Geometry Team Answers and Solutions
January 13, 2024 BC/AHS-PB Statewide Invitational Competition

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<u>ANSWERS</u>:

1.	**A: 24**	**B: 15**	**C:** $30\sqrt{10}$	**D:** $20\sqrt{3}$
2.	**A: 6**	**B: 30**	**C: 12**	**D: 6120**
3.	**A:** $32+16\sqrt{3}$ **or** $16\sqrt{3}+32$		**B:** $16\sqrt{3}+16\sqrt{6}$ **or** $16\sqrt{6}+16\sqrt{3}$	
	C: $8\sqrt{3}+8\sqrt{6}+8$ **(in any order),**		**D: 423**	
4.	**A: 2**	**B:** $\frac{3}{2}\sqrt{2}$ **or** $1.5\sqrt{2}$		
	C: $\frac{9}{4}\sqrt{2}+7$ **or** $2.25\sqrt{2}+7$ **or same in** $a+b\sqrt{c}$ **form**			**D =1**
5.	**A: 36**	**B: 1**	**C: 120**	**D: 48**
6.	**A:** $\frac{13}{2}$ **or 6.5**	**B =115**	**C:** $25\sqrt{3}$	**D: 72**
7.	**A:** $12\sqrt{3}$	**B: 21**	**C:** $6\sqrt{3}$	**D: 3**
8.	**A: 30**	**B:** $10\sqrt{51}-60$	**C:** $80-50\sqrt{2}$	**D: 41**
9.	**A: obtuse**	**B: 3**	**C: 4**	**D: 108**
10.	**A: 27**	**B:** $\frac{2}{3}$ **or 2:3**	**C:** $\frac{65}{9}$	**D:** $\frac{532}{9}$ **(must be in fraction form)**
11.	**A: 155**	**B: 154**	**C: 30**	**D:** $\frac{4}{17}$ **(must be in fraction form)**
12.	**A:** $\frac{148}{3}$	**B:** $\frac{49}{2}$ **or 24.5**	**C: 8**	**D: 7.5 or** $\frac{15}{2}$
13.	**A: 2**	**B: 2.5 or** $\frac{5}{2}$	**C:** $13+5\sqrt{13}$	**D:** $\frac{1}{2}\sqrt{35}$ **or** $0.5\sqrt{35}$

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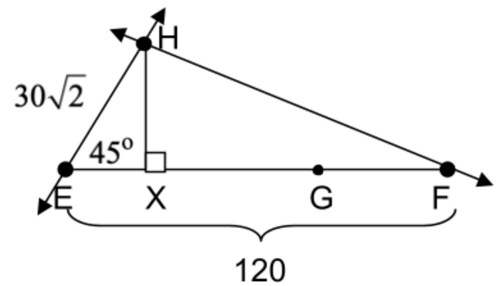
SOLUTIONS:

1. A: 24 B: 15 C: $30\sqrt{10}$ **D:** $20\sqrt{3}$

Part A: : Let ST=x and RS = $\frac{2}{3}x$. $x+\frac{2}{3}x=60$. $\frac{5}{3}x=60$. x = 36. RS = $\frac{2}{3}(36)=24$

Part B: Since XZ = $(4x+20)$ and WZ=120 then XZ=40 and $(4x+20)$=80, and x=15.



Part C: Drop an altitude from H to $\overline{EF}$, and call the point on $\overline{EF}$ that is the foot of the altitude X. EX = 30 so XF=120-30=90. HF = $\sqrt{30^2+90^2}=\sqrt{30^2(1+9)}=30\sqrt{10}$

Part D: Let RS=d, ST=d and TU=2d. RU = 4d=120 so d=30. That means RT=60 and since angle R is 30 degrees, ZT is 60/$\sqrt{3}$ = $20\sqrt{3}$.

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2. A: 6 B: 30 C: 12 D: 6120

Part A: $11a+10+6a=180$. $17a=170$. a = 10. Sides = 360 degrees divided by 6a = 6.

Part B: $180(n-2)=1800$. $n-2=10$. $n=12$. One exterior angle is 360/12 = 30.

Part C: $\frac{1}{2}n(n-3)=54$. $n(n-3)=108$. Since 108 = 12x9 then n=12.

Or you can solve $n^2-3n-108=0$. Answer=12.

Part D: One exterior angle is 360/36=10. One interior = 170. 170x36=6120.

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3. A: $32+16\sqrt{3}$ **or** $16\sqrt{3}+32$ **B:** $16\sqrt{3}+16\sqrt{6}$ **or** $16\sqrt{6}+16\sqrt{3}$
C: $8\sqrt{3}+8\sqrt{6}+8$ **(in any order), D: 423**

Part A: Look at ΔTQU. Since we are told $m\angle TQU=30^{o}$, and TU=8 then QU=$8\sqrt{3}$.
The perimeter of ΔQRT is then $QR+RT+QT=16\sqrt{3}+16+16$ = $32+16\sqrt{3}$

Part B: Since $m\angle SQT = 15^o$, $m\angle SQU = 45^o$. From part A we get QU = $8\sqrt{3}$ so QS= $8\sqrt{6}$. ΔQSR has perimeter $16\sqrt{3}+8\sqrt{6}+8\sqrt{6}=16\sqrt{3}+16\sqrt{6}$.

Part C: The perimeter of ΔQST is TS+QT+QS = $\left(8\sqrt{3}-8\right)+16+8\sqrt{6}=8\sqrt{3}+8\sqrt{6}++8$

Part D: From part B we have SU = $8\sqrt{3}$ and from part A we have QU= $8\sqrt{3}$. So, PU = $2+8\sqrt{3}$. PQ = $\sqrt{(2+8\sqrt{3})^2+\left(8\sqrt{3}\right)^2}$ = $\sqrt{4+32\sqrt{3}+192+192}=\sqrt{388+32\sqrt{3}}$.
The square of QP is $388+32\sqrt{3}$ = $a+b\sqrt{c}$. So, a + b + c = 423.

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4. A: 2 B: $\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$

C: $\frac{9}{4}\sqrt{2}+7$ or $2.25\sqrt{2}+7$ or same in $a+b\sqrt{c}$ form D =1

Part A: The side of square 1 is JK. So, the height to the hypotenuse of ΔJKL is $\frac{1}{2}x$ for JK = x. From F to R is $x+\frac{1}{2}x+\frac{1}{2}x+x=3x$. 3x=6 so x=2. The side of square 1 is 2.

Part B: Two diagonals, of square 3 and square 4 add to 6. So, one diagonal is 3 and the side of square 3 is $\frac{3}{\sqrt{2}}=\frac{3}{2}\sqrt{2}$.

Part C: The perimeter of the shaded pentagon, clockwise from EH, is
3 + 2* + 2 + $\frac{3}{4}\sqrt{2}$ + $\frac{3}{2}\sqrt{2}$** = $\frac{9}{4}\sqrt{2}+7$
*from part A, 6 – 2, divided by 2. **from part B.

Part D: Since square 1 has side length 2 (see part A), and JK = 2 then the height from L in ΔLNP is 1.

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5. A: 36 B: 1 C: 120 D: 48

Part A: Using isosceles triangle PFG, $m\angle P = 108^o$. 180-108=72 so each base angle is 36.

Part B: Since $\angle AFH \cong \angle PFG$ so $m\angle HFG = 180-2(36)=108$, and since the base angles of ΔFHG are congruent, we have by AA~ Postulate, that $\Delta PFG \sim \Delta FHG$, and B = 1.

Part C: Drop the altitude in triangle PFG from P to $\overline{FG}$. Call the base of this altitude X. $\cos 36^{o} = \frac{FX}{3}$. Using a cosine approximation of $\frac{4}{5}$, we have $\frac{4}{5} = \frac{FX}{15}$. FX=12 and FG = 24. The perimeter of the pentagon FGJKH is 24(5) = 120

Part D: Use triangle PAE, and again drop an altitude from P. That makes a triangle similar to triangle PFG, and twice the size. So, AE = 48.

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6. A: $\frac{13}{2}$ or 6.5 B =115 C: $25\sqrt{3}$ D: 72

Part A: $20y + 46 = 6y + 52 + 10y + 20$. $4y = 26$. $y = \frac{13}{2}$

Part B: If PQ=QR then this is an isosceles triangle with vertex angle Q. Angles R and P are base angles so 50 + x + x = 180. x=65. That is the measure of interior angle R, and so $m\angle PRS$ = 180 – 65 = 115.

Part C: Since sides are equal in length, 4x-10 = 2x+20. So, x = 15. One side is 50 and the height is $25\sqrt{3}$

Part D: If XY= XZ then vertex angle is S. $z + 10 + 2(2z + 20) = 180$. $5z = 130$. z = 26. That makes interior angles 36, 72 and 72. Largest is 72.

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7. A: $12\sqrt{3}$ B: 21 C: $6\sqrt{3}$ D: 3

Part A: Draw the altitude from M to $\overline{NH}$. It creates two right triangles, with legs 3 and the altitude. Now we find the height of one hexagon from midpoint of $\overline{NH}$ to the midpoint of the opposite side. It is twice the apothem of the hexagon which gives twice $3\sqrt{3}$ = $6\sqrt{3}$. So, the height of ΔMHN is $12\sqrt{3}$.

Part B: Continue with the right triangles drawn in part A. $3^2 + \left(12\sqrt{3}\right)^2 = (HM)^2$.

$\sqrt{9 + 432} = 3\sqrt{1 + 48} = 3(7) = 21$.

Part C: At G, the angle OGL is 360-120-120= 120. So, we have an isosceles triangle with vertex 120 degrees and base angles 30 each. LO has length $6\sqrt{3}$, from the long leg of a special right triangle formed by the height from G.

Part D: To get PQ we look at ΔMHN and PQ which forms similar triangles MPQ and PMN. $\frac{MP}{PQ}=\frac{MH}{HN}$. From the answers above, we have $\frac{10.5}{PQ}=\frac{21}{6}$ and PQ is half of 6 which is 3.

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8. A: 30 B: $10\sqrt{51}-60$ C: $80-50\sqrt{2}$ D: 41

Part A: The midpoint of $\overline{PQ}$ and $\overline{PG}$ form a segment half of QG. Use the Pythagorean Theorem to get GQ=60 and the answer is 30.

Part B: $\sqrt{100^2-70^2}=10\sqrt{100-49}=10\sqrt{51}$. So, the change in Q is $10\sqrt{51}-60$.

Part C: The ladder is 100 in length so to form an isosceles right triangle we need the ground and wall distances to be $50\sqrt{2}$. P was originally 80 so the change is $80-50\sqrt{2}$.

Part D: GQ will vary from 60 to 100, inclusive. 100 – 59 = 41 integers.

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9. A: obtuse B: 3 C: 4 D: 108

Part A: Sides can have lengths 3, 3, 3 or 3, 2, 4. Since it is not equilateral, we use 2, 3, 4 and since $\sqrt{2^2+3^2}=\sqrt{13}$ we compare 4 to this and see that by the hinge theorem this triangle is obtuse.

Part B: Add the angle measures to get 9x –9=180. x-1=20. x=21. So, angles are 48, 42 and 90. Answer is 3.

Part C: By the Triangle Inequality theorem, $1<k<7$. To be acute, $\sqrt{4^2-3^2}<k<\sqrt{4^2+3^2}$. So, integers in that interval are 3 and 4. The greatest is 4.

Part D: Either x+12 and 2x-12 are base angles, which makes x=24, or
x+12 is the vertex angle and $x+12+2(2x-12)=180$. 5x-12=180. x=38.4,
or 2x-12 is the vertex angle. 2x-12 + 2(x+12)=180 and 4x+12=180. x=42.
In the first case, angles are 36, 36 and y=108. In the second case, angles are 50.4, 64.8, 64.8 and y is 64.8. In the last case, angles are 72, 54, 54 and y is 54. The greatest possible value of y is 108.

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10. A: 27 B: $\frac{2}{3}$ or 2:3 C: $\frac{65}{9}$ D = $\frac{532}{9}$ **(must be in fraction form)**

Part A: Let RS = 2a, ST=3a and TU=4a. RT=15 so 5a=15 and a=3. RU=9a = 27.

Part B: The ratio of RX to XW is equal to the ratio of RS to ST, which is 2/3 or 2:3.

Part C: $\frac{RS}{SX}=\frac{RU}{UV}$. $\frac{2}{2c-10}=\frac{9}{20}$. $\frac{2}{c-5}=\frac{9}{10}$. $9c-45=20$. $c=\frac{65}{9}$.

Part D: $\frac{5}{9}=\frac{TW}{20}$ so TW=$\frac{100}{9}$. 9a=27 so a=3 and RS=6, ST=9, TU=12. 9b=36 so b=4, and RX=8, XW=12 and WV=16. The perimeter of TWVU is 12+$\frac{100}{9}$+16+20 = $\frac{100}{9}+48=\frac{432+100}{9}=\frac{532}{9}$.

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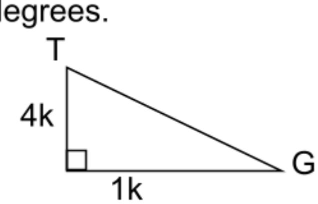
11. A: 155 B: 154 C: 30 D: $\frac{4}{17}$ **(must be in fraction form)**

Part A: $a=30+5(180-a)$. $6a=930$. $a=155$.

Part B: $b=\frac{1}{2}(90-b)-6$. $2b=90-b-12$. $3b=78$. $b=26$. Supplement is 154 degrees.

Part C: $3x=x+20$. x=10. The exterior angle at C then is 30 degrees.

Part D: If $\tan(\angle G)=4$, call the legs 4k and 1k and so GT = $k\sqrt{17}$. $\sin(\angle T)\bullet\cos(\angle T)$ = $\frac{1}{\sqrt{17}}\bullet\frac{4}{\sqrt{17}}=\frac{4}{17}$



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12. A: $\frac{148}{3}$ B: $\frac{49}{2}$ or 24.5 C: 8 D: 7.5 or $\frac{15}{2}$

Part A: $\frac{GM}{MR}=\frac{GN}{NS}$. $\frac{5}{MR}=\frac{6}{8}$. MR = $\frac{20}{3}$. Since NS=SY, then MR=RW. So, perimeter of ΔGWY is 5+6+$\frac{20}{3}+8+\frac{20}{3}+8+9=$36+$\frac{40}{3}=\frac{148}{3}$

Part B: Let TU=2k and UV=k. So, we have TU+UW+WY=2k+2k+3k = 7k. TU=7 so k=7/2 and 7k = 49/2 or 24.5

Part C: $\frac{5}{6}=\frac{MR}{8}$. MR = $\frac{20}{3}$. Now $\Delta GMN \sim \Delta RML$, by AA~ Postulate (and parallel line theorems) $\frac{6}{LR}=\frac{5}{20/3}$. LR = 8.

Part D: PQUT is a parallelogram. UV = 3 so TU=PQ=6. The midline of a trapezoid is the average of the bases, so $\frac{1}{2}(6+9)=7.5$ *or* $\frac{15}{2}$

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13. A: 2 **B: 2.5 or** $\frac{5}{2}$ **C:** $13+5\sqrt{13}$ **D:** $\frac{1}{2}\sqrt{35}$ **or** $0.5\sqrt{35}$

Part A: $9a^2=2a(9)$. Divide by 9a since neither 9 nor a is equal to 0. a = 2.

Part B: $\sqrt{15}^2=b(2b+1)$. $2b^2+b-15=0$. $(2b-5)(b+3)=0$. b = 2.5.

Part C: RS = $\sqrt{4a^2+9a^2}=RS$. $RS=a\sqrt{13}$. $(RS)^2=2a(2a+9)$. $13a^2=2a(2a+9)$. $9a^2=18a$. Divide by 9a: $a=2$. So, RY = 4, YT = 9, SY = 6, RS = $2\sqrt{13}$. Using ΔSTY, ST = $\sqrt{36+81}=\sqrt{117}$ = $3\sqrt{13}$. Perimeter of ΔRST is 4 + 9 + $3\sqrt{13}$ + $2\sqrt{13}$ = $13+5\sqrt{13}$.

Part D: $\sqrt{15}^2=b(2b+1)$. $2b^2+b-15=0$. $(2b-5)(b+3)=0$. b = 2.5. XQ = $\sqrt{\frac{5}{2}\left(\frac{7}{2}\right)}=\frac{1}{2}\sqrt{35}$