Answer Key:

1. D
2. A
3. C
4. B
5. A

6. C
7. D
8. E
9. A
10. B

11. E
12. C
13. A
14. B
15. D

16. C
17. D - Disputed and A, B, C, and D were all accepted.
18. A
19. C
20. A

21. D
22. A
23. A
24. B
25. D

26. C
27. B
28. A
29. D
30. A

Solutions:

1. D: $2(-3)^3 - 3(-3)^2 - 2(-3) + 1 \Rightarrow 2(-27) - 3(9) + 6 + 1 \Rightarrow -54 - 27 + 6 + 1 = -74$

2. A: $-\frac{5}{3} + \frac{7p}{4} = \frac{1}{2}\left(\frac{14p}{3} + 1\right) - \frac{5p}{4} \Rightarrow -\frac{5}{3} + \frac{7p}{4} = \frac{14p}{6} + \frac{1}{2} - \frac{5p}{4} \Rightarrow$
$\left(-\frac{5}{3} + \frac{7p}{4} = \frac{14p}{6} + \frac{1}{2} - \frac{5p}{4}\right) \cdot 12 \Rightarrow -20 + 21p = 28p + 6 - 15p \Rightarrow$
$-20 + 21p = 13p + 6 \Rightarrow 8p = 26 \Rightarrow p = \frac{13}{4}$

3. C: $\frac{c}{a} = \frac{d}{r} \Rightarrow ad = cr \Rightarrow a = \frac{cr}{d}$

4. B: $k + \frac{1}{2} > -\frac{7}{6} \Rightarrow k > -\frac{5}{3}$ and $\frac{5}{3}k + \frac{7}{5} \leq \frac{12}{5} \Rightarrow$ multiply by 15 $\Rightarrow 25k + 21 \leq 36 \Rightarrow$
$25k \leq 15 \Rightarrow k \leq \frac{3}{5}$. So, $-\frac{5}{3} < k \leq \frac{3}{5}$, the only integer solutions are –1 and 0. The sum is –1.

5. A: $\left(\frac{a^{-2}b^2c^{-1} \cdot ab^3}{a^3b^3c^2}\right)^2 \Rightarrow \left(\frac{a^{-1}b^5c^{-1}}{a^3b^3c^2}\right)^2 \Rightarrow \left(\frac{b^2}{a^4c^3}\right)^2 \Rightarrow \frac{b^4}{a^8c^6} \Rightarrow 4 + 8 + 6 = 18.$

6. C: Let $x = percent\ concentration\ of the\ first\ solution$. Take the volume in ml of each solution and multiply by their percent concentration. This gives you: $9(x) + 6(0) = 15(24) \Rightarrow 9x = 360 \Rightarrow x = 40\%$.

7. D: Let $x =$ $average\ speed\ of\ freight\ train$. Find each train's distance by multiplying their rate by their time traveled. $Passenger\ train:\ 8(75)$; $Freight\ train: 12(x)$. Since the passenger train caught up, they both have traveled the same distance. So, $8(75) = 12x \Rightarrow 600 = 12x \Rightarrow x = 50\ km/h$.

8. E: For Bill's integer: the sum of a two-digit integer in the form AB can be written as $10A + B$. So, for Bill's integer, we have $10A + B + 18 = 10B + A \Rightarrow 9A - 9B = -18 \Rightarrow A - B = -2\ and\ A + B = 10$. Solving this system of equations gives $A = 4$ and $B = 6$; so, Bill's integer is 46. For Ted's integer: the sum of the digits of a two-digit integer in the form CD can be written as $10C + D$. So, $10C + D - 36 = 10D + C \Rightarrow 9C - 9D = 36 \Rightarrow C - D = 4\ and\ C + D = 8$. Solving this system gives $C = 6$ and $D = 2$; So, Ted's integer is 62. The product of 46 and 62 is 2852.

9. A: $f(-3) = 2(-3)^2 - 1 = 17$; $f(-1) = 2(-1)^2 - 1 = 1$; $g(-2) = |-(-2) + 2| = 4$; $g(5) = |-5 + 2| = 3$; so, $\frac{f(-3) - g(-2)}{g(5) + f(-1)} = \frac{17-4}{3+1} = \frac{13}{4}$.

10. B: $f(-2) = 2(-2)^2 - 1 = 7$; $g(7) = |-7 + 2| = 5$; $f(5) = 2(5)^2 - 1 = 49$; $g(49) = |-49 + 2| = 47$; $f(47) = 2(47)^2 - 1 = 4417$.

11. E: For a line to be perpendicular to the line $x = 2$, which is a vertical line, the equation of the line must be that of a horizontal line. The slope of a horizontal line is 0. Since the slope of the given line is $m = \frac{B}{2}$, we have, $\frac{B}{2} = 0 \Rightarrow B = 0$. Since B must be zero, then $\frac{B}{C}$ is also zero.

12. C: If you start at the point (0, 0) and move according to the directions you will end up at the point (2, –1). The distance between (0, 0) and (2, –1) is $\sqrt{2^2 + 1^2} = \sqrt{5}$ miles.

13. A: $\frac{a+b}{c-d} = \frac{d-a}{b+c}$ If we cross multiply, we get $ab + b^2 + ac + bc = cd - ac - d^2 + ad$. We can bring all terms with an "a" to one side of the equation: $ab + ac + ac - ad = cd - d^2 - b^2 - bc$. Factor out the a and then divide: $a(b + 2c - d) = cd - d^2 - b^2 - bc \Rightarrow a = \frac{cd-d^2-b^2-bc}{b+2c-d}$.

14. B: $(2x - 5)^4 = 16x^4 - 160x^3 + 600x^2 - 1000x + 625$. The sum of –160 and 600 is 440.

15. D: $\frac{x(x^{2a-3})^{-1}}{(x^{-a+2})(x^{5a})} = \frac{x(x^{-2a+3})}{x^{4a+2}} = \frac{x^{-2a+4}}{x^{4a+2}} = x^{-6a+2}$

16. C: $\left|\left|\left|x - 3\right| - 1\right| - 1\right| - 1\right| = 0$. Since the equation is equal to 0 the outer absolute value bars can be removed and the –1 added to the other side. $\left|\left|\left|x - 3\right| - 1\right| - 1\right| = 1$. This equation has 2 possible solutions: 1: $\left|\left|x - 3\right| - 1\right| - 1 = 1 \Rightarrow \left|\left|x - 3\right| - 1\right| = 2$. OR 2: $\left|\left|x - 3\right| - 1\right| - 1 = -1 \Rightarrow \left|\left|x - 3\right| - 1\right| = 0$

Solving 1: $\left|\left|x - 3\right| - 1\right| = 2 \Rightarrow |x - 3| - 1 = 2$ OR $|x - 3| - 1 = -2$.

$|x - 3| = 3$ OR $|x - 3| = -1$ (which has no solution)

$x - 3 = 3$ OR $x - 3 = -3$

$x = 6$ OR $x = 0$

Solving 2: $\left|\left|x - 3\right| - 1\right| = 0 \Rightarrow |x - 3| - 1 = 0 \Rightarrow |x - 3| = 1$

$x - 3 = 1$ OR $x - 3 = -1$

$x = 4$ OR $x = 2$

The sum of the solutions is $6 + 0 + 4 + 2 = 12$.

17. D: $9x^2 + 38x + 8 = (9x + 2)(x + 4)$; $9x^2 + 68x - 32 = (9x - 4)(x + 8)$. So, $(x - 8)$ is not a factor of either of the two given polynomials. **Disputed and A, B, C, and D were all accepted.**

18. A: $81x^2 - 108xy + 36y^2 - 25 \Rightarrow [(9x)^2 - 2(9x)(6y) + (6y)^2] - 25 \Rightarrow (9x - 6y)^2 - (5)^2 \Rightarrow ((9x - 6y) + 5)((9x - 6y) - 5)$. Fitting this into $(Ax + By + C)(Ax + By - C)$, we get $A = 9, B = -6$, and $C = 5$. The sum $9 + (-6) + 5 = 8$.

19. C: $-4x(2x) - 9(-x + 7) = -x(12x - 5) \Rightarrow -8x^2 + 9x - 63 = -12x^2 + 5x \Rightarrow$

$$4x^2 + 4x - 63 = 0 \Rightarrow (2x - 7)(2x + 9) = 0 \Rightarrow x = \frac{7}{2}, x = -\frac{9}{2} \Rightarrow 4\left(\frac{7}{2}\right) + 2\left(-\frac{9}{2}\right) \Rightarrow 14 - 9 = 5$$

20. A: If we let A = *the ticket price per adult* and B = *the ticket price per student*, then the system of equations modeling this situation is, $8A + 11B = 111$ and $13A + 12B = 151$. Use elimination to solve the system:

$-12(8A + 11B = 111) \Rightarrow -96A - 132B = -1332$ *and* $11(13A + 12B = 151) \Rightarrow 143A + 132B = 1661$

Adding the equations gives:

$$(-96A - 132B = -1332) + (143A + 132B = 1661) \Rightarrow 47A = 329 \Rightarrow A = 7$$

Plug A = 7 back into an original equation to solve for B:

$$8(7) + 11B = 111 \Rightarrow 56 + 11B = 111 \Rightarrow 11B = 55 \Rightarrow B = 5$$

So, the cost for 2 adults and 3 children is: $2(7) + 3(5) = \$29$.

21. D: $[4 \# (1 \& (3 \# 6))] \Rightarrow (3 \# 6) = 4(3) - 3(6) = 12 - 18 = -6$. Then: $(1 \& -6) = 1^2 + (-6)^2 = 37$. Then: $(4 \# 37) = 4(4) - 3(37) = -95$.

22. A: i) The sum of 2 irrational numbers is always irrational. **False**. Counterexample: $\sqrt{2}+(-\sqrt{2})=0$.
ii) The product of 2 irrational numbers is always irrational. **False.** Counterexample: $\sqrt{2}(\sqrt{2})=2$.
iii) Any real number that is expressed using a decimal point is not a rational number. **False.**
Counterexample: $0.5=\frac{1}{2}$.
So, there are 0 true statements.

23. A: Simplify the ordered pairs. $\left(\frac{2^7}{4},\frac{3^5}{9}\right)=(2^{7-2},3^{5-2})=(32,27)$ and $(3!,0!)=(3\cdot 2\cdot 1,1)=(6,1)$. So, the slope of the line between $(32,27)$ and $(6,1)$ is $\frac{27-1}{32-6}=\frac{26}{26}=1$.

24. B: Simplify the equation, $2(ky-3)=\frac{1}{2}(kx-6y+4)\Rightarrow 2ky-6=\frac{k}{2}x-3y+2\Rightarrow$ $2ky+3y=\frac{k}{2}x+8\Rightarrow y(2k+3)=\frac{k}{2}x+8\Rightarrow y=\frac{\frac{k}{2}}{2k+3}x+8$. So, the slope of this line is given by $\frac{\frac{k}{2}}{2k+3}\Rightarrow\frac{k}{4k+6}$. So, we set this slope expression equal to $\frac{3}{2}$: $\frac{k}{4k+6}=\frac{3}{2}$. Cross multiply and solve: $2k=3(4k+6)\Rightarrow 2k=12k+18\Rightarrow -10k=18\Rightarrow k=-\frac{18}{10}=-\frac{9}{5}$.

25. D: 20 questions correct gives a score of 100 points. 1 question blank gives a score of 96 points, and 1 question wrong gives a score of 95 points. So, the scores of 99, 98, and 97 points are unattainable. Now, the next highest score is from getting 2 questions blank and that gives a score of 92 points. So, the scores of 94 and 93 are also unattainable. Thus, these are the 5 highest unattainable scores: 99, 98, 97, 94, 93. Their sum is 99 + 98 + 97 + 94 + 93 = 481.

26. C: Slope of the line containing the points $(2\pi,3)$ and $(-4\pi,6)$ is $\frac{6-3}{-4\pi-2\pi}=\frac{3}{-6\pi}=\frac{-1}{2\pi}$. Then, we can find the y-intercept by plugging into $y=mx+b$: $3=\left(\frac{-1}{2\pi}\right)(2\pi)+b\Rightarrow 3=-1+b\Rightarrow b=4$. The product of 4 and $\frac{-1}{2\pi}$ is $\frac{-2}{\pi}$.

27. B: $(x)+(x+2)=1.50(x+4)\Rightarrow 2x+2=1.5x+6\Rightarrow 0.5x=4\Rightarrow x=8$. So, the integers are 8, 10, and 12. Their sum is 8 + 10 + 12 = 30.

28. A: The sum of the first 50 multiples of 7 is 7 + 14 + 21…. This can be written as $7(1+2+3\ldots)\Rightarrow$ $7(1+2\ldots+50)\Rightarrow 7\left(\frac{50\cdot 51}{2}\right)=7(1275)=8925$.

29. D: If $a-b=20$, then $(a-b)^2=400$. Using the formula $(a-b)^2=a^2-2ab+b^2$, we can substitute in $(a-b)^2=400$ and $a^2+b^2=500$ to get $400=500-2ab$. Solving this equation for ab gives us $-100=-2ab\Rightarrow ab=50$.

30. A: Since $x\cdot x=x^2$ and $x+x=2x$. The product of $x^2\cdot 2x=2x^3$.