Answers:

1.	A: 8	B: -8	C: $\left(-\frac{4}{3}, \frac{7}{3}\right]$	D: -1

NOTE: The answer to Part C must be in correct interval notation!

2.	A: 14	B: 9	C: 2	D: 7
3.	A: ∞	B: $(2,1,1)$	C: $\left(-13, \frac{21}{2}\right)$	D: $\emptyset$

NOTE: The answers to Parts A and D must be in the form ∞ and $\emptyset$, respectively!

4.	A: $\frac{5}{6}$	B: -3	C: 0	D: DNE
5.	A: $\frac{2}{5}$	B: -4	C: 1	D: 36

NOTE: $x = -1$ is an extraneous solution in Part C and should not be included in the final answer!

6.	A: $(8,1)$	B: $30\sqrt{10}$	C: $32\sqrt{5}$	D: $6\sqrt{5}$
7.	A: $[-5,7]$	B: $(-\infty,-5] \cup (0,7]$	C: $[1,5]$	D: $\frac{5}{4}$ *or* 1.25
8.	A: 9	B: 2	C: 3	D: $\left(-1,-\frac{5}{4}\right)$ *or* $(-1,-1.25)$
9.	A: 18	B: 20	C: 12	D: 15
10.	A: 2	B: $\frac{5}{4}$ *or* 1.25	C: 2	D: 36

NOTE: $x = 1$ is an extraneous solution in Part B and should not be included in the final answer!

11.	A: $5+2\sqrt{6}$	B: 6	C: $-4+2i$	D: -1

NOTE: $x = 3$ is an extraneous solution in Part B and should not be included in the final answer!

12.	A: 3	B: $\frac{1}{4}$	C: -1	D: -1
13.	A: -12	B: 2.5 *or* $\frac{5}{2}$	C: II	D: 3

Solutions:

1. A: 8 B: -8 C: $\left(-\frac{4}{3},\frac{7}{3}\right]$ D: -1

A: Let x represent the number. Solve: $x+3=2x-5 \rightarrow x=8$.

B: $\frac{(6-10)^3-(-4^2)}{2+8(2)\div 4}=\frac{(-4)^3+16}{2+16\div 4}=\frac{-64+16}{2+4}=-\frac{48}{6}=-8$

C: $\frac{1}{15}\leq\frac{8-3x}{15}<\frac{4}{5}\rightarrow 1\leq 8-3x<12\rightarrow -7\leq -3x<4\rightarrow \frac{7}{3}\geq x>-\frac{4}{3}\rightarrow -\frac{4}{3}<x\leq\frac{7}{3}\rightarrow x\in\left(-\frac{4}{3},\frac{7}{3}\right]$.

D: By Vieta's Theorem, the sum of the roots of $f(x)=x^n-1$ for even $n\geq 2$ is given by the negative of the coefficient of the x^{n-1} term divided by the leading coefficient and the product of the roots of $f(x)$ is given by the constant term divided by the leading coefficient; thus: $s=\frac{-0}{1}=0$ and $p=\frac{-1}{1}=-1\ \rightarrow s+p=-1$. For those unfamiliar with Vieta's Theorem, one can note that the two real roots of $f(x)=x^n-1$ when n is even are $x=\pm 1$ and that all $n-2$ remaining imaginary roots will appear in conjugate pairs. Thus, if one only considers the two real roots of $f(x)$ to answer the question, they will arrive at the same result.

2. A: 14 B: 9 C: 2 D: 7

A: Noting that the degree of $f(x)=x^2(4x^2-9)(9x^2+4)(x^2-7)(x^2-9)^3=36x^{14}+\cdots$ (when expanded) is 14, then $f(x)$ has 14 <u>total</u> complex roots.

B: The roots of $f(x)$ along with their multiplicities are as follows:

- $x=0$ with multiplicity 2
- $x=\pm\frac{3}{2}$
- $x=\pm\frac{2}{3}i$
- $x=\pm\sqrt{7}$
- $x=3$ with multiplicity 3
- $x=-3$ with multiplicity 3

Thus, $f(x)$ has 9 <u>distinct</u> complex roots.

C: Inspecting the list in the solution to Part B above, we can see $f(x)$ has 2 distinct non-real complex roots.

D: Inspecting the list in the solution to Part B above, we can see $f(x)$ has 7 distinct real roots.

3. A: ∞ B: $(2,1,1)$ C: $\left(-13,\ \frac{21}{2}\right)$ D: $\emptyset$

A: $\begin{cases}2x+6y=8\\3x+9y=12\end{cases}\rightarrow\begin{cases}3(2x+6y)=3(8)\\-2(3x+9y)=-2(12)\end{cases}\rightarrow\begin{cases}6x+18y=24\\-6x-18y=-24\end{cases}\rightarrow\infty$ because there are infinitely many solutions.

B: $\begin{cases}x-y+2z=3\\4x+y-z=8\\3x-y+z=6\end{cases}\rightarrow\begin{cases}5x+z=11\\7x\quad\ =14\end{cases}$ after using the 2nd equation to eliminate y from both the 1st and 3rd equations. Solving $\begin{cases}5x+z=11\\7x\quad\ =14\end{cases}\rightarrow x=2$ and $5(2)+z=11\rightarrow z=1$; and then, $2-y+2(1)=3\rightarrow y=1$. Thus, the unique ordered triple solution is $(2,1,1)$.

C: $\begin{cases} -\frac{1}{2}x-\frac{1}{3}y=3 \\ 0.125x+0.25y=1 \end{cases} \to \begin{cases} 6\left(-\frac{1}{2}x-\frac{1}{3}y\right)=6(3) \\ 8\left(\frac{1}{8}x+\frac{1}{4}y\right)=8(1) \end{cases} \to \begin{cases} -3x-2y=18 \\ x+2y=8 \end{cases} \to -2x=26 \to x=-13.$

Substituting $x=-13$ back in gives us $-13+2y=8 \to 2y=21 \to y=\frac{21}{2} \to \left(-13,\frac{21}{2}\right)$ is the solution.

D: This system of equations is nowhere near as bad as it first appears if one notices that when the 1st equation is multiplied through by $\sqrt{3}$ and the 2nd equation is multiplied through by $\sqrt{2}$, we get the following:

$$\begin{cases} \sqrt{3}\left(2\sqrt{3}x+3\sqrt{2}y=\pi\sqrt{3}\right) \\ \sqrt{2}\left(3\sqrt{2}x+3\sqrt{3}y=\pi\sqrt{2}\right) \end{cases} \to \begin{cases} 6x+3\sqrt{6}y=3\pi \\ 6x+3\sqrt{6}y=2\pi \end{cases}$$

Now, subtracting the 2nd equation from the 1st equation yields the contradiction $0=\pi$, which indicates that there is no solution to the system: $\emptyset$.

4. A: $\frac{5}{6}$ B: -3 C: 0 D: DNE

A: $\begin{vmatrix} \frac{4}{5} & \frac{3}{4} \\ \frac{2}{3} & \frac{5}{3} \end{vmatrix} = \frac{4}{5}\times\frac{5}{3}-\frac{3}{4}\times\frac{2}{3}=\frac{5}{6}$

B: $\begin{vmatrix} 2 & -2 & 1 \\ 4 & 1 & 3 \\ 3 & 1 & 2 \end{vmatrix} = 2(2-3)-(-2)(8-9)+1(4-3)=-3$

C: Since 4th row of the given 6×6 matrix consists of all 0's, the determinant of the matrix is 0.

D: Since the given matrix is of the order 3×4, the determinant is not defined; therefore: DNE.

5. A: $\frac{2}{5}$ B: -4 C: 1 D: 36

A: $\log_5\left(\log_2 32+\log_{32}4-\log_5\sqrt[5]{25}\right)^{2/5} = \log_5\left(\log_2 2^5+\log_{32}2^2-\log_5 5^{2/5}\right)^{2/5} = \log_5\left(5+\frac{2}{5}-\frac{2}{5}\right)^{2/5} = \frac{2}{5}$

B: $4^{3x-12}=\left(\frac{1}{64}\right)^{-2x} \to 4^{3x-12}=(4^3)^{2x} \to 4^{3x-12}=4^{6x} \to 3x-12=6x \to -3x=12 \to x=-4.$

C: $\ln(x^3+1)-\ln(x+1)=\ln(-x+2) \to \frac{x^3+1}{x+1}=-x+2 \to \frac{(x+1)(x^2-x+1)}{x+1}=-x+2 \to x^2-x+1=-x+2 \to x^2-1=0 \to x=\pm 1$. However, when we check each potential solution in the original logarithmic equation, we will find that $x=-1$ is an extraneous solution. So, $x=1$ is the only real solution.

D: To solve the equation $(x^2-8)^{x^2-4}=1$ for all real values of x, we must first realize that any solution must do one of two things: either make the base of $x^2-8=1$ (while at the same time making the exponent of $x^2-4\neq 0$); or, make the exponent of $x^2-4=0$ (while at the same time making the base of $x^2-8=0$). Thus, solving both $x^2-4=0$ and $x^2-8=1$ give us a solution set for x of $\{-2,2,-3,3\}$ and the product of these four values is 36 since all four values meet one of the aforementioned criteria.

6. A: $(8, 1)$ B: $30\sqrt{10}$ C: $32\sqrt{5}$ D: $6\sqrt{5}$

A: Since the graph of the given equation of $x = -2y^2 + 4y + 6$ is that of a parabola which opens sideways and to the left, the y-coordinate of the vertex is given by $y = -\frac{4}{2(-2)} = 1$ and so the x-coordinate is given by $x = -2(1)^2 + 4(1) + 6 = 8$. Thus, the vertex is $(8, 1)$ as an ordered pair.

B: Completing the square for both the x-terms and the y-terms and converting into standard form we get: $x^2 + y^2 + 10y + 6x - 6 = 0 \to (x^2 + 6x + 9) + (y^2 + 10y + 25) = 6 + 9 + 25 \to (x+3)^2 + (y+5)^2 = 40$ Thus, the center is $(-3, -5)$ and the radius is $r = 2\sqrt{10}$. Thus, $abr = (-3)(-5)(2\sqrt{10}) = 30\sqrt{10}$.

C: $5x^2 + 4y^2 = 80 \to \frac{x^2}{16} + \frac{y^2}{20} = 1$ and so the length of the minor axis is $2a = 2\sqrt{16} = 8$ and the length of the major axis is $2b = 2\sqrt{20} = 4\sqrt{5}$. The product of these two values is $8(4\sqrt{5}) = 32\sqrt{5}$.

D: $5x^2 - 4y^2 = 100 \to \frac{x^2}{20} - \frac{y^2}{25} = 1$ and so the distance between the two foci is $2\sqrt{20+25} = 2\sqrt{45} = 6\sqrt{5}$.

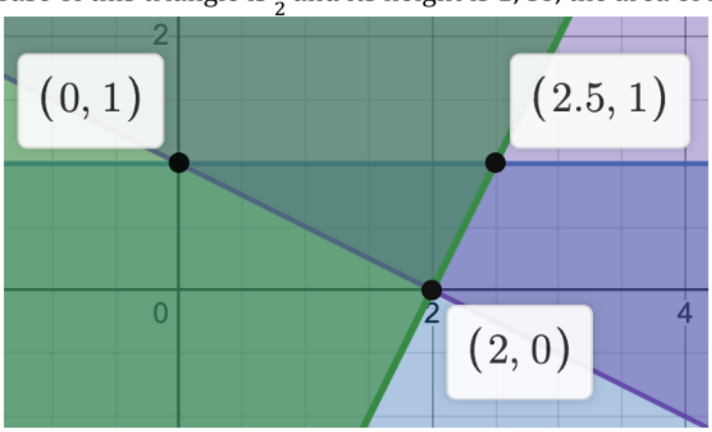
7. A: $[-5, 7]$ B: $(-\infty, -5] \cup (0, 7]$ C: $[1, 5]$ D: $\frac{5}{4}$ or 1.25

A: $2x + 35 \geq x^2 \to x^2 - 2x - 35 \leq 0 \to (x+5)(x-7) \to x = -5, x = 7$ are the critical values. Testing values in each of the three subintervals of the real number line formed by these two critical values results in $[-5, 7]$ as the solution set. Likewise, since the graph of $y = x^2 - 2x - 35$ is that of a parabola opening up with x-intercepts located at $x = -5$ and $x = 7$, the graph of the parabola is less than or equal to 0 (i.e., below or equal to the x-axis) between the x-intercepts of $x = -5$ and $x = 7$.

B: $x - 2 \leq \frac{35}{x} \to x = 0$ is a critical value and since $x - 2 \leq \frac{35}{x} \to x^2 - 2x - 35 \leq 0 \to (x+5)(x-7) \to$ $x = -5$ and $x = 7$ are critical values as well. Testing points within each of the four subintervals of the real number line formed by these three critical values yields the solution set of $(-\infty, -5] \cup (0, 7]$.

C: $\left|\frac{3x-5}{x}\right| \leq 2 \to \frac{3x-5}{x} \leq 2$ and $\frac{3x-5}{x} \geq -2 \to 3x - 4 \leq 2x$ and $3x - 4 \geq -2 \to x \leq 5$ and $x \geq 1$ which makes the solution set $[1, 5]$.

D: The solution region defined in the question forms a triangle with vertices located at $(0, 1)$, $(2, 0)$, and $\left(\frac{5}{2}, 1\right)$. The length of the base of this triangle is $\frac{5}{2}$ and its height is 1; so, the area of it is $\frac{1}{2}(5)(1) = \frac{5}{4}$.



8. A: 9 B: 2 C: 3 D: $\left(-1,-\frac{5}{4}\right)$ *or* $(-1,-1.25)$

A: We are given $x+y=10$; so, $(x+y)^2=x^2+2xy+y^2=100$. Also, $(x-y)^2=x^2-2xy+y^2=64$. Subtracting the two equations gives: $4xy=36$. So, $xy=9$.

B: $x=-\frac{x}{1-\frac{1}{1-\frac{1}{1-\frac{1}{x}}}} \rightarrow -1=1-\frac{1}{1-\frac{1}{x}} \rightarrow 2=\frac{1}{1-\frac{1}{x}} \rightarrow \frac{1}{2}=1-\frac{1}{x} \rightarrow -\frac{1}{2}=-\frac{1}{x} \rightarrow x=2.$

C: $f(x)=\frac{x+5}{x^2-2x-3}+\frac{x}{x-3}-\frac{x}{x+1}=\frac{x+5+x(x+1)-x(x-3)}{(x-3)(x+1)}=\frac{5x+5}{(x-3)(x+1)}=\frac{5(x+1)}{(x-3)(x+1)}=\frac{5}{x-3}$. Therefore, $f(x)$ has a vertical asymptote located at $x=3$ and a horizontal asymptote at the x-axis, or $y=0$, because the degree of the denominator of $f(x)$ is greater than the degree of the numerator. Thus, $3+0=3$.

D: Since the factor of $x+1$ cancels between the numerator and the denominator of $f(x)$ as noted in the algebraic simplification in Part C above, $f(x)$ has a hole in the graph located at $x=-1$ and $y=\frac{5}{-1-3}=-\frac{5}{4}$.

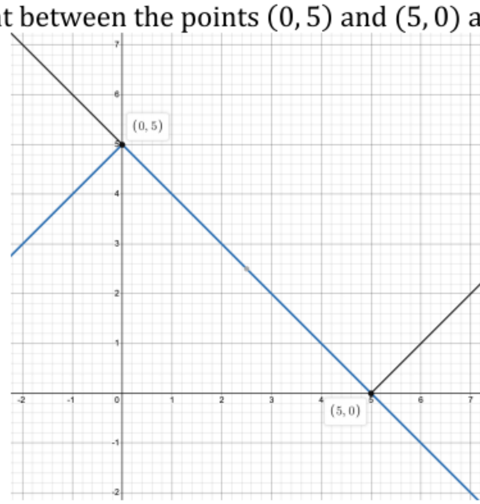
9. A: 18 B: 20 C: 12 D: 15

A: $|x-2|<5 \rightarrow x-2<5$ or $x-2>-5 \rightarrow x>-3$ or $x<7$. Thus, the sum of the integral solutions is $-2-1+0+1+2+3+4+5+6=18$.

B: $|x^2-25|<24 \rightarrow x^2-25<24$ or $x^2-25>-24$. Now, with *x* being restricted to positive integral values, we have $x^2-25<24 \rightarrow x^2<49$ and $x^2>1 \rightarrow x>1$ and $x<7 \rightarrow x=2,3,4,5,$ or 6. The sum of these positive integral solutions is $2+3+4+5+6=20$.

C: $0<|x^2-25|<25 \rightarrow |x^2-25|<25 \rightarrow x^2<50 \rightarrow |x|<\sqrt{50}\leq 7$ which gives us an integral solution set for *x* of $\{-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7\}$. However, noting that letting $x=\pm 5$ makes the given expression of $|x^2-25|=0$ and $x=0$ makes the given expression of $|x^2-25|=25$, we need to eliminate those three values from the integral solution set of *x*. This leaves us with the following solution set for *x* $\{-7,-6,-4,-3,-2,-1,1,2,3,4,6,7\}$ which has 12 elements.

D: $|x-5|=-|x|+5 \rightarrow x-5=-x+5 \rightarrow 2x=10 \rightarrow x=5$ or $-(x-5)=-(-x)+5 \rightarrow 2x=0 \rightarrow x=0$. However, note that another alternative is $x-5=-(-x)+5 \rightarrow 0=0$ which indicates that there are infinitely many real number solutions to the given inequality between $x=0$ and $x=5$, inclusive. This is seen when we graph the two sides of the given equality separately as $y=|x-5|$ and $y=-|x|+5$ and note that they share a line segment between the points $(0,5)$ and $(5,0)$ as shown below:



This makes the sum of the integral solutions $0+1+2+3+4+5=15$.

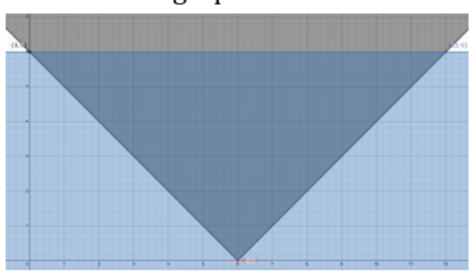
10. A: 2 B: $\frac{5}{4}$ *or* 1.25 C: 2 D: 36

A: The slope of the given line is $\frac{2-4}{1+3}=-\frac{1}{2}$. So, the perpendicular line's slope is the opposite reciprocal: 2.

B: $\frac{4x^2-24x}{3x^2-x-2}+\frac{3}{3x+2}=\frac{-4}{x-1} \rightarrow \frac{4x^2-24x}{(3x+2)(x-1)}+\frac{3}{3x+2}=\frac{-4}{x-1} \rightarrow 4x^2-24x+3(x-1)=-4(3x+2) \rightarrow$
$4x^2-24x+3x-3=-12x-8 \rightarrow 4x^2-9x+5=0 \rightarrow (4x-5)(x-1)=0 \rightarrow x=\frac{5}{4}$ or $x=1$.
However, $x=1$ is an extraneous solution because it makes two of the denominators in the original equation equal to 0. Therefore, the only valid solution is $x=\frac{5}{4}$ or 1.25.

C: $\begin{cases} x^2+(y-2)^2=4 \\ -x^2+y=-2 \end{cases} \rightarrow (y-2)^2+y=2$ by adding the two equations. Expanding the binomial and putting everything into the standard form of a quadratic equation, we obtain $y^2-3y+2=0$ which becomes $(y-1)(y-2)=0 \rightarrow y=1$ or $y=2$. Solving $-x^2+y=-2$ for x gives us $x=\pm\sqrt{y+2}$ which leads to the four points of intersection being $(\pm\sqrt{3},1)$ and $(\pm 2,2)$. Thus, $\frac{X}{Y}=\frac{(-\sqrt{3})(\sqrt{3})(-2)(2)}{1+1+2+2}=\frac{12}{6}=2$.

D: Graphing $|x-6|\le y$ results in shading the region above the absolute value function $y=|x-6|$ and graphing the solution set to $-2\le y\le 6$ results in shading the region in the coordinate plane between the lines $y=-2$ and $y=6$. Setting $|x-6|=6$ results in the points of intersection between $|x-6|\le y$ and $y=6$ being $(0,6)$ and $(12,6)$. Now, since the vertex of $y=|x-6|$ is the point $(6,0)$, the resulting bounded region is a triangle with vertices at $(0,6)$, $(6,0)$ and $(12,6)$. This triangle has a base of 12 and a height of 6 for an area of $\frac{1}{2}(6)(12)=36$. This is seen in the graph below:



11. A: $5+2\sqrt{6}$ B: 6 C: $-4+2i$ D: -1

A: $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \rightarrow \frac{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}=\frac{3+2\sqrt{6}+2}{3-2}=5+2\sqrt{6}$

B: $x-\sqrt{x-2}=4 \rightarrow (x-4)^2=\left(\sqrt{x-2}\right)^2 \rightarrow x^2-8x+16=x-2 \rightarrow x^2-9x+18=0 \rightarrow$
$(x-3)(x-6)=0 \rightarrow x=3$ or $x=6$. However, when we check each of these two potential solutions in the original equation, we realize that $x=3$ is extraneous which leaves us with only $x=6$ as a solution.

C: $\frac{6}{1+i}\times\frac{10}{2+3i}\times\frac{13i}{6-9i}\times\frac{1+2i}{5-5i}=\frac{6}{1+i}\times\frac{10}{2+3i}\times\frac{13i}{3(2-3i)}\times\frac{1+2i}{5(1-i)}$. Rearranging some of the factors, we get:
$\left[\frac{6}{(1+i)}\times\frac{1+2i}{5(1-i)}\right]\times\left[\frac{10}{(2+3i)}\times\frac{13i}{3(2-3i)}\right]=\frac{6(1+2i)}{5(2)}\times\frac{13(10i)}{3(13)}=2i(1+2i)=-4+2i$.

D: Since n is some non-negative integer, then we have the following:
$i^{4n}=(i^4)^n=1^n=1, i^{4n+1}=(i^4)^n i=1^n i=i, i^{4n+2}=(i^4)^n i^2=1^n(-1)=-1$, and $i^{4n+3}=(i^4)^n i^3=1^n i^3=-i$, etc. Thus, the given expression simplifies as follows:

$$(i^{4n}+i^{4n+1}+i^{4n+2}+i^{4n+3}+i^{4n+4}+i^{4n+5}+i^{4n+6}+i^{4n+7}+i^{4n+8}+i^{4n+9}+i^{4n+10})^{4n+2}=$$
$$(1+i+(-1)+(-i)+1+i+(-1)+(-i)+1+i+(-1))^{4n+2}=i^{4n+2}=(i^4)^n(i)^2=-1$$

12. A: 3 B: $\frac{1}{4}$ C: -1 D: -1

A: By the Rational Root Theorem, the set of potential rational roots of $f(x)$ is $\left\{\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}\right\}$. Using synthetic division on the two easiest options of $x = \pm 1$ yields $x = -1$ as a rational root of $f(x)$ which we can now factor as follows:
$f(x) = 4x^5 + 4x^4 + 3x^3 + 3x^2 - x - 1 = (x+1)(4x^4 + 3x^2 - 1) = (x+1)(4x^2 - 1)(x^2 + 1) =$
$(x+1)(2x+1)(2x-1)(x^2+1)$ and so, the roots are $\left\{-1, -\frac{1}{2}, \frac{1}{2}, -i, i\right\}$ of which 3 are rational.

B: Using the set of roots found in the solution to Part A, the product of the roots is $\frac{1}{4}$.

C: Using the set of roots found in the solution to Part A, the sum of the roots is -1.

D: Using the set of roots found in the solution to Part A, the sum of the reciprocals of the roots is -1.

13. A: -12 B: 2.5 *or* $\frac{5}{2}$ C: II D: 3

A: Let $x = 0$; then, $p = 2(0)^2 - 5(0) - 12 = -12$.

B: $y = 2x^2 - 5x - 12 = (2x+3)(x-4) = 0$ when $x = -\frac{3}{2}, x = 4$. Then $m + n = -\frac{3}{2} + 4 = 2.5$ or $\frac{5}{2}$.

C: The vertex of both parabolas is $(-1, 4)$ where one opens up and the other down. The sole intersection point is in the second quadrant.

D: Two parabolas need not intersect, such as $y = x^2 + 1$ and $y = -x^2 - 1$, so $r = 0$. Two parabolas can intersect a maximum of two times, such as $y = x^2 - 1$ and $y = 1 - x^2$, so $s = 2$. Then
$r^s + s^r + r + s = 2^0 + 0^2 + 2 + 0 = 3$.