<u>FAMAT Calculus Standards – June 1, 2014</u>

Distribution of Topics

January Tests: At least 75% of topics from Sections 2 to 9 and no more than three questions with topics from Sections 10 to 12. Questions containing topics from Section 1 should also contain a topic from another section, and should be of at least Moderate difficulty.

February Tests: At least 75% of topics from Sections 2 to 11 and no more than three questions with topics from Sections 12 and 13. Questions containing topics from Section 1 should also contain a topic from another section, and should be of at least Moderate difficulty. Questions containing topics from Section 13 should be of Moderate-Hard or Hard difficulty.

March Tests: At least 75% of topics from Sections 2 to 12, and no more than four questions with topics from Section 13. Questions containing topics from Section 1 should also contain a topic from another section, and should be of at least Moderate difficulty. Questions containing topics from Section 13 should be of at least Moderate difficulty.

1. Standard Algebra and Precalculus Concepts
1.01 Functions, Conics, and Polynomials
1.02 Trigonometry
1.03 Matrices & Vectors
1.04 Non-Calculus Sequences & Series
1.05 Probability and Counting
2. Demonstrate the ability to apply the concept of limits to functions.
2.01 An intuitive understanding of the limiting process
2.02 Calculating limits using algebra
2.03 Estimating limits from graphs or tables of data
2.04 Limit definition of e.
3. Demonstrate the ability to identify asymptotic and unbounded behavior.
3.01 Understanding asymptotes in terms of graphical behavior
3.02 Describing asymptotic behavior in terms of limits involving infinity
3.03 Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)
4. Demonstrate an understanding of continuity.
4.01 An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
4.02 Understanding continuity in terms of limits
4.03 Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)
5. Apply the concept of the derivative.
5.01 Derivative presented graphically, numerically, and analytically
5.02 Derivative interpreted as an instantaneous rate of change
5.03 Derivative defined as the limit of the difference quotient
5.04 Relationship between differentiability and continuity
6. Demonstrate the ability to apply derivatives to find the slope of a curve and tangent and normal lines to a curve and as an instantaneous rate of change.
6.01 Slope of a curve at a point.
6.02 Tangent line to a curve at a point and local linear approximation
6.03 Normal line to a curve at a point

6.04 Instantaneous rate of change as the limit of average rate of change
6.05 Approximate rate of change from graphs and tables of values
7. Demonstrate the ability to compute derivatives of algebraic; trigonometric; exponential; and logarithmic functions.
7.01 Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
7.02 Derivative rules for sums, products, and quotients of functions
7.03 Chain rule and implicit differentiation
7.04 Find the derivative of the inverse of a function
7.05 Find higher order derivatives
8. Demonstrate the ability to identify increasing and decreasing functions; relative and absolute maximum and minimum points; concavity; and points of inflection.
8.01 Corresponding characteristics of graphs of f and f'
8.02 Relationship between the increasing and decreasing behavior of f and the sign of f'
8.03 The Mean Value Theorem and its geometric interpretation
8.04 Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.
8.05 Corresponding characteristics of the graphs of f, f', and f''
8.06 Relationship between the concavity of f and the sign of f''
8.07 Points of inflection as places where concavity changes
9. Demonstrate an understanding of applications of the derivative.
9.01 Analysis of curves, including the notions of monotonicity and concavity
9.02 Optimization, both absolute (global) and relative (local) extrema
9.03 Modeling rates of change, including related rates problems
9.04 Use of implicit differentiation to find the derivative of an inverse function
9.05 Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
10. Demonstrate the ability to interpret definite integrals and use properties of definite integrals
10.01 Basic properties of definite integrals (examples include additivity and linearity)
10.02 Numerical approximations to definite integrals. Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values
10.03 Definite integral as a limit of Riemann sums
10.04 Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:
11. Demonstrate the ability to use the techniques of integration
11.01 Use of the Fundamental Theorem to evaluate definite integrals
11.02 Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined
11.03 Antiderivatives following directly from derivatives of basic functions, including algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions
11.04 Antiderivatives by substitution of variables (including change of limits for definite integrals)
12. Demonstrate the ability to apply antiderivatives to solve problems including growth and decay, particle motion, and finding areas and volumes
12.01 Finding specific antiderivatives using initial conditions, including applications to motion along a line
12.02 Solving separable differential equations and using them in modeling
12.03 Find the average value of a function
12.04 Find the total distance traveled by a particle along a line
12.05 Find the accumulated change from a given rate of change
12.06 Find the area under a curve using integration

12.07 Find the volume of solids of revolution using disc, washer, or shell methods
12.08 Find the volume of solids with known cross sections
12.09 Use integration appropriately to model physical, biological, or economic situations or other similar applications

13. Calculus BC Topics

13.01 L'Hospital's Rule
13.02 Derivatives of parametric, polar, and vector functions
13.03 Area of regions bounded by polar curves
13.04 Numerical solution of differential equations using Euler's method
13.05 Antiderivatives by parts and by partial fractions
13.06 Improper integrals (as limits of definite integrals)
13.07 Series convergence tests, including those related to the harmonic series, alternating series with error bound, integral and p-tests, ratio and root tests, and comparison tests
13.08 Functions defined by power series
13.09 Radius and Intervals of convergence of power series
13.10 Taylor's theorem and the Maclaurin series of basic functions (exponential, sine, cosine, 1/(1-x))
13.11 Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.

Distribution of Problem Difficulty for All Calculus Tests

Problem difficulty will be gauged on the approximate percentage of students that would get a particular question correct over a Combined Regional testing. It should be noted that Statewide competition percentages correct tend to be higher than Combined Regional percentages correct, so problem difficulty may be slightly higher for Statewide competitions in order to maintain the below distribution of scores, at the discretion of the editors.

For a 30 question test:

10 Questions should be considered **Easy**, with **≥ 75%** of students getting these questions correct in the Combined Regional format.

5 Questions should be considered **Moderate-Easy**, with **50-75%** of students getting these questions correct in the Combined Regional format.

5 Questions should be considered **Moderate**, with **25-50%** of students getting these questions correct in the Combined Regional format.

5 Questions should be considered **Moderate-Hard**, with **5-25%** of students getting these questions correct in the Combined Regional format.

5 Questions should be considered **Hard**, with **≤ 5%** of students getting these questions correct in the Combined Regional format.

Please see **Example Problems** below as a **guideline** for the **difficulty level** of questions, based upon past Combined Regional results.

Please note that these are ranges of difficulty level, and within each difficulty level problems should be distributed across the range of percentage correct listed above.

A Couple of Notes On Problem Solutions:

(1) A problem that is more difficult requires a more detailed solution. Solutions should skip no steps and should be thought of as a written version of how one would teach how to do the given problem in a classroom.

(2) Many problems have multiple ways to solve them. Some of those ways may be easier, but outside the Curriculum. Therefore, it is important to note that the solution to a given problem must use only content that is in the Curriculum, and that the difficulty of a problem will be based on that solution, even if advanced knowledge renders a question trivial. Therefore, it is recommended that such questions not be included among the Hard questions of a test, since they will not challenge the most upper-level students effectively.

Example Problems

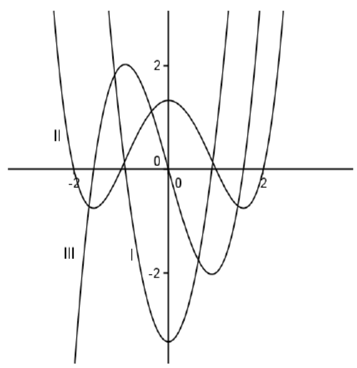
<u>10 Questions: Easy (≥ 75% Correct)</u>

January, 2012 (75.7%)

25. Three graphs, labeled I, II, and III are displayed in the figure to the right. One of the graphs is $f(x)$, one is $f'(x)$, and one is $f''(x)$. Which of the following identifies each of the three graphs?

	$f(x)$	$f'(x)$	$f''(x)$
A)	I	II	III
B)	II	I	III
C)	III	I	II
D)	II	III	I

E) NOTA



February, 2011 (76.2%)

17. At what value of *x* does the function f (x) = $x^3 - 3x^2 - 9x$ change from decreasing to increasing?

A. 3 B. -3 C. -1 D. 1 E. NOTA

January, 2010 (84.5%)

3) What is the antiderivative of cos(x)?

A. –cos(x) + C B. –sin(x) + C C. cos(x) + C D. sin(x) + C E. NOTA

February, 2011 (86.4%)

14. Find f '(x) if f (x) = $\sqrt[3]{5x^2 - x + 4}$.

A. $\dfrac{10x-1}{3\sqrt[3]{(5x^2-x+4)^2}}$ B. $\dfrac{10x-1}{2\sqrt[3]{(5x^2-x+4)^2}}$ C. $\dfrac{10x}{3\sqrt[3]{(5x^2-x+4)^2}}$ D. $\dfrac{10x-1}{3\sqrt[3]{(5x^2-x+4)}}$

E. NOTA

February, 2012 (91.9%)

1) Evaluate $\lim_{x\to 5} 2$

A) -1 B) 0 C) 1 D) 2 E) NOTA

<u>5 Questions: Moderate – Easy (50-75% Correct)</u>

February, 2011 (50.4%)

2. Find all the points on the curve $x^2y^2 + xy = 2$, where the slope of the tangent line is -1.

A. (-1, -1), (-1, 1) B. (1, -1), (1, 1) C. (1, 1), (1, 2) D. (-1, -1), (1, 1) E. NOTA

January, 2012 (57.0%)

2. Find the slope of the line normal to $f(x) = x^2 \cdot 2^x$ at x=2.

A) $\dfrac{-1}{8+8\ln 2}$ B) $\dfrac{-1}{16+16\ln 2}$ C) $\dfrac{1}{16+16\ln 2}$ D) $-\dfrac{1}{32}$ E) NOTA

March, 2013 (60.4%)

6. There exists a function such that $P(x) = (R-C)(x)$. If $P(x) = x^2 + 4x + 3$ and $C(x) = x^2 - 9x + 6$. What is $R'(10)$?

A. 143 B. 35 C. 24 D. 9 E. NOTA

February, 2013 (67.2%)

4. If $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ for all $a \in \mathbb{R}$, then $f(x)$ ***must*** be:

A. even B. odd C. both even and odd D. neither even nor odd E. NOTA

February, 2009 (73.4%)

24. Evaluate: $\int_{-2}^{2} (4x^3 - 3x^2 + 2x - 1)dx.$

A. -20 B. -12 C. 20 D. 60 E. NOTA

<u>5 Questions: Moderate (25-50% Correct)</u>

February, 2009 (26.2%)

13. Grant's Hope Powered Dream Machine outputs dreampower, d, based on the amount of hope, h, for the economy as well as the number of supporters in millions, s (i.e. if 2 million people support him, $s = 2$). The relation is $d = \frac{(h^2+2)^s - 2^s}{(h^2+2)^{s-2} - 2^{s-2}}$. If he has 5 million supporters, as the economy becomes hopeless (i.e. $h \to 0$) what does the dreampower output approach?

A. 4 B. $\frac{20}{3}$ C. 8 D. $\frac{40}{3}$ E. NOTA

February, 2011 (26.6%)

15. If $x^y = y^x$, find y'.

A. y/x B. $\frac{x(\ln x^y - y)}{y(\ln x^y - x)}$ C. $\frac{y(\ln x^y + y)}{x(\ln x^y - x)}$ D. $\frac{y(\ln y^x - y)}{x(\ln x^y - x)}$ E. NOTA

March, 2013 (38.9%)

13. $\int_{-3}^{\frac{3\pi}{2}} \frac{dx}{x^2+6x+10} =$

A. $\arctan\left(\frac{3\pi+6}{2}\right)$ B. $\arctan\left(\frac{3\pi-6}{2}\right)$ C. $\arctan\left(\frac{3\pi-3}{2}\right)$ D. $\arctan\left(\frac{3\pi+3}{2}\right)$ E. NOTA

March, 2013 (41.2%)

12. For some function $f(x)$ over an interval I, $f'(x) < 0, f''(x) > 0$. Order the following from least to greatest area:

1. The left hand Riemann sum approximation.
2. The Trapezoidal sum approximation
3. The right hand Riemann sum approximation.
4. The definite integral over I.

A. 1,2,3,4
B. 4,3,2,1
C. 1,2,4,3
D. 3,4,2,1
E. NOTA

March, 2010 (48.4%)

17. Alex is throwing darts at a rectangular dartboard in the *x*-*y* plane bounded by the lines $x = 0$, $x = 3$, $y = 0$, and $y = 9$. On this dartboard there is a target, which is the region below the curve $y = x^2$. Alex has bad aim and for any two points *P* and *Q* on the dartboard, the probability that he hits *P* is equal to the probability that he hits *Q*. If Alex always hits the dartboard, then what is the probability that he hits the target?

A. $\frac{1}{3}$ B. $\frac{7}{20}$ C. $\frac{3}{8}$ D. $\frac{1}{2}$ E. NOTA

5 Questions: Moderate – Hard (5-25% Correct)

March, 2012 (5.9%)

16. Daniel and Payal are playing a game with $f(x) = \dfrac{x-3}{x-2}$. They take turns reciting the value of $f^{(n)}(1)$; that is, Daniel gives the value of $f'(1)$, Payal gives the value of $f''(1)$, Daniel gives the value of $f'''(1)$, and so on. Diego is computing the sequence $\{s_k\}$ with kth term $s_k = \dfrac{\sum_{i=1}^{k} \frac{P_i}{D_i}}{k^2}$, where P_i is the ith value Payal gives and D_i is the ith value Daniel gives. Compute $\lim_{k\to\infty} s_k$, the value to which $\{s_k\}$ converges.

A) 1/2 B) 1 C) $\pi^2/6$ D) e E) NOTA

February, 2011 (6.4%)

23. Find G' (x) given G(x) = $\int_x^1 x^2\sqrt{u^2+1}du$.

A. - x $\sqrt{x^2+1}$ B. - x$^2\sqrt{x^2+1}$ C. - x$^2\sqrt{x^2+1}-2x\int_x^1\sqrt{u^2+1}du$

D. - x$^2\sqrt{x^2+1}+2x\int_x^1\sqrt{u^2+1}du$ E. NOTA

March, 2013 (13.8%)

21. $\frac{dy}{dx}=(xy+x+y+1)^2$. If y = 0 when x = 2, what is y when x = - 4?

A. $-\frac{3}{4}$ B. $\frac{7}{9}$ C. $\frac{12}{13}$ D. $-\frac{18}{19}$ E. NOTA

February, 2013 (14.0%)

9. Find the maximum value of $\sum_{k=1}^{n}\left(\frac{(-1)^k}{k!}\cdot\int_0^k 3x^2\,dx\right)$ where n is a positive integer.

A. 2 B. 3 C. 4 D. $\sqrt{e^3}$ E. NOTA

February, 2009 (22.9%)

14. Let $f(x)=\ln(\cos(\operatorname{arccot}(x)))$. Find $|f'(\sqrt{3})|$.
A. $\frac{\sqrt{3}}{12}$ B. $\frac{\sqrt{3}}{8}$ C. $\frac{1}{4}$ D. $\frac{\sqrt{3}}{4}$ E. NOTA

5 Questions: Hard (≤ 5% Correct)

February, 2009 (5.0%)

20. Let $a(x)=x^4+3x^2+6$. Let b be the rate of change of the y-intercept of the tangent line to $a(x)$. Find b when $x=2$.
A. -108 B. -54 C. 54 D. 108 E. NOTA

February, 2013 (4.7%)

12. Find $f(e)+f\left(\frac{1}{e}\right)$ where $f(x)=\int_1^x \frac{\ln(t)}{1+t}\,dt$ for $x>0$. *Hint:* It may be helpful to first calculate, more generally, $f'(x)+f'\left(\frac{1}{x}\right)$, and then use that result to find $f(x)+f\left(\frac{1}{x}\right)$.

A. $\frac{1}{e}$ B. $\frac{1}{2}$ C. 1 D. e E. NOTA

February, 2013 (3.9%)

26. When

$$\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$$

is written in the form $\frac{a\pi^b}{c}$, where a, b, and c are positive integers such that a and c are relatively prime, what is the value of $a+b+c$? *Hint:* Substitute $u=\pi-x$.

A. 4 B. 5 C. 6 D. 7 E. NOTA

March, 2013 (3.2%)

29. $$\int_2^{2\sqrt{3}} \left(\sqrt{\frac{4}{x^2}+1}\right)dx$$

a. $4+\sqrt{2}-\ln(1+\sqrt{2})$ b. $4-2\sqrt{2}+\ln\left(\frac{1+\sqrt{2}}{3}\right)$

c. $4+\sqrt{2}-\ln\left(1+\frac{\sqrt{2}}{2}\right)$ d. $4-2\sqrt{2}+\ln\left(1+\frac{2\sqrt{2}}{3}\right)$

e. NOTA

February, 2009 (1.3%)

30. Evaluate: $\int_0^{\frac{\pi}{4}} \ln(\tan(x)+1)dx$. (Hint: Work backwards with angle addition formulas)

A. $\frac{\pi\ln 2}{8}$ B. $\frac{\pi\ln 2}{4}$ C. $\frac{\pi\ln 2}{2}$ D. $\pi\ln(2)$ E. NOTA