy the inequality: $0<|x^2-25|<25$?

D: What is the sum of the integral solutions to: $|x-5|=-|x|+5$?
HINT: Graph the two sides of the equation on the same plane and something should jump out at you!

BC January Invitational **Algebra 2 Team** **Question #9.**

A: What is the sum of the integral solutions to: $|x-2|<5$?

B: What is the sum of the positive integral solutions to: $|x^2-25|<24$?

C: How many integers satisfy the inequality: $0<|x^2-25|<25$?

D: What is the sum of the integral solutions to: $|x-5|=-|x|+5$?
HINT: Graph the two sides of the equation on the same plane and something should jump out at you!

.
..

BC January Invitational — Algebra 2 Team — Question #10.

A: What is the slope of the line perpendicular to the line passing through points $(-3, 4)$ and $(1, 2)$?

B: Solve the following equation for x:

$$\frac{4x^2 - 24x}{3x^2 - x - 2} + \frac{3}{3x + 2} = \frac{-4}{x - 1}$$

C: Find the four distinct ordered pair solutions to the following system of equations:

$$\begin{cases} x^2 + (y - 2)^2 = 4 \\ -x^2 + y = -2 \end{cases}$$

Let X equal the product of the four x-coordinates of the four distinct ordered pair solutions to the system above and let Y equal the sum of the four y-coordinates of the four distinct ordered pair solutions to the system above. What is the value of $\frac{X}{Y}$?

D: What is the area of the region in the coordinate plane bounded by the graphs of $|x - 6| \leq y$ and $-2 \leq y \leq 6$?

BC January Invitational — Algebra 2 Team — Question #10.

A: What is the slope of the line perpendicular to the line passing through points $(-3, 4)$ and $(1, 2)$?

B: Solve the following equation for x:

$$\frac{4x^2 - 24x}{3x^2 - x - 2} + \frac{3}{3x + 2} = \frac{-4}{x - 1}$$

C: Find the four distinct ordered pair solutions to the following system of equations:

$$\begin{cases} x^2 + (y - 2)^2 = 4 \\ -x^2 + y = -2 \end{cases}$$

Let X equal the product of the four x-coordinates of the four distinct ordered pair solutions to the system above and let Y equal the sum of the four y-coordinates of the four distinct ordered pair solutions to the system above. What is the value of $\frac{X}{Y}$?

D: What is the area of the region in the coordinate plane bounded by the graphs of $|x - 6| \leq y$ and $-2 \leq y \leq 6$?

BC January Invitational — Algebra 2 Team — Question #11.

A: Rationalize the denominator of the following expression:

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

B: Solve the following equation for the real values of *x*:

$$x-\sqrt{x-2}=4$$

C: Simplify the following expression and write your final answer in standard form $a+bi$ where $i=\sqrt{-1}$:

$$\frac{6}{1+i}\times\frac{10}{2+3i}\times\frac{13i}{6-9i}\times\frac{1+2i}{5-5i}$$

D: Suppose *n* is some randomly selected non-negative integer. Simplify the following expression and write your final answer in standard form $a+bi$ where $i=\sqrt{-1}$:

$$(i^{4n}+i^{4n+1}+i^{4n+2}+i^{4n+3}+i^{4n+4}+i^{4n+5}+i^{4n+6}+i^{4n+7}+i^{4n+8}+i^{4n+9}+i^{4n+10})^{4n+2}$$

BC January Invitational — Algebra 2 Team — Question #11.

A: Rationalize the denominator of the following expression:

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

B: Solve the following equation for the real values of *x*:

$$x-\sqrt{x-2}=4$$

C: Simplify the following expression and write your final answer in standard form $a+bi$ where $i=\sqrt{-1}$:

$$\frac{6}{1+i}\times\frac{10}{2+3i}\times\frac{13i}{6-9i}\times\frac{1+2i}{5-5i}$$

D: Suppose *n* is some randomly selected non-negative integer. Simplify the following expression and write your final answer in standard form $a+bi$ where $i=\sqrt{-1}$:

$$(i^{4n}+i^{4n+1}+i^{4n+2}+i^{4n+3}+i^{4n+4}+i^{4n+5}+i^{4n+6}+i^{4n+7}+i^{4n+8}+i^{4n+9}+i^{4n+10})^{4n+2}$$

BC January Invitational **Algebra 2 Team** **Question #12.**

Answer each of the questions that follow for the following polynomial function:

$$f(x) = 4x^5 + 4x^4 + 3x^3 + 3x^2 - x - 1$$

A: How many rational roots $f(x)$ have?

B: What is the product of the roots of $f(x)$?

C: What is the sum of the roots of $f(x)$?

D: What is the sum of the reciprocals of the roots of $f(x)$?

BC January Invitational **Algebra 2 Team** **Question #12.**

Answer each of the questions that follow for the following polynomial function:

$$f(x) = 4x^5 + 4x^4 + 3x^3 + 3x^2 - x - 1$$

A: How many rational roots $f(x)$ have?

B: What is the product of the roots of $f(x)$?

C: What is the sum of the roots of $f(x)$?

D: What is the sum of the reciprocals of the roots of $f(x)$?

BC January Invitational **Algebra 2 Team** **Question #13.**

A: The graph of the quadratic equation $y = 2x^2 - 5x - 12$ has a y-intercept at $(0, p)$. What is p?

B: The graph of the quadratic equation $y = 2x^2 - 5x - 12$ has two x-intercepts at $(m, 0)$ and $(n, 0)$. What is $m + n$?

C: In what quadrant (I, II, III, or IV) do the graphs of $y = 2(x + 1)^2 + 4$ and $y = 4 - 5(x + 1)^2$ intersect?

D: Let r and s be the minimum and maximum number of times, respectively, that the parabolic graphs of two different quadratic functions, $y = f(x)$ and $y = g(x)$, can intersect. What is the value of $r^s + s^r + r + s$?

BC January Invitational **Algebra 2 Team** **Question #13.**

A: The graph of the quadratic equation $y = 2x^2 - 5x - 12$ has a y-intercept at $(0, p)$. What is p?

B: The graph of the quadratic equation $y = 2x^2 - 5x - 12$ has two x-intercepts at $(m, 0)$ and $(n, 0)$. What is $m + n$?

C: In what quadrant (I, II, III, or IV) do the graphs of $y = 2(x + 1)^2 + 4$ and $y = 4 - 5(x + 1)^2$ intersect?

D: Let r and s be the minimum and maximum number of times, respectively, that the parabolic graphs of two different quadratic functions, $y = f(x)$ and $y = g(x)$, can intersect. What is the value of $r^s + s^r + r + s$?