Diagrams below may not be to scale. All points are coplanar.

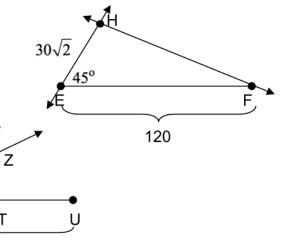
<u>Part A</u>: S is on \overline{RT} so that $RS = \frac{2}{3}(ST)$. If RT = 60, find RS.



Part B: X and Y are on \overline{WZ} so that WX=XY=YZ. If XZ = (4x + 20) and WZ = 120, then find the value of x



- Part C: HE = $30\sqrt{2}$. $m\angle HEF = 45^{\circ}$. If EF=120, find HF.
- Part D: S and T are on \overline{RU} . RS:ST:TU = 1 : 1 : 2. RU=120 and $m \angle ZRU = 30^{\circ}$. $\overrightarrow{ZT} \perp \overline{RU}$ and find the distance from **Z** to \overline{RU} .



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30°

S

S

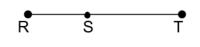
120

120

Question #1.

Diagrams below may not be to scale. All points are coplanar.

<u>Part A</u>: S is on \overline{RT} so that $RS = \frac{2}{3}(ST)$. If RT = 60, find RS.

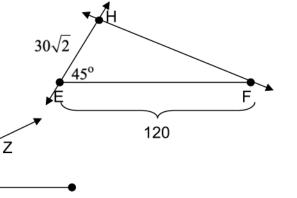


Part B: X and Y are on \overline{WZ} so that WX=XY=YZ. If XZ=(4x+20) and WZ=120, then find the value of x



HE = $30\sqrt{2}$. $m\angle HEF = 45^{\circ}$. Part C: If EF=120, find HF.

Part D: S and T are on \overline{RU} . RS:ST:TU = 1 : 1 : 2. RU=120 and $m \angle ZRU = 30^{\circ}$. $\overline{ZT} \perp RU$ and find the distance from Z to RU.



"Exterior angle" is an angle which forms a linear pair with an interior angle of a polygon.

- Part A: A regular polygon has one interior angle with measure $(11a+10)^{\circ}$ and one exterior angle with measure (6a). How many sides does the polygon have?
- Part B: A regular polygon has interior angles with measures which total 1800 degrees. Give the measure of one exterior angle of the polygon.
- Part C: A regular polygon has 54 diagonals. How many sides does the polygon have?
- Part D: A regular polygon has 36 sides. Give the sum of the interior angles of the polygon.

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"Exterior angle" is an angle which forms a linear pair with an interior angle of a polygon.

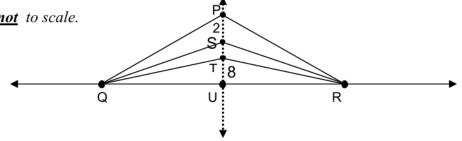
- Part A: A regular polygon has one interior angle with measure $(11a+10)^{\circ}$ and one exterior angle with measure (6a). How many sides does the polygon have?
- Part B: A regular polygon has interior angles with measures which total 1800 degrees. Give the measure of one exterior angle of the polygon.
- Part C: A regular polygon has 54 diagonals. How many sides does the polygon have?
- Part D: A regular polygon has 36 sides. Give the sum of the interior angles of the polygon.

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Question #3.

Diagram is <u>not</u> to scale.



In the diagram above, $\overrightarrow{PU} \perp \overrightarrow{QR}$, S and T lie on \overline{PU} as shown, PS=2, TU=8, $m\angle TQU=30^{\circ}$ and $m\angle SQT=15^{\circ}$.

- Part A: Find the perimeter of $\triangle QRT$.
- Part B: Find the perimeter of $\triangle QSR$.
- Part C: Find the perimeter of $\triangle QST$.
- <u>Part D</u>: The square of the length of \overline{QP} is $a+b\sqrt{c}$ for c a prime integer and a and b rational. Give the value of (a+b+c).

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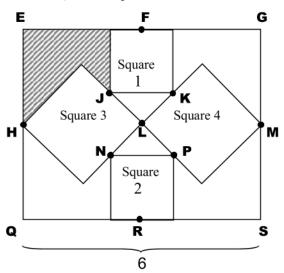
Question #3.

Diagram is <u>not</u> to scale.

Q
U
R

In the diagram above, $\overrightarrow{PU} \perp \overrightarrow{QR}$, S and T lie on \overline{PU} as shown, PS=2, TU=8, $m\angle TQU=30^{\circ}$ and $m\angle SQT=15^{\circ}$.

- Part A: Find the perimeter of $\triangle QRT$.
- Part B: Find the perimeter of $\triangle QSR$.
- Part C: Find the perimeter of $\triangle QST$.
- Part D: The square of the length of \overline{QP} is $a+b\sqrt{c}$ for c a prime integer and a and b rational. Give the value of (a+b+c).



In the diagram, which is not drawn to scale, EGSQ is a square. Four smaller squares are drawn: Squares 1 and 2 are congruent and share a midpoint on one side with EGSQ (F and R). Squares 3 and 4 are congruent and have a vertex (H and M) on the midpoint of \overline{EQ} and \overline{GS} as shown. L is the center of EGSQ. J, K, N and P are midpoints of one side of squares 3 and 4. **QS = 6**.

Part A: Find the length of one side of square 1.

Part B: Find the length of one side of square 3.

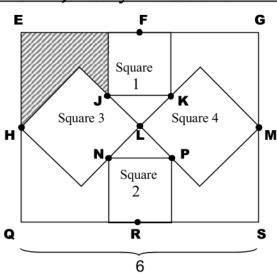
<u>Part C</u>: Find the perimeter of the shaded pentagon.

<u>Part D</u>: Find the height of ΔLNP from L to \overline{NP} .

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Question #4.



In the diagram, which is not drawn to scale, EGSQ is a square. Four smaller squares are drawn: Squares 1 and 2 are congruent and share a midpoint on one side with EGSQ (F and R). Squares 3 and 4 are congruent and have a vertex (H and M) on the midpoint of \overline{EQ} and \overline{GS} as shown. L is the center of EGSQ. J, K, N and P are midpoints of one side of squares 3 and 4. **QS = 6**.

<u>Part A</u>: Find the length of one side of square 1.

<u>Part B</u>: Find the length of one side of square 3.

Part C: Find the perimeter of the shaded pentagon.

<u>Part D</u>: Find the height of ΔLNP from L to \overline{NP} .

Regular pentagon PENTA has midpoints of its sides G, J, K, H and F. EN = 30.

Part A: Find $m \angle PFG$.

Part B: If $\triangle PFG \sim \triangle FHG$ then the answer to part B is 1. If $\triangle PFG$, $\triangle FHG$ are not similar, then the answer to part B is 0. If it cannot be determined, then the answer to part B is -1.

E H T K N

Part C: If $\frac{4}{5}$ is used as an approximation of $\cos 36^{\circ}$, find the perimeter of pentagon FGJKH.

Part D: If $\frac{4}{5}$ is used as an approximation of $\cos 36^{\circ}$, find the length of diagonal \overline{AE} .

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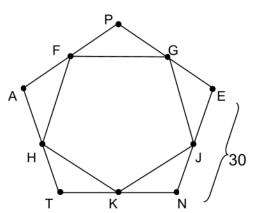
Geometry Team

Question #5.

Regular pentagon PENTA has midpoints of its sides G, J, K, H and F. EN = 30.

Part A: Find $m \angle PFG$.

Part B: If $\Delta PFG \sim \Delta FHG$ then the answer to part B is 1. If ΔPFG , ΔFHG are not similar, then the answer to part B is 0. If it cannot be determined, then the answer to part B is -1.



Part C: If $\frac{4}{5}$ is used as an approximation of $\cos 36^{\circ}$, find the perimeter of pentagon FGJKH.

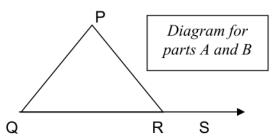
Part D: If $\frac{4}{5}$ is used as an approximation of $\cos 36^{\circ}$, find the length of diagonal \overline{AE} .

Part A: For $\triangle PQR$, \overrightarrow{QS} is drawn so that Q, R and S are collinear.

$$m\angle PQR = (6y + 52)^{o},$$

$$m \angle RPQ = (10y + 20)^{0}$$
, and

$$m \angle PRS = (20y + 46)^{\circ}$$
. Find y.



Part B: For $\triangle PQR$, \overrightarrow{QS} is drawn so that Q, R and S are collinear. If PQ = QR and $m\angle Q = 50^{\circ}$, then find $m\angle PRS$.

<u>Part C</u>: An equilateral triangle RST (not shown) has RS = (4x-10) cm and ST = (2x+20) cm. Give the height of RST in cm.

<u>Part D</u>: For ΔXYZ (not shown), XY = XZ. $m \angle X = (z+10)^{\circ}$ and $m \angle Y = (2z+20)^{\circ}$. Give the measure of the largest (interior) angle of ΔXYZ .

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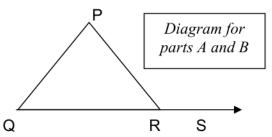
Question #6.

Part A: For ΔPQR , \overrightarrow{QS} is drawn so that Q, R and S are collinear.

$$m\angle PQR = (6y + 52)^{o},$$

$$m \angle RPO = (10v + 20)^{0}$$
, and

$$m\angle PRS = (20y + 46)^{\circ}$$
. Find y.



<u>Part B</u>: For $\triangle PQR$, \overrightarrow{QS} is drawn so that Q, R and S are collinear. If PQ = QR and $m\angle Q = 50^{\circ}$, then find $m\angle PRS$.

Part C: An equilateral triangle RST (not shown) has RS = (4x-10) cm and ST = (2x+20) cm. Give the height of RST in cm.

<u>Part D</u>: For ΔXYZ (not shown), XY = XZ. $m \angle X = (z+10)^{\circ}$ and $m \angle Y = (2z+20)^{\circ}$. Give the measure of the largest (interior) angle of ΔXYZ .

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Question #7.

Regular hexagons HEXGON and XYZKLG are congruent. M is the midpoint of \overline{ZK} . \overline{HM} and \overline{NM} intersect side \overline{GX} at P and Q as shown. Each side of the hexagons has **length 6**.

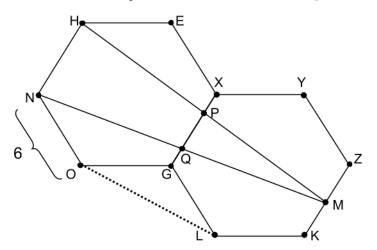
 $\underline{\underline{\mathsf{Part}}\; \mathsf{A}} \text{: The length of the a} \underline{\underline{\mathsf{ltitude}}} \; \mathsf{of}$

 ΔMHN from M to \overline{HN} .

Part B: The length of \overline{HM} .

Part C: The length of \overline{LO} .

Part D: The length of \overline{PQ} .



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Regular hexagons HEXGON and XYZKLG are congruent. M is the midpoint of \overline{ZK} . \overline{HM} and \overline{NM} intersect side \overline{GX} at P and Q as shown. Each side of the hexagons has **length 6**.

<u>Part A</u>: The length of the altitude of

 ΔMHN from M to HN.

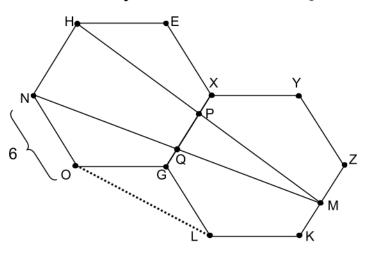
Part B: The length of \overline{HM} .

Part C: The length of \overline{LO} .

Part D: The length of \overline{PQ} .

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Question #7.

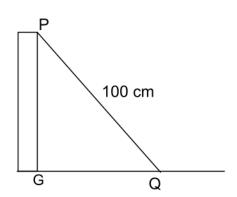


A 100 cm long ladder is leaning against a vertical wall as shown. The top of the ladder is P and the bottom is at ground level, Q. P starts 80 cm above ground on the wall. The diagram is not to scale. When P slides down the wall, the base of the ladder slides along the ground.

Part A: If P is 80 cm high, tell the distance from the midpoint of PQ to the wall, in cm.

Part B: If the top of the ladder slides down the wall so that P is on the wall 10 cm lower than it was originally, tell the distance in cm that Q moves because of the change in P's position.

Part C: Find the number of cm that P must slide down the wall from its original position so that the angle of elevation at O becomes 45 degrees.



Part D: As P slides down the wall from its original position, maintaining contact with the wall always, the height of P above ground (G) changes from 80 cm to 0 cm. How many integer lengths (in cm) can the base GO have during that slide? (Include the integer lengths which occur when PG is 80 cm and 0 cm.)

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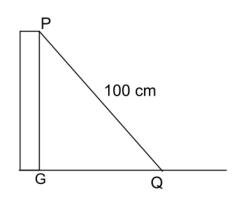
Question #8.

A 100 cm long ladder is leaning against a vertical wall as shown. The top of the ladder is P and the bottom is at ground level, Q. P starts 80 cm above ground on the wall. The diagram is not to scale. When P slides down the wall, the base of the ladder slides along the ground.

Part A: If P is 80 cm high, tell the distance from the midpoint of PO to the wall, in cm.

Part B: If the top of the ladder slides down the wall so that P is on the wall 10 cm lower than it was originally, tell the distance in cm that Q moves because of the change in P's position.

Part C: Find the number of cm that P must slide down the wall from its original position so that the angle of elevation at O becomes 45 degrees.



Part D: As P slides down the wall from its original position, maintaining contact with the wall always, the height of P above ground (G) changes from 80 cm to 0 cm. How many integer lengths (in cm) can the base GQ have during that slide? (Include the integer lengths which occur when PG is 80 cm and 0 cm.)

- Part A: ΔRST has integer length sides. One side length is 3 and the perimeter is 9. If ΔRST is not equilateral, then determine whether it is a(n) acute, obtuse or right triangle. Answer with one word: acute, obtuse or right. You must spell the word correctly!
- Part B: $\triangle RST$ has angle measures $(2x+6)^{\circ}$, $(3x-21)^{\circ}$, and $(4x+6)^{\circ}$.

If
$$\triangle RST$$
 is
$$\begin{cases} acute, & let \ B=1 \\ obtuse, & let \ B=2 \\ right, & let \ B=3 \\ not \ possible, & let \ B=4 \end{cases}$$

Give the value of B.

- Part C: $\triangle RST$ has side lengths 3, 4 and k. Find the greatest possible integer length for k so that ΔRST is acute.
- Part D: $\triangle RST$ is isosceles with angle measures $(x+12)^{\circ}$, $(2x-12)^{\circ}$, and v° . Give the greatest possible value of v.

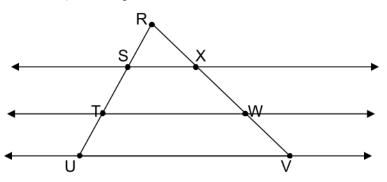
BC/AHS-PB January Invitational Geometry Team Question #9.

- Part A: $\triangle RST$ has integer length sides. One side length is 3 and the perimeter is 9. If ΔRST is not equilateral, then determine whether it is a(n) acute, obtuse or right triangle. Answer with one word: acute, obtuse or right. You must spell the word correctly!
- Part B: $\triangle RST$ has angle measures $(2x+6)^{\circ}$, $(3x-21)^{\circ}$, and $(4x+6)^{\circ}$.

If
$$\triangle RST$$
 is
$$\begin{cases} acute, & let \ B=1 \\ obtuse, & let \ B=2 \\ right, & let \ B=3 \\ not \ possible, & let \ B=4 \end{cases}$$

Give the value of B.

- Part C: $\triangle RST$ has side lengths 3, 4 and k. Find the greatest possible integer length for k so that ΔRST is acute.
- Part D: $\triangle RST$ is isosceles with angle measures $(x+12)^{\circ}$, $(2x-12)^{\circ}$, and y° . Give the greatest possible value of v.



 $\overrightarrow{XS} \parallel \overrightarrow{WT} \parallel \overrightarrow{VU} \; .$

RS:RT:TU = 2:5:4.

UV = 20.

Diagram is not drawn to scale.

Part A: If RT = 15, then give the length of \overline{RU} .

Part B: If RX = 5, then give the ratio of RX to XW.

Part C: If SX = (2c-10), then find the value of c.

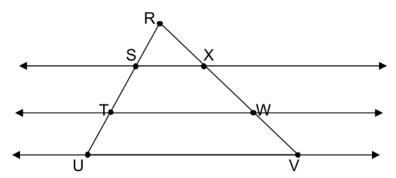
Part D: If RU = 27 and RV = 36, then give the perimeter of

quadrilateral TWVU in fraction form.

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Question #10.



 $\overrightarrow{XS} \parallel \overrightarrow{WT} \parallel \overrightarrow{VU}$.

RS:RT:TU = 2:5:4.

UV = 20.

Diagram is not drawn to scale.

Part A: If RT = 15, then give the length of \overline{RU} .

Part B: If RX = 5, then give the ratio of RX to XW.

Part C: If SX = (2c-10), then find the value of c.

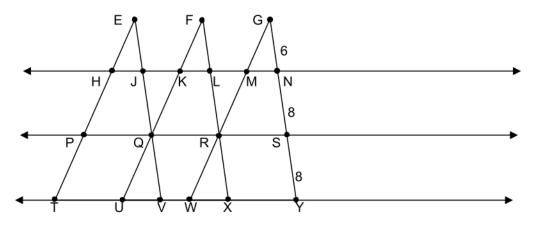
Part D: If RU = 27 and RV = 36, then give the perimeter of

quadrilateral TWVU in fraction form.

- Part A: The measure of $\angle A$ is 30 degrees more than five times the measure of its supplement. Find the measure of $\angle A$.
- Part B: The measure of $\angle B$ is six degrees less than half of its complement. Find the measure of the supplement of $\angle B$.
- Part C: The exterior angle at C to ΔCUT has a measure that is three times one of the remote interior angles to C. If the other remote interior angle to C has measure 20°, then find the measure of the exterior angle at C.
- Part D: Right ΔTUG has hypotenuse \overline{GT} . If $\tan(\angle G) = 4$, then find $\sin(\angle T) \cdot \cos(\angle T)$ in fraction form.

BC/AHS-PB January Invitational Geometry Team **Question #11.**

- Part A: The measure of $\angle A$ is 30 degrees more than five times the measure of its supplement. Find the measure of $\angle A$.
- Part B: The measure of $\angle B$ is six degrees less than half of its complement. Find the measure of the supplement of $\angle B$.
- Part C: The exterior angle at C to ΔCUT has a measure that is three times one of the remote interior angles to C. If the other remote interior angle to C has measure 20°, then find the measure of the exterior angle at C.
- Part D: Right ΔTUG has hypotenuse GT. If $\tan(\angle G) = 4$, then find $\sin(\angle T) \cdot \cos(\angle T)$ in fraction form.



ΔVTE, ΔXUF, and ΔYWG are congruent. **TU: UV=2:1. UW: WX=2:1.** $\overrightarrow{HN} \parallel \overrightarrow{PS} \parallel \overrightarrow{TY}$. H, J, K, L, M, N are collinear; P, Q, R, S are collinear; T, U, V, W, X, Y are collinear. GN=6, NS=8, SY=8.

Part A: If GM = 5 and WY = 9 then find the perimeter of Δ GWY.

<u>Part B</u>: If TU = 7 then give the length of \overline{TY} .

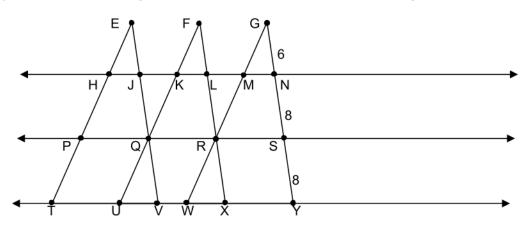
Part C: If GM = 5, then find the length of \overline{LR} .

Part D: If UV = 3, then find the length of the median (midline) of trapezoid PQVT.

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Question #12.



ΔVTE, ΔXUF, and ΔYWG are congruent. **TU: UV=2:1. UW: WX=2:1.** $\overrightarrow{HN} \parallel \overrightarrow{PS} \parallel \overrightarrow{TY}$. H, J, K, L, M, N are collinear; P, Q, R, S are collinear; T, U, V, W, X, Y are collinear. GN=6, NS=8, SY=8.

Part A: If GM = 5 and WY = 9 then find the perimeter of Δ GWY.

<u>Part B</u>: If TU = 7 then give the length of \overline{TY} .

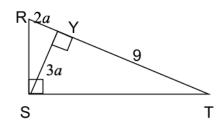
Part C: If GM = 5, then find the length of \overline{LR} .

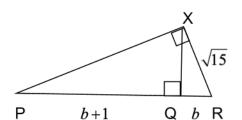
Part D: If UV = 3, then find the length of the median (midline) of trapezoid PQVT.

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Question #13.





In $\triangle RST$, $\angle RST$ is a right angle, and \overline{SY} is the altitude to the hypotenuse. In $\triangle PXR$, $\angle PXR$ is a right angle, and \overline{XQ} is the altitude to the hypotenuse. Diagrams are not drawn to scale.

Part A: If RY= 2a, YT = 9 and SY = 3a then find the value of a.

Part B: In $\triangle PXR$, $XR = \sqrt{15}$, PQ = (b+1) and QR = b. Find the value of b.

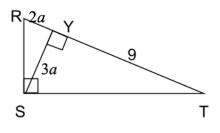
Part C: If RY= 2a, YT = 9 and SY = 3a, then find the perimeter of ΔRST .

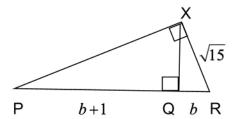
Part D: In $\triangle PXR$, $XR = \sqrt{15}$, PQ = (b+1) and QR = b. Find the length of \overline{XQ} .

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Question #13.





In $\triangle RST$, $\angle RST$ is a right angle, and \overline{SY} is the altitude to the hypotenuse. In $\triangle PXR$, $\angle PXR$ is a right angle, and \overline{XQ} is the altitude to the hypotenuse. Diagrams are not drawn to scale.

Part A: If RY= 2a, YT = 9 and SY = 3a then find the value of a.

<u>Part B</u>: In ΔPXR , $XR = \sqrt{15}$, PQ = (b+1) and QR = b. Find the value of b.

<u>Part C</u>: If RY= 2a, YT = 9 and SY = 3a, then find the perimeter of ΔRST .

Part D: In $\triangle PXR$, $XR = \sqrt{15}$, PQ = (b+1) and QR = b. Find the length of \overline{XQ} .