

Barbra Nunn Solutions December 2022

1. D $RT = D \rightarrow (R-10)2T = \frac{3}{2}D \rightarrow 2RT - 20T = \frac{3}{2}RT$
 $2R - 20 = \frac{3}{2}R \rightarrow R = 40$

2. B Low numbers are easy to get so start high and check. 29, 28, 26, 25, 22, and 19.
 A total of 6 are not possible.

3. B

C	A	T
1	1	1
1	$\frac{-1}{3}$	$\frac{-1}{3}$
0	$\frac{1}{3}$	0
$\frac{2}{3}$	1	$\frac{2}{3}$

 You repeat this 2 more times so $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

4. A Draw a good picture. Angles SON and NSO are congruent, so they are each 36 degrees. Since L is the midpoint, it creates an altitude, so it is 90 degrees. LSO is 72 degrees since SON is 36 degrees. Therefore, our angle is 18 degrees

5. E (111) The total is 210 and the 4th number is 30. Make all other numbers as small as possible except the largest. You get 1, 2, 3, 30, 31, and 32. This sums to 99.
 $210 - 99 = 111$

6. C $(W+3)(2W+3) = 3 \cdot 2W^2 \rightarrow 2W^2 + 9W + 9 = 6W^2$
 $4W^2 - 9W - 9 = 0 \rightarrow (4W+3)(W-3) \rightarrow W = 3 \rightarrow 2 \cdot 3 + 3 = 9$

7. D $\frac{3}{2}x \cdot 6 + x \cdot 3 = 12x \rightarrow \frac{12x}{1.5x+x} = \frac{24}{5}$

8. C $\frac{3}{4} \cdot 300 = 225 \rightarrow 3+6+12+13+16+23+28+43+51 = 195$. All the scores in the 9's finally passes a total of 225 so the answer is 10 to be higher than 75%

9. B Draw a 6x6 and a 1x6. On the 6x6 add up the number of ways to get a sum of 4 all the way up to a sum of 9. $3+4+5+6+5+4=27$

10. C $28 \cdot 72 \cdot (x+y) = 30 \cdot 73 \rightarrow x+y = 2190 - 2016 = 174$
 $x+y = 174 \rightarrow x-y = 22 \rightarrow x = 98 \rightarrow y = 76 \rightarrow 7+6 = 13$

11. D List out the numbers and then divide by 4 until you see a pattern. The pattern is 1, 1, 2, 3, 1, 0. So it recycles every 6 times. Divide 100 by 6 and check the remainder and you get the 4th number, which is 3.

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12. B You could probably guess and check but Algebra would be.

$$\frac{2x^2 + 2y^2}{x^2 - y^2} \rightarrow \frac{2x^2 - 2y^2 + 4y^2}{x^2 - y^2} = 2 + \frac{4y^2}{x^2 - y^2} \rightarrow y = 0 \rightarrow x = 1 \rightarrow 2$$
13. D Time to do some angle chasing. Draw a good picture. Let angle IWG=x and angle IGW=y. Angle UIL=2y and Angle WIL=180-2x so x-3y=0 and y=22.5. We want x so 67.5
14. D $x^3 = x^2 + 4x \rightarrow x^3 - x^2 - 4x = 0 \rightarrow x(x^2 - x - 4) = 0$. This yields 3 real solutions.
15. C The points of the triangle become (3,4), (3,-4), and (-4,3) $\frac{1}{2}bh = \frac{1}{2} \cdot 8 \cdot 7 = 28$
16. C If you draw the altitude outside the rhombus, it creates a 30-60-90 triangle with the height opposite the 30-degree angle. Opposite the 90-degree angle is a side of the rhombus and it will have length of 18 so perimeter is 72.
17. E(-21) You might be able to guess and check this one but don't forget negative numbers. Here is the Algebra

$$\frac{21}{k} = \frac{\sqrt{k+21} + \sqrt{21-k}}{\sqrt{k+21} - \sqrt{21-k}} \cdot \frac{\sqrt{k+21} + \sqrt{21-k}}{\sqrt{k+21} + \sqrt{21-k}}$$

$$\frac{k+21+21-k+2\sqrt{-k^2+21^2}}{k+21-(21-k)} = \frac{42+2\sqrt{21^2-k^2}}{2k}$$

$$\frac{21+\sqrt{21^2-k^2}}{k} \rightarrow k = -21$$
18. B Best to work backwards on a problem like this. E must give up 1/6 to get to 30 so E gives up 6 to D which means D must give up a 1/4 to get to 24. Keep going and you get E=36, D=32, C=33, B=38, and A=11
19. C Big circle has diameter of 6 so 3 across and 2 above and 2 below for a total of 7
20. B SSS is one unique triangle and it meets triangle inequality so no traps
21. A $y = (x^2 - 8x + 16) - 3 \rightarrow (x - 4)^2 - 3 \rightarrow (4, -3)$
 $y = (x^2 - 10x + 25) + 2 \rightarrow (x - 5)^2 + 2 \rightarrow (5, 2)$
 $\frac{y-2}{0-5} = \frac{2-3}{5-4} \rightarrow y-2 = -25 \rightarrow y = -23$
22. A $\frac{x^{-\frac{1}{2}} - x^{\frac{3}{2}}}{x^2} \cdot \frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = \frac{1-x^2}{x^{\frac{5}{2}}} = \frac{1}{\sqrt{x^5}} - \frac{1}{\sqrt{x}}$

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23. D

$$f(x) = \frac{2x^2 - x - 15}{3x^2 - 5x - 12} = \frac{(2x+5)(x-3)}{(3x+4)(x-3)} \rightarrow x = \frac{-4}{3}$$

24. A

$$(1-0)^2 + (y-1)^2 = (5-0)^2 + (y+5)^2$$
$$1 + y^2 - 2y + 1 = 25 + y^2 + 10y + 25 \rightarrow 12y = -48$$
$$y = -4 \rightarrow \sqrt{(0-(-1))^2 + (-4-(-6))^2} = \sqrt{5}$$

25. D

Same line so consistent, which means at least one solution and dependent since each line depends on the other

26. B

$$\sqrt{223 + \frac{1}{225}} = \sqrt{\left(15 - \frac{1}{15}\right)^2} = 15 - \frac{1}{15} = 14\frac{14}{15} \rightarrow 14 + 14 + 15 = 43$$

27. B

$$f(2022)f(2021) = 2021$$
$$f(2023)f(2022) = 2022 \rightarrow \frac{f(2021)}{f(2023)} \cdot f(2024) \cdot f(2023)$$
$$= \frac{2021}{2022} \cdot 2023 \approx 2022$$

28. B

There are 25. Count them!!

29. C

Triangle inequality. There are 9 of them. (3,5,7), (3,11,13), (5,7,11), (5,11,13), (5,13,17), (7,11,13), (7,11,17), (7,13,17), (11,13,17)

30. B

${}_6C_2 = 15$. Only (0, 3) and (1, 2) work.